HANDBOOK
OF
ELECTRICAL
TESTING

H. R. KEMPE

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BY

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NOTE

In the present Edition I have not only taken advantage, as far as possible, of the many friendly suggestions which have been made to me for the improvement of the original work, but have added a considerable amount of new matter, besides thoroughly revising the old. I have to thank Messrs. Elliott Brothers, Mr. B. Pell (of Messrs. Johnson and Phillips's), Messrs. Nalder Brothers, and Mr. P. Jolin, for many of the illustrations of apparatus. I am also particularly indebted to Dr. A. Muirhead, Mr. W. J. Murphy, Mr. H. W. Sullivan, and Mr. J. Rymer Jones, for much useful information, and to Mr. Herbert Taylor for the Table (XII.) of data of recent Submarine Cables.

H. R. K.

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HANDBOOK

OF

ELECTRICAL TESTING.

CHAPTER I.

SIMPLE TESTING.

1. In order to be able to make measurements of any kind, it is necessary to have certain standard units with which to make comparisons. For example, in the case of length, or weight, we have as standards the foot and the pound. Some of the units are dependent upon two of the other units; the unit of "work," for example, is the foot-pound, or the work done in raising a pound one foot high. Now in electrical measurements we require units of a like character. Those with which we have to deal chiefly are *electromotive force*, the unit of which is called the *volt*; *resistance*, the unit of which is the *ohm*; and *capacity*, the unit of which is the *farad*; also we have the unit of *current*, which is dependent upon the volt and the ohm, and is called the *ampère*.*

Certain of these units are too large for practical use, and sub-multiples of them are employed; thus for practical purposes the *microfarad*, that is the one-millionth part of a farad, is the sub-multiple used as a standard. For electric light and electric power purposes, the *ampère* is a practical standard of current; but for telegraphic work the *milliampère*, that is the one-thousandth part of an ampère, is more convenient, and is therefore generally employed.

2. If the two poles of a battery be joined by a conductor a current will flow, and the strength of this current will vary directly as the electromotive force of the battery, and inversely as the total resistance in the circuit. This relation is known as

* The value of these units is given in the Appendix.

"Ohm's law." If the electromotive force is expressed in volts and the resistance in ohms, then the resulting current will be in ampères.

3. Suppose now a battery of a resistance r and electromotive force E, a galvanometer of a resistance G, and a wire of a resistance ρ, be joined up in circuit, as shown by Fig. 1. By the foregoing law, the strength of current C, which will flow out of the battery and through the galvanometer, will be

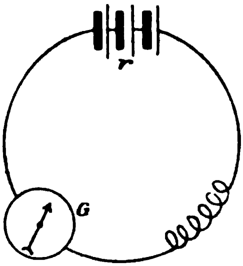


Fig. 1.

$$C = \frac{E}{\rho + r + G}.$$

The current, in flowing through the galvanometer, produces a deflection of its needle, which deflection will remain constant provided the electromotive force of the battery and also the resistances remain constant. If now ρ be a wire whose resistance we require to find, and which we can replace by another wire the value of whose resistance can be varied at pleasure, then by adjusting this latter so that the deflection of the galvanometer needle becomes the same as it was before the change of resistances was made, this resistance gives the value of our unknown resistance ρ.

This method of testing, known as the *substitution* method, although exceedingly simple, is a very good and accurate one if a little ordinary care be taken in making it. Its correctness is only limited by the sensibility of the galvanometer to small changes of strength in the current affecting it, and by the accuracy with which the variable resistance can be adjusted.

It should be mentioned, however, that for reasons which will become obvious when the subject of testing is gone further into, the resistance of the battery and galvanometer used in making a test of the kind should be small compared with the resistance being measured.

4. Next, suppose the galvanometer to have its scale so graduated that the number of divisions on it will, by the deflection of the needle, accurately represent the comparative strength (C) of currents which may pass through it. Let the battery, galvanometer and resistance be joined up as at first, then, as before

$$C = \frac{E}{\rho + r + G}; \quad \text{or,} \quad E = C(\rho + r + G).$$

Now remove ρ, and insert any other known resistance R, in its place. Calling the new strength of current, C_1, then

$$C_1 = \frac{E}{R + r + G}; \quad \text{or,} \quad E = C_1 (R + r + G).$$

But we have seen that $E = C(\rho + r + G)$, therefore

$$C(\rho + r + G) = C_1 (R + r + G),$$

or

$$\rho + r + G = \frac{C_1}{C}(R + r + G),$$

that is,

$$\rho = \frac{C_1}{C}(R + r + G) - (r + G). \qquad [1]$$

Now, as we have supposed the deflections of the galvanometer needle to be directly proportional to the strengths of current which produce them, we may, instead of C and C_1, write in our formulæ the deflections of the galvanometer needle which those strengths produce. Calling, then, a the deflection obtained with the strength C, and a_1 that with the strength C_1, our formula [1] becomes

$$\rho = \frac{a_1}{a}(R + r + G) - (r + G). \qquad [2]$$

In order to find ρ, it is necessary to know G, which is usually marked on the galvanometer by the manufacturer. r also must be known, but as it is difficult to determine its value accurately, it is best to use a battery whose resistance is very small in comparison with the other resistances in the circuit, and which may consequently be neglected; in this case we may write our formula

$$\rho = \frac{a_1}{a}(R + G) - G. \qquad [3]$$

Having then obtained a with ρ and a_1 with R, we can find the value of ρ.

For example.

With a galvanometer whose resistance was 100 ohms, and a battery whose resistance could be neglected, we obtained with an unknown resistance ρ a deflection of 30 divisions (a), and with a resistance of 320 ohms (R) in place of ρ, a deflection of 20 divisions (a_1). What was the unknown resistance ρ?

$$\rho = \frac{20}{30}(320 + 100) - 100 = 180 \text{ ohms}.$$

5. Next, suppose it is required to find the resistance of a galvanometer.

From equation [3], by multiplying up, we find that

$$\rho\, a = R\, a_1 + G\, a_1 - G\, a,$$

by arranging the quantities and changing signs we get

$$G\, a - G\, a_1 = R\, a_1 - \rho\, a,$$

or

$$G\,(a - a_1) = R\, a_1 - \rho\, a;$$

therefore

$$G = \frac{R\, a_1 - \rho\, a}{a - a_1}. \qquad [4]$$

If, then, with a known resistance ρ, we obtain a deflection of a divisions, and with another known resistance R we obtain a deflection of a_1 divisions, we can determine G.

For example.

With a galvanometer (G) and a battery whose resistance could be neglected, we obtained with a resistance of 200 ohms (ρ) a deflection of 30 divisions (a), and with a resistance of 350 ohms (R) a deflection of 20 divisions (a_1). What was the resistance of the galvanometer?

$$G = \frac{350 \times 20 - 200 \times 30}{30 - 20} = 100 \text{ ohms.}$$

6. Lastly, when the resistance of our battery is considerable, and it is required to find its value, from equation [2] (page 3) by multiplying up, we find

$$\rho\, a = R\, a_1 + r\, a_1 + G\, a_1 - r\, a - G\, a,$$

by arranging the quantities and changing signs we get

$$r\, a - r\, a_1 = R\, a_1 - \rho\, a + G\, a_1 - G\, a,$$

or

$$r\,(a - a_1) = R\, a_1 - \rho\, a - G\,(a - a_1),$$

that is

$$r = \frac{R\, a_1 - \rho\, a}{a - a_1} - G. \qquad [5]$$

For example.

With a galvanometer whose resistance was 100 ohms (G), and a battery (r), we obtained with a resistance in circuit of 150 ohms (ρ) a deflection of 40 divisions (a), and with a resistance in circuit

of 300 ohms (R) a deflection of 30 divisions (a_1). What was the resistance of the battery?

$$r = \frac{300 \times 30 - 150 \times 40}{40 - 30} - 100 = 200 \text{ ohms.}$$

7. The foregoing formulæ may be considerably simplified if we so adjust our resistances that one deflection becomes half the other, or, in other words, if we make $a_1 = \frac{a}{2}$. Formula [3] (page 3) for determining any resistance then becomes

$$\rho = \frac{\frac{a}{2}}{a}(R + G) - G = \frac{R + G}{2} - G;$$

therefore

$$2\,\rho = R + G - 2\,G = R - G,$$

or

$$\rho = \frac{R - G}{2}.$$

8. Similarly we should find that formula [4] (page 4) for determining the resistance of a galvanometer becomes

$$G = R - 2\,\rho;$$

and formula [5] (page 4) for determining the resistance of a battery,

$$r = R - (2\,\rho + G);$$

ρ being in all cases the resistance which gives the large deflection, and R being the larger resistance which halves it.

9. Still further simplifications of the two latter formulæ may be obtained if, when observing the first deflection a, we have no resistance in circuit (except, of course, that of the galvanometer, or of the battery, the resistance of either of which we are measuring), that is to say, if we have $\rho = 0$; we then get

$$G = R \qquad [A]$$

and

$$r = R - G, \qquad [B]$$

that is to say, in case [A] the resistance inserted in order to halve the deflection is the resistance of the galvanometer, and in case [B] the resistance inserted, minus the resistance of the galvanometer, is the resistance of the battery.

In the latter case, by using a galvanometer of negligible resistance, we get

$$r = R. \qquad [C]$$

The formulæ [A] and [C], although they have been arrived at by mathematical analysis, are really quite obvious, since it must be clear that the halving of the deflection, that is of the current, must be effected by doubling the resistance of the circuit, that is by adding a resistance equal to the only resistance which is in the circuit in the first instance, that is the resistance of the galvanometer or of the battery, as the case may be.

10. When the resistance we have to measure is very high compared with the resistance of the galvanometer and battery used for measuring, then in our equation (page 3)

$$\rho = \frac{a_1}{a}(R + r + G) - (r + G),$$

we may practically, especially when great accuracy of measurement is not required, put G as well as r equal to 0, in which case

$$\rho = \frac{a_1}{a} R.$$

To measure a resistance according to this formula, we should first join up, as shown by Fig. 1 (page 1), our battery, galvanometer and standard resistance (as it is called), which in our formula is R; and having noted the deflection a_1, should multiply the latter by R; this gives us what is called the *constant*. ρ (the resistance to be determined) is then inserted in the place of R; a new deflection a is obtained, by which we divide the constant, and thus get the value of ρ.

This method of measuring resistances is one which is sometimes employed in making tests for *insulation* resistance of telegraph lines, the standard resistance R being 1000 ohms.

When the insulation resistances of several lines are to be measured, the constant would first be taken and worked out, and the several lines to be measured being inserted one after the other in the place of the resistance R, the deflections are noted; then the constant being divided by the several deflections, the resistances are thus obtained.

For example.

With a battery, a galvanometer and a resistance of 1000 ohms (R) in circuit, we obtained a deflection of 20 divisions (a_1), then

$$\text{Constant} = 1000 \times 20 = 20{,}000.$$

Taking away our resistance and inserting

Wire No.	1, we obtained	a deflection of	5	divisions
,,	2, ,,	,,	6	,,
,,	3, ,,	,,	12	,,
,,	4, ,,	,,	3	,,

The resistance of our wires would then be

$$\begin{aligned} \text{No. } 1,\ 20{,}000 \div 5 &= 4000 \text{ ohms.} \\ ,,\ 2,\ 20{,}000 \div 6 &= 3333\ ,, \\ ,,\ 3,\ 20{,}000 \div 12 &= 1666\ ,, \\ ,,\ 4,\ 20{,}000 \div 3 &= 6666\ ,, \end{aligned}$$

These results are the *total* insulation resistances of the wires, which may be of various lengths. To get comparative results, it is necessary to obtain the insulation resistance of some unit length of each wire, such as a mile.

Now, it will be readily seen that the greater the length of the wire the *greater* will be the leakage, and consequently the less will be the insulation resistance, or, in other words, this resistance will vary *inversely* as the length of the wire. To obtain, then, the insulation resistance, or "insulation" as it is simply called, all we have to do is to multiply the *total* insulation by the length of wire. Thus, for example, if No. 1 wire were 100 miles long, its insulation per mile would be $4000 \times 100 = 400{,}000$ ohms.

It is usual to fix a standard insulation per mile, and if the result is below the same, the line is considered faulty. 200,000 ohms per mile is the standard adopted by the Postal Telegraph Department.

11. The rule of multiplying the total insulation by the mileage of the wire to get the insulation per mile is not strictly correct, more especially for long lines, as it assumes that the leakage is the same at every point along the line. This, however, is clearly not the case, as a little of the current leaking out at one point leaves a smaller quantity to leak out at the next. In fact, we really measure the last portion of the line with a weaker battery than we do the first. The true law is, however, somewhat complex, and will be considered hereafter.*

12. We have hitherto considered the galvanometer deflections to be directly proportional to the currents producing them, but in no galvanometer is this the case if the deflections are measured in *degrees*; in such a case the deflections are proportional to some

* See Appendix.

function of those degrees, such as the *tangent*. Thus, if we were reading off the scale of degrees on a *tangent* galvanometer, that is to say, a galvanometer in which the strengths of currents are directly proportional to the *tangents* of the angle of deflection which those currents produce, we should have to find the tangents of those degrees of deflection before multiplying and dividing.

For example.

If with a tangent galvanometer we obtained with our standard resistance of 1000 ohms a deflection of 20°, and with the unknown resistance (ρ) a deflection of 15°, we should have

$$\rho = \frac{\tan 20^\circ \times 1000}{\tan 15^\circ} = \frac{\cdot 364 \times 1000}{\cdot 268} = 1358 \text{ ohms.}$$

When measuring the insulation resistance of a line of telegraph, having taken the constant, we should join up our instruments and line, as shown by Fig. 2. In making a measurement

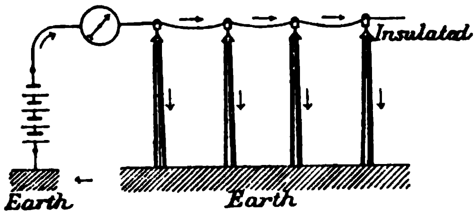


FIG. 2.

of this kind, it is usual to have the positive pole of the battery to earth, so that a negative (zinc) current flows out to the line, as a zinc current will show best any defective insulation in the wire, a positive current having the effect, to a certain extent, of *sealing* a fault up, more especially if the defect is in any underground work which may be in the circuit.

The foregoing method of measurement is, as a rule, sufficiently accurate for all practical purposes. Greater accuracy may, however, be obtained with but little extra trouble by allowing for the resistance of our battery and galvanometer in the following manner :—

Instead of multiplying the *constant* deflection by the 1000 ohms standard resistance, multiply it by 1000 plus the resistance of the galvanometer and battery, and having divided the result by the deflection obtained with the line wire in circuit, subtract from the result the resistance of the galvanometer and battery.

For example.

With a standard resistance of 1000 ohms, a tangent galvanometer of a resistance of 50 ohms, and a battery of a resistance of 100 ohms, we obtained a deflection of 30°, and with the line wire in circuit a deflection of 10°. What was the exact insulation resistance of the line?

$$\left.\begin{array}{l}\text{Insulation}\\ \text{resistance}\end{array}\right\} = \frac{\tan 30^\circ\,(1000 + 50 + 100)}{\tan 10^\circ} - (50 + 100)$$

$$= \frac{\cdot 577 \times 1150}{\cdot 176} - 150 = 3760 \text{ ohms.}$$

In order to save calculation it is very convenient to have a table constructed on the following plan:—

EARTH READINGS.

Constant Readings through 1000 ohms.	1°	2°	3°	4°
20°	20852	10423	6945·0	5205·0
21°	21992	10993	7324·6	5489·5
22°	23146	11570	7709·3	5777·9
23°	24318	12155	8099·5	6070·2
24°	25507	12750	8495·5	6367·1

In this table the first vertical column represents the deflections in degrees obtained with a tangent galvanometer through a standard resistance of 1000 ohms, and the top row of degrees are the deflections obtained with the line wire in circuit. The numbers at the points of intersection of a vertical with a horizontal column give the resistances corresponding to those deflections, these resistances being calculated from the formula

$$\left.\begin{array}{l}\text{Insulation}\\ \text{resistance}\end{array}\right\} = \frac{\tan \textit{constant reading} \times 1000}{\tan \textit{earth reading}}.$$

Thus the *constant* deflection, or reading, with the 1000 ohms standard resistance being 22°, and the deflection with the line wire (the earth reading) being 2°, the resistance required is seen at a glance to be 11,570 ohms.

Before proceeding to the more intricate systems of measurement, we will consider some of the instruments which would be used in making measurements such as we have described.

CHAPTER II.

RESISTANCE COILS.

13. The essential points of a good set of resistance coils are, that they should not vary their resistance appreciably through change of temperature, and that they should be accurately adjusted to the standard units, which adjustment ought to be such that not only should each individual coil test according to its marked value, but the total value of all the coils together should be equal to the numerical sum of their marked values. It will be frequently found in imperfectly adjusted coils that although each individual coil may test, as far as can be seen, correctly, yet when tested all together their total value will be perceptibly more or less than the sum of their individual values; because, although an error of a fraction of a unit may not be perceptible in testing each coil individually, yet the accumulated error may be comparatively large.

The wire of the coils is either of platinum-silver alloy or of German silver; the former material has the advantage that its resistance changes but very slightly by variation of temperature; this variation not amounting to more than ·031 per cent. per degree Centigrade (·017 per degree Fahrenheit). Platinum-silver is, however, rather expensive, and consequently, where the highest possible accuracy is not of great importance, German silver, whose percentage of resistance variation per degree Centigrade is ·044 (·024 per degree Fahrenheit), is used. The alloy discovered by Mr. F. W. Martino, called *platinoid*, which is a combination of tungsten, copper, nickel and zinc, besides being very inexpensive, has a lower coefficient of resistance variation by change of temperature than even platinum-silver, this percentage being as low as ·021 per degree Centigrade (·012 per degree Fahrenheit), it is therefore largely used for resistance coils. The alloy called *manganin*, which has within the last few years come into considerable use, has also a very small *temperature coefficient;* some specimens of the alloy in fact have, it is stated, been found to be practically

unaffected as regards resistance by change (within certain limits) of temperature.

The wire is usually insulated by two coverings of silk, and is wound double on ebonite bobbins, the object of the double winding being to eliminate the extra current which would be induced in the coils if the wire were wound on single. By double winding, the current flows in two opposite directions on the bobbin, the portion in one direction eliminating the inductive effect of the portion in the other direction. When wound, the bobbins are saturated in hot paraffin wax, which thoroughly preserves their insulation, and prevents the silk covering from becoming damp, which would have the effect of partially short-circuiting the coils and thereby reducing their resistance.

The small resistances are made of thick wire, the higher ones of thin wire, to economise space.

When bulk and weight are of no consequence, it is better to have all the coils made of thick wire, more especially if high battery power is used in testing, as there is less liability of the coils becoming heated by the passage of a current through them.

The individual resistances of a set of coils are generally of such values that, by properly combining them, any resistance from 1 to 10,000 can be obtained. One arrangement in general use has coils of the following values: 1, 2, 2, 5, 10, 10, 20, 50, 100, 100, 200, 500, 1000, 1000, 2000, 5000 ohms. These numbers enable any resistance from 1 to 10,000 to be obtained, using a minimum number of coils without fractional values. As, however, it is a matter of some little difficulty to see at once what coils

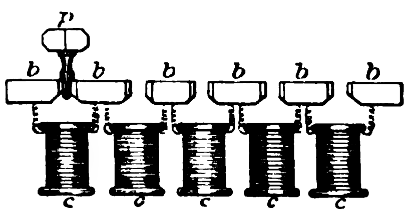


FIG. 3.

it is necessary to put into circuit in order to obtain a particular resistance, and as it is often necessary to be quick in changing the resistances, the following numbers are frequently used: 1, 2, 3, 4, 10, 20, 30, 40, 100, 200, 300, 400, 1000, 2000, 3000, 4000, which enables any particular resistance, that is required to be inserted, to be seen almost at a glance.

The way in which the different coils are put in circuit is shown by Fig. 3. The ends of the several resistances, *c*, *c*, *c*,···· are connected between the brass blocks, *b*, *b*, *b*,···· Any of the coils can then be cut out of the circuit between the first and last blocks, by inserting plugs, *p*, as shown, which short-circuit the coils between them; thus, if all the plugs were inserted, there would be no resistance in circuit, and if all the plugs were out, all the coils would be in circuit.

14. There are various ways of arranging the coils in sets; one of the most common is that shown in outline by Fig. 4, and in general view by Fig. 5. This form is much used in submarine

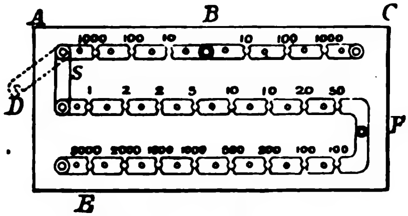


Fig. 4.

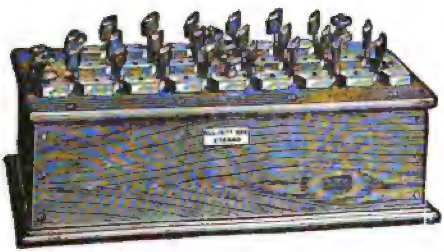


Fig. 5.

cable testing. The brass blocks, here shown in plan, are screwed down to a plate of ebonite, which forms the top of the box in which the coils are enclosed. The ebonite bobbins are fixed to the lower surface of the ebonite top, the ends of the wires being fixed to the screws which secure the brass blocks. The holes shown in the middle of the brass blocks are convenient for holding the plugs that are not in use.

It will be seen that six terminals, A, B, C, D, E, F, are provided; when we only require to put a resistance in circuit, the two terminals D and E would be used. The use of the other terminals and of the movable brass strap S, will be explained hereafter.

15. In using a set of resistance coils, one or two precautions are necessary.

First of all, it should be seen that the brass shanks of the plugs are clean and bright, as the insertion of a dirty plug will not entirely short-circuit the coil it is intended to cut out. It is a good plan, occasionally, to give the plugs a scrape with a piece of glass or emery paper, taking care to rub off any grains of grit which may remain sticking to them after this has been done.

When a plug is inserted it should not be simply pushed into the hole, but a twisting motion should be given it in doing so, that good contact may be ensured; too much force should not be used, as the ebonite tops may thereby be twisted off in extracting the plugs. Care should also be taken that the neighbouring plugs are not loosened by the fingers catching them during the operation of shifting a plug.

Before commencing work it is as well to give all the plugs a twist in the holes, so as to ensure that none of them are left loose. On no account must the plugs be greased to prevent their sticking, and their brass shanks should be touched as little as possible with the fingers.

16. For taking the insulation resistance of a line in the manner described in the last chapter, such an elaborate set of coils is not of course wanted. A single coil of a resistance of 1000 ohms in a box with two terminals, to which the ends of the coil are attached, is all that is required.

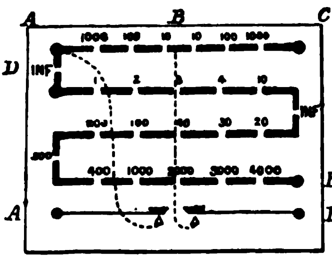


FIG. 6.

17. One of the most useful sets of coils for general purposes, known as the "Post Office pattern," is represented in outline by Fig. 6, and in general view by Fig. 7. The general arrangement of resistances, it will be seen, is the same as that shown by Fig. 4. Two keys, however, are provided (drawn in Fig. 6 in elevation). The contact point of the right-hand key is connected, as shown by the dotted line, with the middle brass block of the upper set of resistances, the terminal B′ at the end of the key corresponding in fact, when the key is pressed down, with the terminal B shown in Fig. 4. In like manner the terminal A′ corresponds with the terminal A. In the place of the movable strap of brass between A and D (Fig. 4), a plug marked INF. (infinity) is provided, which

answers the same purpose; an infinity plug is also placed at the first bend of the coils on the right hand of the figure.*

FIG. 7.

When we require simply to insert a resistance in a circuit, we should use the terminals A′ and E, the left-hand key being

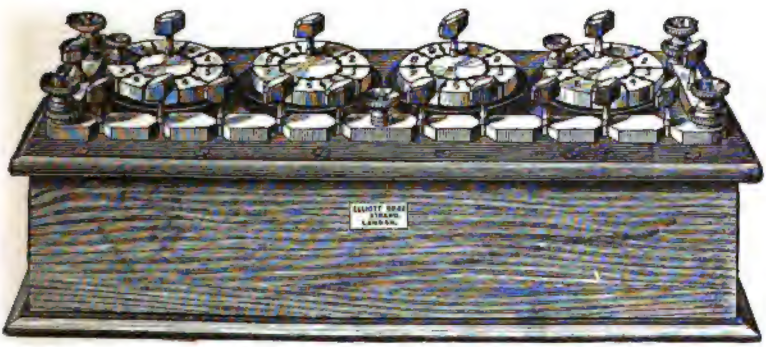


FIG. 8.

pressed down when the deflection of the galvanometer needle is to be noted. The current can thus be conveniently cut off or

* A reversing switch, the use of which will be explained hereafter, is attached to the right-hand side of the box.

put on when required, by releasing or depressing the key. Care should be taken that the two *infinity* plugs are firmly in their places, to ensure their making good contact. For the same purpose the key contacts should be occasionally cleaned by drawing between them a piece of clean paper, the key being held firmly down.

Another set of coils, known as the "Dial" pattern, is represented in general view by Fig. 8; these will be again referred to

FIG. 9.

hereafter (Chapter VIII). In this pattern (as will be seen from the figure) ten brass blocks are arranged radially around a central circular block. One disadvantage of the arrangement is that it is difficult to clean the surface of the ebonite on which the brass blocks are mounted; in a somewhat similar pattern, designed by

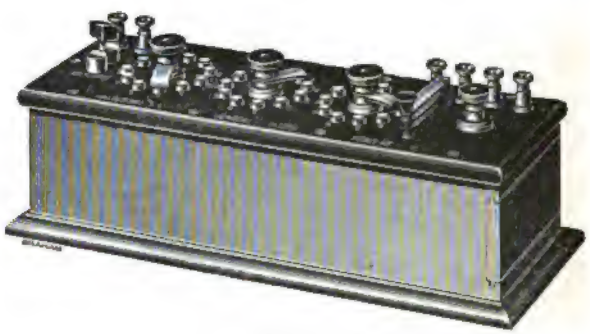


FIG. 10.

Dr. Muirhead and shown by Fig. 9, this disadvantage is got over by substituting a rectangular bar for the central circular block, and arranging five of the brass blocks in a row on one side and five on the other side of the same. By this arrangement a piece of rag can easily be passed between the blocks and the central

bar, and the surface of the ebonite on which the blocks and bar are mounted be readily cleaned.

18. Fig. 10 shows a set of coils similar in general arrangement to Fig. 8, but in which the various resistances are brought into circuit by turning finger knobs instead of shifting plugs. For quickness of manipulation this arrangement is very satisfactory, and it is doubtful whether the objection raised against the same, viz. that the lever arms turned by the finger knobs are liable not to make good contact with the studs against which they press, is really a valid one.

Slide Resistance Coils.

19. Fig. 11 shows the principle of this method of arranging Resistance Coils.

The coils, which are generally all of equal value, are connected between brass blocks, as in Fig. 3, but instead of plugs being inserted between the blocks to cut the various coils out of circuit, a sliding piece, B, is provided, which can be moved along a rod with which it is in connection. The slider has a spring fixed to it which presses against the brass blocks; it is evident, then, that any required resistance can be inserted between A and B, that is, between A and a terminal fixed to the end of the rod, by simply sliding the piece B along the rod.

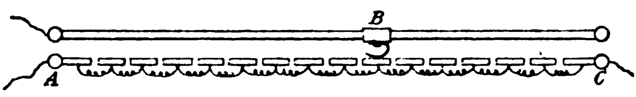


Fig. 11.

The object of arranging the coils in this manner is more particularly to enable the ratio of A B to B C to be varied, whilst the sum of the two, that is to say the whole length, A C, remains constant; this is sometimes required to be done.

These coils are sometimes set in a circle instead of a straight line, the contact-piece B being a spring forming a radius of the circle. This is a very compact and useful arrangement.

20. For some tests a long straight wire of German silver or other metallic compound is employed in the place of the resistance coils. It is important that this wire should be made of a perfectly uniform alloy, and should be of the same diameter throughout, so that its resistance may be directly proportional to its

length; thus, if the slider were at the middle point of the wire, the resistance on each side should be exactly the same.

If, as is sometimes the case, it is required to use a long wire of this kind, it would be inconvenient to have it straight; in such a case, therefore, the wire is wound spirally on a cylinder of ebonite or other insulating material, the two ends being connected to the metal axes, these latter being in connection with terminals. The sliding contact-piece is moved along parallel with the axes of the cylinder by a screw which gears with the cylinder, and which is therefore revolved by the handle which turns the latter; the contact of the slider with the wire is made when required by pressing the former with the finger. The arrangement, in fact, is a modified form of Jacobi's Rheostat.

The Thomson-Jolin Rheostat.

21. This apparatus, which is shown by Fig. 12, was devised by Lord Kelvin (Sir William Thomson) and Mr. P. Jolin, of Bristol, and is a valuable modification of the original rheostat of Wheatstone, the apparatus being entirely free from the defects which characterised the latter instrument.

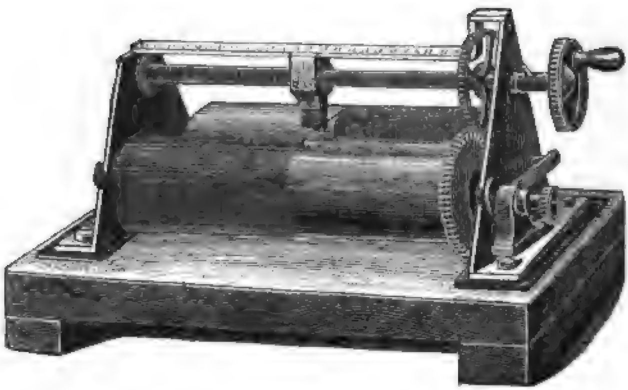


FIG. 12.

In this rheostat the wire is guided between the cylinders, so as to be laid on them spirally, by means of a travelling nut on a long screw. The screw is turned by the handle, and carries a toothed wheel which gears into two other toothed wheels; one of the latter turns one of the cylinders, and the other a loose shaft carrying the other cylinder; a spring fixed to this shaft acts on

the last-named cylinder which surrounds it on the principle of the main spring of a watch. By this arrangement the wire is kept tightly stretched, and the barrels can be turned backwards or forwards without the wire becoming slack. The guiding nut is also arranged to stop the motion of the screw shaft at each end of the range, and so prevent the possibility of over-winding; it also carries an index, which moves along a graduated scale and indicates the number of turns of wire on the insulating cylinder.

The conducting cylinder and the wire are both of "platinoid," which has properties which make it specially suitable for the purpose. It has very high electric resistance, very small temperature variation of resistance (as has previously been pointed out on p. 11), and it remains with its surface almost, if not altogether, untarnished in the air. On account of the last-named property, the contact between the wire and the conducting-cylinder is as perfect as can be desired; and continuity of action, which was a great difficulty in the old Wheatstone instrument, is (according to Lord Kelvin) absolutely complete.

22. It is evident that a much finer adjustment of resistance can be obtained by the slide wire than by the slide resistance coils, but inasmuch as the length of the wire and the smallness of its diameter must be limited, it does not admit of very large variations of resistance being obtained. By combining, however, a slide-wire resistance with plug resistance coils, this difficulty can be got over, though in tests which we shall describe, it is preferable to use the slide coils.

23. Slide resistance coils, though very convenient, are not absolutely necessary for varying the ratio of the resistances in the manner described; for it is evident that A B and B C (Fig. 11) could be two sets of resistance coils in which, to adopt the slide resistance principle, the resistances would have to be increased in one set and diminished in the other, or *vice versâ*, care being taken that the same resistance is added in one set as is taken out in the other.

CHAPTER III.

GALVANOMETERS.

24. For the class of tests in which it is required, by adjusting certain resistances, to bring the needle to zero, and where great sensitiveness is not required, the simple form of galvanometer shown by Fig. 13 is very useful; it is a pattern used in the Postal Telegraph Department in connection with testing by the Wheatstone Bridge (Chapter VIII.). In this instrument there is but one needle (within a coil), a light index pointer being fixed at right angles to the same, and the whole having a compass suspension.

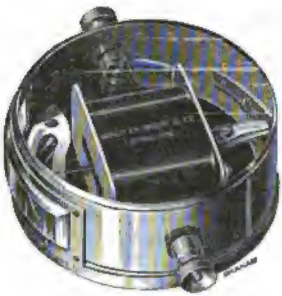


Fig. 13.

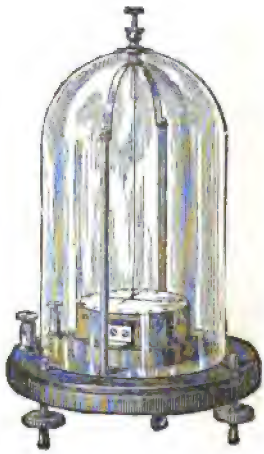


Fig. 14.

25. In cases, however, where tests of the kind described on p. 2 have to be made, a different class of instrument is necessary; the form shown by Fig. 14 is a convenient and useful one for such purposes, the scale being graduated to degrees. The instrument is provided with an astatic pair of needles suspended by a cocoon fibre, the end of the latter being attached

to a piece of metal connected to a screw, by the twisting of which the needles can be lowered down on to the coil, so that there would be no danger of the fibre being fractured when the instrument has to be moved about.

When the galvanometer is to be used it should be placed on a firm table, and the screw connected to the fibre turned until the needles swing clear of the coil. The instrument should then be placed in such a position that the top needle stands as nearly as possible over the zero points. It should next be carefully levelled by means of the levelling screws attached to its base, until the metal axis which connects the two needles together is exactly in the centre of the hole in the scale-card through which it passes.

The adjustment of the needles to zero is much facilitated in the instrument by making the coil movable about the centre of the scale-card by means of a simple handle attached direct to the coil. The final touch can thus be given without shaking the needles, which would render exact adjustment difficult.

In some galvanometers there is a scale graduated to degrees attached to the coil, so that the angle through which it is turned can be seen if required. This scale is employed when using the instrument as a *Sine* galvanometer.

The Sine Galvanometer.

26. We before stated that the strengths of currents producing certain deflections are not directly proportional to those deflections, but to some function of them, such as the *tangent*. In measuring strengths of currents by means of a *sine* galvanometer we proceed as follows:—

The needle is first adjusted to zero. The current whose strength is to be measured is then allowed to flow, and a deflection of the needle produced. The *coil* is now turned round; this causes the needle to diverge still more with respect to the stand of the instrument, but the angle which it makes with the coil becomes less the farther the latter is turned, and finally a point is reached at which the needle is again parallel to the coil—that is, its ends are again over the zero points on the scale-card. The reason of this is, that the deflective action of the coil on the needle is always the same, provided the current strength does not vary, but the farther the needle moves from the magnetic meridian, the greater becomes its tendency to return to that meridian, and finally when the needle becomes parallel to the coil, the defective force of the latter exactly balances the reactive force of the earth's magnetism.

The strength of the current which produces the deflection of the needle will then be directly proportional to the *sine* of the angle through which the coil has been turned.

The sine galvanometer, though but rarely used, is a very accurate instrument, in so far that its results are entirely independent of the shape of the coil, size of the needle, &c. The only precaution necessary is to see that when the needle is at zero at starting it is brought back exactly at zero. Indeed, it is not absolutely necessary that the starting point be zero—the law of the *sines* holds good if the needle be at, say, 5° when commencing, but in this case, by the turning of the coil, the needle must be brought back to 5°, and not to zero.

The Tangent Galvanometer.

27. The tangent galvanometer, which is perhaps the most useful and convenient instrument for general purposes, consists essentially of coils of wire wound in the deep groove in the circumference of a circular ring, a magnetic needle being placed at the centre of the latter over a graduated circle. The length of this needle must be small compared with the diameter of the coils so as to ensure, as far as possible, the magnetic influence of the current on the needle being the same at whatever angle the needle may be with respect to the coil. Theoretically, to effect this result, the magnet should be a mere point, but this is of course impossible, and practically it is sufficient for the coil to be eight or ten times as large in diameter as the length of the needle. Upon the influence of the coil on the needle being the same, whatever angle the needle takes up with respect to it, depends the truth of the proposition, that the *strength of currents circulating in the coil are directly proportional to the tangents of the angles of deflection of the needles.* For a 6- or 7-inch ring, a needle about three-quarters of an inch in length is a convenient size, and gives sufficiently accurate results for all practical purposes. The needle must be so placed that its central point is at the axis of the coils, and also in the same plane with them.

28. The principle of the instrument is as follows:—

Let $n\,s$ (Fig. 15) be the needle in its normal position, i.e. the position where it is parallel to the magnetic meridian, and also parallel to the ring or coils. Let $n_1\,s_1$ be the position the needle takes up when deflected by the action of the coils. Draw $c\,d$ at right angles to $n_1\,s_1$ making $c\,n_1$ equal to $n_1\,d$; draw $a\,c$ and $d\,a_1$ each at right angles to $c\,d$; also draw $n_1\,a$ parallel to $n\,s$ and $n_1\,a_1$

at right angles to $n s$. Now the position which the needle takes up is due to the fact that the deflective action of the coils, and the directive force of the earth's magnetism when resolved at right angles to the needle, are equal and opposite in effect. The first of these forces f_2, acts at right angles to $n s$, and the second, f_1, acts parallel to $n s$; then if $a n_1$ and $a_1 n_1$ represent the forces f_1 and f_2 respectively, $c n_1$ and $d n_1$ will represent the resolved forces at right angles to $n_1 s_1$, which forces are equal since equilibrium is produced; let their value be f. Now since $a n_1$ is parallel to n,o, and $a c$ parallel to $n_1 o$, the angle $c a n_1$ is equal to the angle a°;* also since $n_1 a_1$ is perpendicular to $n o$, and $n_1 d$ is perpendicular to $n_1 o$, the angle $a_1 n_1 d$ is equal to the angle a°. We consequently have

$$f = f_2 \cos a^\circ, \text{ and } f = f_1 \sin a^\circ;$$

therefore

$$f_2 \cos a^\circ = f_1 \sin a^\circ,$$

or

$$f_2 = f_1 \frac{\sin a^\circ}{\cos a^\circ} = f_1 \tan a^\circ;$$

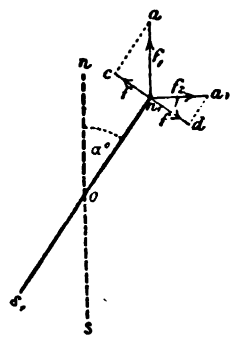


FIG. 15.

but f_1 (the directive force of the earth's magnetism) is constant, therefore f_2 (the deflective force of the coils) is proportional to tan a°, that is to say, the current strength, C, in the ring or coils is proportional to tan a°,† or

$$C = \tan a^\circ \times \text{a constant}.$$

29. Fig. 16 shows a pattern of tangent galvanometer which is used by the Postal Telegraph Department.‡ The magnetic needle

* Euclid, Book i. prop. 34.

† Professor J. P. Joule and Professor Jack point out in vol. vi. pp. 135, 147, and 151, of the 'Proceedings of the Manchester Literary and Philosophical Society,' that if the needle be of a considerable length, then if a° be the angle of deflection, l the *magnetic* length of the needle (generally about $\frac{5}{6}$ of the actual length), and d the magnetic diameter of the coil, the correction to be supplied to the tangent of the angle of deflection is

$$\tfrac{3}{4}(4 \tan^2 a^\circ - 1)\frac{l^2}{d^2} \sin 2 a^\circ,$$

which correction is additive at great deflections, and subtractive at small ones. At a certain deflection this correction vanishes; that is to say, we have

$$4 \tan^2 a^\circ - 1 = 0,$$

or

$$\tan a = \tfrac{1}{2} = \tan 26^\circ 30'.$$

‡ The exact arrangement of this instrument is described in the Appendix.

(which is $\frac{3}{4}$ of an inch long) has a long pointer of gilt copper, about 5 inches long, fixed at right angles to it; when the needle is parallel to the coil, each end of this pointer is over the zero of a graduated scale. One of these scales is divided to true degrees, and the other to numbers proportional to the tangent of those degrees, so that if we read off two deflections from the degrees scale, the other extremity of the pointer will indicate, approxi-

FIG. 16.

mately, numbers proportional to the tangents of those two degrees of deflection.

Now, as the strengths of currents producing certain deflections are proportional to the tangents of the degrees of those deflections, if we read off from the degrees scale we must, as we have explained in Chapter I. (§ 12, page 7), reduce the degrees to tangents, from a table of tangents,* before working out a formula which has

* Table I.

reference to the strengths of currents. If, however, we read off from the tangent scale, no reduction is necessary, and the numbers can be at once inserted in the formula.

To avoid parallax error, in consequence of the pointer being elevated above the scale, a mirror-glass is fixed under each end of the pointer, so that when we look at the end of the latter and see that its reflected image coincides with the pointer itself, we know that we are looking at the end perpendicularly with the scale.

30. Before using the galvanometer it should be seen that the pointer has not become bent, but stands at right angles to the magnet, and that when suspended it turns freely. On no account should the magnet suspension be oiled, as quite the opposite effect to what is intended will be produced by so doing. Care should be taken that the scale is in its proper position, so that when the two ends of the pointer are over the zero points, the pointer stands at right angles to the coils. The correct setting of the position of the scale with reference to the coil is a mechanical adjustment which when once properly effected is not likely to alter; it is, however, as well to verify its correctness by means of a set square before the instrument is brought into general use. The pointer attached to the magnetic needle is very subject to accident, and is often badly adjusted. The correctness of the setting can be verified by placing the galvanometer so that the pointer stands at zero, and then sending a current through the coil first in one direction and then in the other. The deflections on either side of zero in this case should be equal; if they are not, the position of the pointer relative to the needle should be readjusted until the required equality of deflections on either side of zero is obtained. Care should be taken when making this adjustment to place the instrument clear of any unequally distributed masses of iron, otherwise unequal deflections may be obtained although the pointer and magnet are correctly set.

Angle of Maximum Sensitiveness.

31. In using the tangent galvanometer it is always as well to avoid obtaining either very high or very low deflections. The reason of this is, that any small change in the strength of a current traversing the galvanometer will produce the greatest effect on the needle when the latter stands at some deflection which is neither very high nor very low. The galvanometer is, in fact, most sensitive when the needle points, under the influence of a current, at that deflection.

Thus, for example, suppose we had a current which produced a deflection of 5°, and this current was increased say by $\frac{1}{10}$th, then the deflection would be increased to 5° 30′, because

$$\tan 5^\circ : \tan 5^\circ\ 30' :: 1 : 1\tfrac{1}{10}.$$

Next, suppose the needle stood at 80°, and the current was, as before, increased by $\frac{1}{10}$th, then the deflection would be increased to 80° 54′, for

$$\tan 80^\circ : \tan 80^\circ\ 54' :: 1 : 1\tfrac{1}{10}.$$

Lastly, let us suppose the needle stood at 43°, then by the increase in the current the deflection would have changed to 45° 43′, for

$$\tan 43^\circ : \tan 45^\circ\ 43' :: 1 : 1\tfrac{1}{10}.$$

In the first case, then, when the deflection was low, the increase was

$$5^\circ\ 30' - 5^\circ = 30';$$

in the second case, when the deflection was high,

$$80^\circ\ 54' - 80^\circ = 54';$$

and in the third case, when the deflection was of a medium value,

$$45^\circ\ 43' - 43^\circ = 2^\circ\ 43'.$$

What, then, is the deflection at which this increase is greatest?

The point to be determined is, what deflection is increased most by any small alteration in the current producing that deflection?

If C be a current giving a deflection of $a_1{}^\circ$, and C_1 a current a little stronger, say, which increases this deflection to $(a_1{}^\circ + \delta^\circ)$, we have to find what value given to $a_1{}^\circ$, makes δ° as large as possible when C and C_1 are very nearly and ultimately equal.

We have

$$C : C_1 :: \tan a_1{}^\circ : \tan (a_1 + \delta^\circ),$$

therefore

$$\tan (a_1{}^\circ + \delta^\circ) = \frac{C_1}{C} \tan a_1{}^\circ. \qquad \text{[A]}$$

Now we have to make δ° a maximum, supposing that the foregoing equation holds good.

Since δ° is to be a maximum, tan δ° must also be a maximum.

Now

$$\tan (a_1{}^\circ + \delta^\circ) = \frac{\tan a_1{}^\circ + \tan \delta^\circ}{1 - \tan a_1{}^\circ \tan \delta^\circ} = \frac{C_1}{C} \tan a_1{}^\circ,$$

therefore

$$\tan \alpha_1{}^\circ + \tan \delta^\circ = \frac{C_1}{C} \tan \alpha_1{}^\circ (1 - \tan \alpha_1{}^\circ \tan \delta^\circ),$$

therefore

$$\tan \delta^\circ \left(1 + \frac{C_1}{C} \tan^2 \alpha_1{}^\circ\right) = \tan \alpha_1{}^\circ \left(\frac{C_1}{C} - 1\right),$$

therefore

$$\tan \delta^\circ = \frac{\tan \alpha_1{}^\circ \left(\frac{C_1}{C} - 1\right)}{1 + \frac{C_1}{C} \tan^2 \alpha_1{}^\circ} = \frac{\frac{C_1}{C} - 1}{\frac{1}{\tan \alpha^\circ} + \frac{C_1}{C} \tan \alpha_1{}^\circ}. \qquad [B]$$

We have then to find what value of $\tan \alpha_1{}^\circ$ makes this fraction a maximum, and this we shall do by finding what value makes the denominator of the fraction a minimum. Now

$$\frac{1}{\tan \alpha_1{}^\circ} + \frac{C_1}{C} \tan \alpha_1{}^\circ = \left(\frac{1}{\sqrt{\tan \alpha_1{}^\circ}} - \sqrt{\frac{C_1}{C} \tan \alpha_1{}^\circ}\right)^2 + 2\sqrt{\frac{C_1}{C}},$$

and this will be a minimum when

$$\frac{1}{\sqrt{\tan \alpha_1{}^\circ}} - \sqrt{\frac{C_1}{C}} \tan \alpha_1{}^\circ = 0,$$

that is, when

$$1 = \sqrt{\frac{C_1}{C}} \tan^2 \alpha_1{}^\circ, \quad \text{or,} \quad \tan \alpha_1{}^\circ = \sqrt{\frac{C}{C_1}};$$

but as C_1 and C are ultimately equal, $\frac{C}{C_1}$ becomes equal to 1, therefore

$$\tan \alpha_1{}^\circ = \sqrt{1} = 1 = \tan 45^\circ.$$

32. We see, then, that in order to make the tangent galvanometer as sensitive as possible we should obtain the deflection of its needle as near 45° as possible; 45° is, in fact, the *angle of maximum sensitiveness.*

Every galvanometer has an angle of maximum sensitiveness, although it is not the same in all. The angle can, however, be found experimentally (see 'Calibration of Galvanometers'), and should be marked on the instrument for future reference.

33. If we require to adjust two currents in two different measurements so that they shall be equal in both cases, it is evident that the needle of the galvanometer employed to measure them should in each case show the same deflection. In

making the two measurements, we take the deflection obtained by one current as the standard, and then in making the second measurement we adjust the current until the same deflection is obtained. Now the accuracy with which this current can be adjusted depends upon the sensitiveness of the galvanometer to a change in the strength of the current, and we have seen that this sensitiveness is at a maximum when the deflection is 45°. If, therefore, we employ a tangent galvanometer for such a test as that just mentioned, we should endeavour in both measurements to bring the needle to 45°.

34. In what way can the property of the galvanometer be taken advantage of when comparing two deflections?

We must in such a case endeavour to obtain both deflections as near to 45° as possible. To do this we should have to get one deflection on one side, and the other deflection on the other side, of 45°. But then the question arises, should we get the deflections at an equal distance on either side, or one closer to the 45° than the other, and if so, should the higher or the lower deflection be the closer of the two?

Now, a little consideration will make it clear that if the two deflections in question are taken either near 0° or 90°, they will be much closer together than if they are taken near 45°, for the reason that the tangents of high or low deflections differ more widely from one another than do the tangents of medium deflections. But we have shown that when deflections are high or low, any increase or decrease in the strength of the current producing those deflections has less effect than when the deflections are of a medium value. It is therefore evident that it is most advantageous to get the deflections as wide apart as possible.

Let then $\tan \theta^\circ$ represent the stronger, and $\tan \phi^\circ$ the weaker current, and let one current be n times as strong as the other. We then have to find what values of θ° and ϕ° make

$$\theta^\circ - \phi^\circ$$

a maximum, supposing that

$$\tan \theta^\circ = n \tan \phi^\circ.$$

If in the last investigation we substitute $\theta^\circ - \phi^\circ$ for δ°, ϕ° for $a_1{}^\circ$, and n for $\frac{C_1}{C}$, we can see that in order to get the required result we must make

$$\tan \phi^\circ = \frac{1}{\sqrt{n}},$$

and, since $\tan \theta^\circ = n \tan \phi^\circ$,

$$\tan \theta^\circ = \frac{n}{\sqrt{n}} = \sqrt{n}.$$

If one current strength is to be twice as great as the other, then $n = 2$; consequently,

$$\tan \theta^\circ = \sqrt{2} = 1{\cdot}41421 = \tan 54^\circ\ 44' = \tan 54\tfrac{3}{4}^\circ,$$

and

$$\tan \phi^\circ = \frac{1}{\sqrt{2}} = {\cdot}70711 = \tan 35^\circ\ 16' = \tan 35\tfrac{1}{4}^\circ.$$

These, then, are the deflections that theoretically it is best to obtain in making a test with a tangent galvanometer in which one current is to be twice as strong as the other. But practically we may make the deflections 55° and $35\frac{1}{2}^\circ$, as these are more convenient to adjust to, and $\tan 55^\circ$ is, within $1'$, exactly double $\tan 35\frac{1}{2}^\circ$.

If we examine the theoretical deflections $54^\circ\ 44'$ and $35^\circ\ 16'$ it will be seen that

$$54^\circ\ 44' - 45^\circ = 9^\circ\ 44',$$

and

$$45^\circ - 35^\circ\ 16' = 9^\circ\ 44',$$

or in other words, the angular deflections on either side of 45° are in this case the same. Let us then see whether they are so when n has any value other than 2.

The angular deflection between 45° and θ° will be

$$\theta^\circ - 45^\circ,$$

that between 45° and ϕ°,

$$45^\circ - \phi^\circ;$$

now $\tan(\theta^\circ - 45^\circ) = \dfrac{\tan \theta^\circ - 1}{1 + \tan \theta^\circ}$,

and $\tan(45^\circ - \phi^\circ) = \dfrac{1 - \tan \phi^\circ}{1 + \tan \phi^\circ}$;

but we know, since

$$\tan \theta^\circ = \sqrt{n} \text{ and } \tan \phi^\circ = \frac{1}{\sqrt{n}},$$

that

$$\tan \phi^\circ = \frac{1}{\tan \theta^\circ},$$

that is

$$\tan(45^\circ - \phi^\circ) = \frac{1 - \dfrac{1}{\tan\theta^\circ}}{1 + \dfrac{1}{\tan\theta^\circ}} = \frac{\tan\theta^\circ - 1}{1 + \tan\theta^\circ},$$

that is

$$\tan(45^\circ - \phi^\circ) = \tan(\theta^\circ - 45^\circ),$$

or

$$45^\circ - \phi^\circ = \theta^\circ - 45^\circ;$$

showing that these angular deflections *are* the same whatever be the value of n.

This is a very useful fact, as it shows that when we are making a test in which two deflections are involved whose relative values are unknown, we should so adjust the resistances, &c., that the deflections are obtained, as near as possible, at equal distances, on either side of 45°.

To sum up, then we have

Best Conditions for using the Tangent Galvanometer.

35. When a test is made in which only one deflection is concerned, then that deflection should be as near 45° as possible.

If there are two deflections to be dealt with, then these should be as nearly as possible at equal distances on either side of 45°. If one of these deflections is to be double the other, then 55° and 35½° are the most convenient to employ.

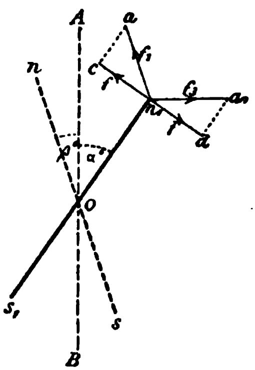


Fig. 17.

Skew Zero.

36. Although it is usual to take the readings on the tangent galvanometer, starting with the pointer at the ordinary zero, i.e. with the needle parallel to the plane of the ring or coils, yet it is not absolutely necessary that this arrangement should be adopted; the instrument can be used when the needle in its normal position makes an angle with the plane of the ring. Under the latter conditions, however, the current strength will not be in direct proportion to the tangent of the angle of deflection.

Let the dotted line A B (Fig. 17) represent the plane of the

coils, and let $n\,s$ be the needle in its normal position, i.e. in the plane of the magnetic meridian; also let $n_1\,s_1$ be the position which the needle takes up under the influence of the current. Let β° be the angle which the needle makes normally with the coils, and let $a^\circ + \beta^\circ$ be the angle through which the needle turns when deflected to the position $n_1\,s_1$.

Draw $c\,d$ at right angles to $n_1\,s_1$, making $c\,n_1$ equal to $n_1\,d$; draw $c\,a$ and $d\,a_1$ each at right angles to $c\,d$; also draw $n_1\,a$ parallel to $n\,s$, and $n_1\,a$ at right angles to A B. Now since $a\,n_1$ is parallel to $n\,o$, and $a\,c$ parallel to $n_1\,o$, the angle $c\,a\,n_1$ is equal to the angle $a^\circ + \beta^\circ$; also since $n_1\,a_1$ is perpendicular to A o, and $n_1\,d$ is perpendicular to $n_1\,o$, the angle $a_1\,n_1\,d$ is equal to the angle a°. We consequently have

$$f = f_3 \cos a^\circ, \text{ and } f = f_1 \sin (a^\circ + \beta^\circ),$$

therefore

$$f_3 \cos a^\circ = f_1 \sin (a^\circ + \beta^\circ),$$

or

$$f_3 = f_1 \frac{\sin (a^\circ + \beta^\circ)}{\cos a^\circ}. \text{*} \qquad \text{[A]}$$

$$= f_1 \frac{\sin a^\circ \cos \beta^\circ + \sin \beta^\circ \cos a^\circ}{\cos a^\circ}$$

$$= f_1 (\tan a^\circ \cos \beta^\circ + \sin \beta^\circ).$$

$$= f_1 \cos \beta^\circ \left(\tan a^\circ + \frac{\sin \beta^\circ}{\cos \beta^\circ}\right)$$

$$= f_1 \cos \beta^\circ (\tan a^\circ + \tan \beta^\circ).$$

So that $\cos \beta^\circ$ being a constant quantity, the strength of a current is directly proportional to $(\tan a^\circ + \tan \beta^\circ)$, which is the reading on the tangent scale (§ 29) if the figures on the latter are re-arranged so that the zero is at the division at which the needle points in its normal position.† Fig. 18 shows a scale so

* If the angle β° had been on the right- instead of the left-hand side (as in Fig. 17) of the coils A B, the angle a° still being the angle A $o\,n_1$, then we should have had

$$f_3 = \frac{\sin (a^\circ - \beta^\circ)}{\cos a^\circ}.$$

† It should be pointed out that if the needle does not deflect from the zero position to the opposite side of the ring or coils, then the angle, a°, which it makes with A B, being on the left-hand side of A B, must be taken as negative. In this case we get

$$f_3 = f_1 \cos \beta^\circ (\tan \beta^\circ - \tan a^\circ).$$

$(\tan \beta^\circ - \tan a^\circ)$ is in this case also the reading on the tangent scale if the figures on the latter are re-arranged so that the zero is at the division at which the needle points in its normal position.

re-arranged, the new figures being additional to the old ones; such a scale has been adopted in the tangent instruments used for testing purposes in the Postal Telegraph Department.

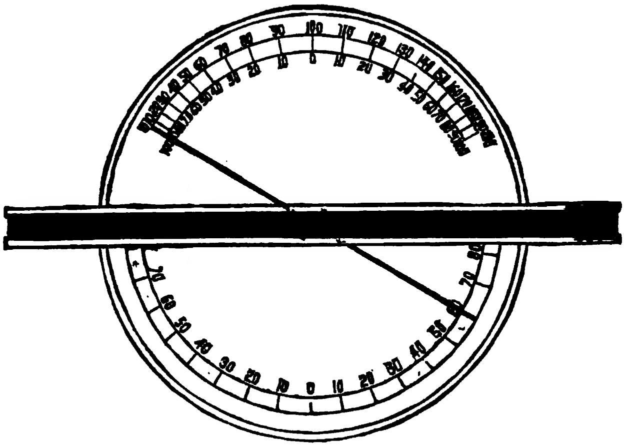


FIG. 18.

Apart from the fact that the adoption of the foregoing "skew" method of using the tangent galvanometer gives an increased range to the instrument, a considerable increase of sensitiveness in the case of high deflections is also obtained by it, i.e. a current which would move the needle of the instrument through a given angle from the old zero, will move it through a much larger angle from the new or "skew" zero. This, however, is only the case if the first angular deflection in question (the one from the old zero) exceeds a certain value; if it is less than this value, then the deflection for a given current will be less from the skew than from the old zero.

Let ζ° be the angular deflection obtained with a given current when the needle is deflected from the ordinary zero, then

$$f = f_1 \tan \zeta^\circ;$$

but if the needle had been at the skew zero, then with the same current we should have had

$$f = f_1 \frac{\sin (a^\circ + \beta^\circ)}{\cos a^\circ},$$

therefore

$$\tan \zeta^\circ = \frac{\sin (a^\circ + \beta^\circ)}{\cos a^\circ},$$

Suppose we have $\beta^\circ = 60^\circ$, and suppose the current to be of such a strength as to turn the needle through an angle of 120°, then in this case $\alpha^\circ = 60^\circ$, and we consequently have

$$\tan \zeta^\circ = \frac{\sin 120^\circ}{\cos 60^\circ};$$

but $\sin 120^\circ = \sin (180^\circ - 60^\circ) = \sin 60^\circ$, therefore,

$$\tan \zeta^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \tan 60^\circ,$$

or

$$\zeta^\circ = 60^\circ;$$

that is to say, the angle through which the needle would turn if the zero were 60° to one side of zero, would be twice what it would be if it were deflected from the ordinary zero.

The relative values of the deflections, with a given current, from the ordinary and from the skew zero, approach nearer to an equality in proportion as the deflections become smaller; at a certain point they become equal, and then the relative values become reversed, i.e. for the same current the deflection from the skew zero becomes less than the deflection from the ordinary zero. Let us determine at what point the deflections from the two zeros become the same. We have in this case

$$\frac{\sin (\alpha^\circ + \beta^\circ)}{\cos \alpha^\circ} = \tan \alpha^\circ = \frac{\sin \alpha^\circ}{\cos \alpha^\circ},$$

therefore

$$\sin (\alpha^\circ + \beta^\circ) = \sin \alpha^\circ;$$

if now the angle α° is negative, that is to say, if the angular deflection from the skew zero is less than the angle β°, then we have

$$\sin (\beta^\circ - \alpha^\circ) = \sin \alpha^\circ,$$

or

$$\beta^\circ - \alpha^\circ = \alpha^\circ,$$

that is

$$\beta^\circ = 2\,\alpha^\circ,$$

or

$$\alpha^\circ = \frac{\beta^\circ}{2};$$

that is to say, whatever be the angle β° (the angular distance of the skew from the ordinary zero) then a current sufficient to move the needle a distance of $\frac{\beta^\circ}{2}$ from the ordinary zero would move the

needle the same distance from the skew zero. *If the deflection from the old zero be less than* $\frac{\beta^\circ}{2}$, *then the deflection from the skew zero will be less still*, so that there is no advantage in the use of the skew zero unless the deflections exceed $\frac{\beta^\circ}{2}$.

From what has been proved, it is obvious that the greater we make β° the greater will be the deflection obtained with a given current, but there is a practical limit to increasing β°, for the larger we make the latter the more does the deflective action of the coil tend to act in a direction parallel but opposite to the earth's magnetism, the consequence being that the resultant of the two forces is a comparatively small quantity, and the friction of the pivot, &c., prevents the needle from settling down to the true angle representing the force of the current. Under such conditions large errors in the readings may result. Were it not for this fact the instrument would increase in actual sensitiveness up to the point at which $\beta^\circ = 90^\circ$, at which point the needle would not move unless acted upon by a current exceeding in deflective force the intensity of the earth's magnetism; when the current exceeded this value the needle would swing completely round through an angle of 180°.

37. What is the *angle of maximum sensitiveness* in the case of a tangent galvanometer with a skew zero? Referring to page 26, it is obvious, since the current strength is in proportion to tan a_1° + tan β°, that equation (A) on the page referred to becomes

$$\tan (a_1^\circ + \delta^\circ) + \tan \beta^\circ = \frac{C_1}{C} (\tan a_1^\circ + \tan \beta^\circ),$$

or

$$\tan (a_1^\circ + \delta^\circ) = \frac{C_1}{C} (\tan a_1^\circ + \tan \beta^\circ) - \tan \beta.$$

Now we have to make δ° a maximum, supposing that the foregoing equation holds good.

Since δ° is to be a maximum, tan δ° must also be a maximum.

Now

$$(\tan a_1 + \delta^\circ) = \frac{\tan a_1^\circ + \tan \delta^\circ}{1 - \tan a_1^\circ \tan \delta^\circ} = \frac{C_1}{C} (\tan a_1^\circ + \tan \beta^\circ) - \tan \beta^\circ,$$

therefore

$$\tan a_1^\circ + \tan \delta^\circ = \left[\frac{C_1}{C} (\tan a_1^\circ + \tan \beta^\circ) - \tan \beta^\circ\right] [1 - \tan a_1^\circ \tan \delta^\circ],$$

therefore

$$\tan \delta^\circ \left[1 + \frac{C_1}{C} \tan \alpha_1{}^\circ (\tan \alpha_1{}^\circ + \tan \beta^\circ) - \tan \alpha_1{}^\circ \tan \beta^\circ\right]$$
$$= (\tan \alpha_1{}^\circ + \tan \beta^\circ)\left(\frac{C_1}{C} - 1\right),$$

therefore

$$\tan \delta^\circ = \frac{(\tan \alpha_1{}^\circ + \tan \beta^\circ)\left(\frac{C_1}{C} - 1\right)}{1 - \tan \alpha_1{}^\circ \tan \beta^\circ + \frac{C_1}{C} \tan \alpha_1{}^\circ (\tan \alpha_1{}^\circ + \tan \beta^\circ)}$$

$$= \frac{\frac{C_1}{C} - 1}{\frac{1 - \tan \alpha_1{}^\circ \tan \beta^\circ}{\tan \alpha_1{}^\circ + \tan \beta^\circ} + \frac{C_1}{C} \tan \alpha_1{}^\circ}.$$

We have then to find what value of $\tan \alpha_1{}^\circ$ makes this fraction a maximum, and this we shall do by finding what value makes the denominator of the fraction a minimum. Let $\tan \alpha_1{}^\circ = a$, $\tan \beta^\circ = b$, and $\frac{C_1}{C} = \kappa$, then we have to determine what value of a makes

$$\frac{1 - a\,b}{a + b} + \kappa\,a$$

a minimum.

Now

$$\frac{1 - a\,b}{a + b} + \kappa\,a = \kappa\,(a + b)\left[1 - \frac{\sqrt{\frac{1 + b^2}{\kappa}}}{a + b}\right]^2$$
$$+ 2\sqrt{\kappa\,(1 + b^2)} - b\,(\kappa + 1),$$

and this will be a minimum when

$$1 - \frac{\sqrt{\frac{1 + b^2}{\kappa}}}{a + b} = 0,$$

that is, when

$$a + b = \sqrt{\frac{1 + b^2}{\kappa}};$$

but as C_1 and C are ultimately equal, $\frac{C_1}{C}$, that is κ, becomes equal to 1, therefore

$$a + b = \sqrt{1 + b^2},$$

therefore

$$a^2 + b^2 + 2\,a\,b = 1 + b^2,$$

therefore

$$2\,a\,b = 1 - a^2,$$

therefore

$$b = \frac{1 - a^2}{2\,a},$$

that is

$$\tan\beta^\circ = \frac{1 - \tan^2 a_1^\circ}{2\tan a_1^\circ} = \cot 2\,a_1^\circ,$$

or

$$\cot(90^\circ - \beta^\circ) = \cot 2\,a_1^\circ,$$

therefore

$$90^\circ - \beta^\circ = 2\,a_1^\circ,$$

or

$$a_1^\circ = 45^\circ - \frac{\beta^\circ}{2}.$$

Since β° cannot be greater than 90°, or less than 0° (unless it has a negative value), we see that a_1° must lie between the ordinary zero and 45° from it. In the case of a galvanometer where $\beta^\circ = 60^\circ$ we have

$$a_1^\circ = 45^\circ - \frac{60^\circ}{2} = 15^\circ,$$

that is $(60^\circ + 15^\circ)$, or 75°, from the skew zero.

38. In order that a tangent galvanometer when used in the ordinary way may give accurate results, it is obviously necessary that the magnetic needle, or rather the magnetic axis of the same, be strictly parallel to the magnetic plane of the coils, that is to say, the angle β° must be equal to nothing. When the latter is the case, the angular deflection for a given current should be the same to whichever side of zero the needle is deflected. If it is found that these deflections are different, we can determine from the two results what is the magnitude of the angle β° (Fig. 17, page 30). Referring to this figure, let θ_1° be the angular movement of the needle from its position of rest, then

$$\theta_1^\circ = a^\circ + \beta^\circ,$$

or

$$a^\circ = \theta_1{}^\circ - \beta^\circ;$$

therefore from equation (A) (page 31), we can see that in this case

$$f_3 = f_1 \frac{\sin \theta_1{}^\circ}{\cos(\theta_1{}^\circ - \beta^\circ)}.$$

If the same current is now sent in the reverse direction, and the angular movement of the needle from its position of rest is $\theta_2{}^\circ$, we have

$$f_3 = f_1 \frac{\sin \theta_2{}^\circ}{\cos(\theta_2{}^\circ + \beta^\circ)},$$

therefore

$$\frac{\sin \theta_1{}^\circ}{\cos(\theta_1{}^\circ - \beta^\circ)} = \frac{\sin \theta_2{}^\circ}{\cos(\theta_2{}^\circ + \beta^\circ)},$$

therefore

$$\frac{\sin \theta_1{}^\circ}{\cos \theta_1{}^\circ \cos \beta^\circ + \sin \theta_1{}^\circ \sin \beta^\circ} = \frac{\sin \theta_2{}^\circ}{\cos \theta_2{}^\circ \cos \beta^\circ - \sin \theta_2{}^\circ \sin \beta^\circ},$$

therefore

$$\cot \theta_1{}^\circ \cos \beta^\circ + \sin \beta^\circ = \cot \theta_2{}^\circ \cos \beta^\circ - \sin \beta^\circ,$$

therefore

$$\cot \theta_1{}^\circ + \tan \beta^\circ = \cot \theta_2{}^\circ - \tan \beta^\circ,$$

that is

$$2 \tan \beta^\circ = \cot \theta_2{}^\circ - \cot \theta_1{}^\circ,$$

or

$$\tan \beta^\circ = \frac{\cot \theta_2{}^\circ - \cot \theta_1{}^\circ}{2},$$

that is, β° must be an angle whose tangent is $\dfrac{\cot \theta_2{}^\circ - \cot \theta_1{}^\circ}{2}$.

To make then the instrument read correctly, the graduated dial plate would have to be turned round through the angle β°, in the direction in which the needle moved when the largest of the two deflections was obtained; the zero point will then be correctly set, and the tangent of the angle of deflection taken from this zero will represent directly the current strength. When the needle is provided with a pointer, the simplest method of making the correction is to bend the pointer as explained in § 30 (page 25), until equal readings are obtained, with the same current, on both sides of zero.

THE GAUGAIN GALVANOMETER.

39. A form of tangent galvanometer, which is in very general use for lecture and educational purposes, is shown by Fig. 19. This instrument is known as the *Gaugain* galvanometer, though actually, it is a modification by Helmholtz of the original instrument of Gaugain. It was shown by the latter, that if the magnetic needle were suspended, not at the centre of the coil, but at

FIG. 19.

a point on the axis at a distance from the centre equal to half the radius of the coil, then the chief error due to the magnetic needle not being infinitely short, disappears. Helmholtz improved upon this arrangement by placing a second coil, similar to the first, at an equal distance on the other side of the magnet; by this means, the error due to the centre of the magnetic needle not being truly at the point indicated by Gaugain, is got rid of. In order that the ratio between the diameter of the coil to its distance from the centre of the magnet may be preserved with reference to every turn of which the coil is composed, these turns should be wound on a conical surface as in the instrument shown by Fig. 19. It is pointed out by Clerk Maxwell, however,* that

* 'Electricity and Magnetism,' by J. Clerk Maxwell, vol. ii. p. 318.

such a method of winding is quite unnecessary, as the conditions may be satisfied by coils of a rectangular section, which can be constructed with far greater accuracy than coils wound on an obtuse cone.

THE TROWBRIDGE OR OBACH GALVANOMETER.*

40. In this galvanometer, which is shown by Fig. 20, the ring instead of being fixed as in the ordinary tangent instrument is

FIG. 20.

movable about an horizontal axis; by this means the deflective action of the ring on the needle can be reduced from the full

* This instrument, although generally known as the "Obach" Galvanometer, and indeed invented independently by the late Dr. E. Obach, was fully described by Professor John Trowbridge in the 'American Journal of Science and Arts,' in August 1871, and called by that gentleman a "Cosine" galvanometer.

effect (when the ring is in the usual vertical position) down to zero (when the ring is in an horizontal position), so that the instrument has a very wide range, a range which in practice is 100 times as great as that of an ordinary tangent galvanometer, thus enabling either weak or very powerful currents to be measured.

The effect of setting the ring at an angle to the vertical position is as follows:—

In Fig. 21, let $a\,b$ be the vertical position of the ring, and $a_1\,b_1$, the latter when inclined at an angle $\psi°$. Draw $a_1\,c$ at right angles to $a_1\,b_1$, and $a_1\,d$ at right angles to $a\,b$, then the angle $c\,a_1\,d$ equals the angle $\psi°$.

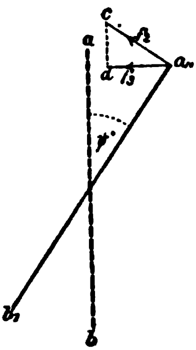


Fig. 21.

Now if $c\,a_1$, that is f_2, represents the magnetic force of the ring when the latter is traversed by a current, this force being at right angles to the ring, then $a_1\,d$, that is f_3, will be the resolved force at right angles to the vertical $a\,b$. We have then

$$\frac{f_2}{f_3} = \sec \psi°,$$

or

$$f_2 = f_3 \sec \psi°;$$

that is to say, the magnetic force of the ring is equal to its deflective force on the needle multiplied by the *secant* of the angle at which the ring is set. But the magnetic force of the ring is in direct proportion to the current strength, and the resolved deflective force is in direct proportion to the tangent of the angle of deflection ($a°$) of the needle of the instrument. Hence *the strengths of currents circulating in the ring are directly proportional to the tangents of the angles of deflection of the needle multiplied by the respective secants of the angles of inclination of the ring;* or we may say

$$C = \tan a° \times \sec \psi° \times \text{a constant}.$$

It must be obvious that there are several ways in which the instrument can be used. In the first place it can be made use of as an ordinary tangent galvanometer, the ring being set at such an angle as would cause the deflection obtained to be brought as nearly as possible in the neighbourhood of 45° (the angle of maximum sensitiveness); the current strengths in this case would of course be directly proportional to the *tangents* of the angles of deflection.

Again, the ring could be moved so that the same deflection of the needle is obtained with each current being measured; in this case, the current strengths will, of course, be directly proportional to the *secants* of the angles at which the ring had to be set in the different cases. Inasmuch as the adjustment of the position of the ring is dependent upon the observation of the movement of needle, it is best to arrange that the latter shall point as nearly as possible at the angle of maximum sensitiveness, i.e. at 45°.

The "equality" method of using the instrument consists in moving the ring until it is found that the angular deflection of the needle, and the angles through which the coil has been turned are the same; in this case we get

$$C = \tan \psi^\circ \times \sec \psi^\circ \times \text{a constant.}$$

As only a single angle has to be dealt with for a particular measurement, the products of tangents and secants can be calculated beforehand and embodied in a table.

In the ordinary tangent galvanometer, the deflective action of the ring acts in the same plane as that in which the needle turns;

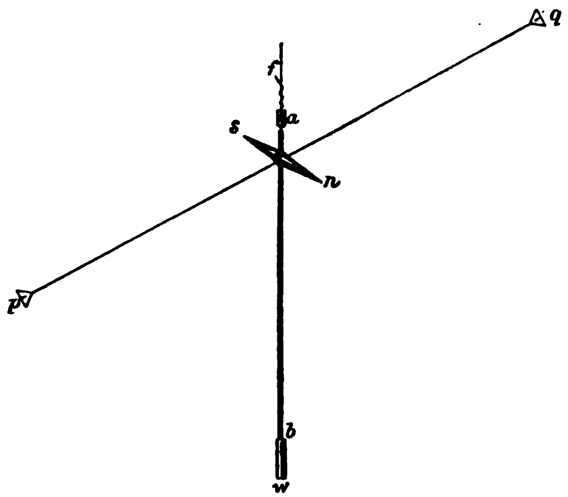


Fig. 22.

but in the Obach instrument, the deflective force, being at an angle with this plane, tends to make the needle dip when the ring is inclined. In order to avoid this tendency, the arrangement shown by Fig. 22 is adopted by Dr. Obach. The needle *n s* is

fixed near to the upper end of a thin vertical axis *a b*, the lower end of the latter being provided with a cylindrical brass weight *w*. This weight offers but little additional momentum to the whole system round the vertical axis, whilst the movement round the horizontal axis is completely prevented. The aluminium pointer *p q*, is situated in the same plane as the scale; the ends are flattened and provided with a fine slit, which serves as an index for reading the deflections; the bottom of the box in which the needle turns being blackened, the reading can be taken without parallax, and therefore very accurately. The magnetic needle *n s*, has a biconical shape, which entirely prevents the shifting of the magnetic axis from its original position, as was sometimes found to be the case with the old broad needles. Adjustments are provided by which the cocoon fibre *f*, serving to suspend the needle, can be raised or lowered, as well as accurately centred.

In order to damp the oscillations of the needle, a shallow cylindrical box, about 8 centimetres in diameter and 1¼ centimetres deep, is provided; this box has two radial partitions which can be slid in or out; the axle of the needle passing through the centre of this box, carries a light and closely fitting vane. By sliding the partitions more or less into the box, various degrees of damping can be obtained; and if they are right in, the motion is practically dead beat.

The scale over which the needle turns is provided with degree and also with tangent divisions. The scale fixed to the ring enables the inclination of the latter to be read to $\frac{1}{10}$th of a degree; this scale is also engraved with secant divisions, so as to avoid the necessity of reducing the degrees to secants by means of a table. In order to enable the "constant" of the instrument, i.e. the deflection due to a given current, to be made the same at any place when the instrument is being used, an auxiliary magnet (seen in the figure) is placed at the side of the instrument; this magnet can be turned round an horizontal axis passing through its neutral point and the centre of the needle, and is at right angles to the diameter on which the ring is turned. This magnet does not affect the zero position, and, moreover, if placed exactly vertical with its magnetic axis, it does not alter the original constant, which then only depends upon the horizontal terrestrial component, more or less modified by the surroundings; but if it is dipped, the horizontal force acting on the needle is either augmented or diminished, according to the direction in which the magnet is turned, and to the amount of dip given.

The ring of the instrument, it should be mentioned, is of gun-

metal, and serves for the purpose of measuring strong currents, whilst fine wire wound in a groove in the ring enables weaker currents also to be measured. The relative values of the deflective actions of the ring and of the fine wire upon it, are so adjusted that a current of 1 ampère through the ring gives exactly the same deflection as an electromotive force of 1 volt at the terminals of the fine wire.

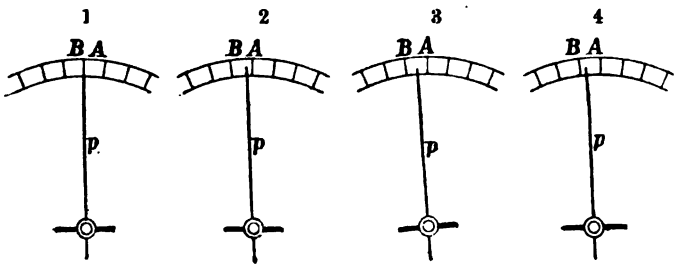
Since we have

$$C = \tan \alpha^\circ \times \sec \psi^\circ \times \text{a constant}$$

we can easily see if any particular instrument is properly made and the scales correctly graduated; for if we pass a constant current through the ring, and set the latter at different inclinations, then the products of the secants of the angles of inclination of the ring and the tangents of the corresponding angles of deflection obtained, should be the same in every case.

Method of Reading Galvanometer Deflections.

41. The reading of galvanometer deflections requires considerable method, in order that accurate results may be obtained in making measurements.



Deflection = A. Deflection = A¼. Deflection = A½. Deflection = A¾.

Fig. 23.

Let A and B (Fig. 23) be two contiguous division marks on the galvanometer scale. Now, by observation, we can always determine without difficulty whether the pointer lies exactly over A or over B, or whether it lies exactly midway between the two; and further, if it does not occupy either of these exact positions, we can judge without difficulty whether it lies nearest to A or to B. This is equivalent to saying that we can be certain of the

magnitude of the deflection within a *quarter* of a degree. Thus, supposing the pointer stood between A and B, but nearer to A than to B, then we should call the deflection "A¼," and supposing the deflection was actually *very* nearly equal to A, then A¼ would be a quarter of a division, or degree, too much; if, on the other hand, the deflection was *very* nearly equal to A½, then A¼ would be a quarter of a division, or degree, too little. In one case the error would be a plus one, and in the other a minus one; but in either case its *maximum* value would be ¼ only. We have, in fact, the rule that—if A be the smaller of two contiguous deflections A and B, then when the pointer is exactly over A, the deflection should be called "A"; if nearer to A than to B, then it should be called "A¼"; if exactly midway between A and B, it should be called "A½"; and lastly, if the pointer is nearer to B than to A, then the deflection should be called "A¾"—; thus, for example, if A and B (Fig. 23) were the 57° and 58° division marks respectively on the scale; then in case 1 the deflection would be taken as 57°; in case 2 the deflection would be taken as 57¼°; and again, in cases 3 and 4 the deflections would be taken as 57½° and 57¾° respectively. By keeping to these instructions, then, we can be sure of the magnitude of a deflection within ¼ of a division or degree.

42. If we are making a measurement with a tangent galvanometer and we read from the *degrees* scale, and if we have two deflections to deal with, one of which is to be a proportional part of the other (usually one-half), then, after the first deflection has been observed it has to be reduced to a tangent,* and then the latter being divided, say, by two, the corresponding deflection is ascertained from the tangent table; the resistances, &c., are then adjusted till the required second deflection is as nearly as possible obtained. If we find that the halved tangent does not exactly correspond to a deflection in the table, then we must take, say, the nearest deflection *below* the exact value, and then take care to adjust so that the deflection of the pointer is a litte *above* that angle. Thus suppose the first deflection to be 58°, then the tangent of 58° is 1·6003, and $\frac{1 \cdot 6003}{2} = \cdot 8001$; now the nearest number below this in the table is ·7954, which is the tangent of 38½; in adjusting the deflection, therefore, we should take care that we get it rather *more* than 38½.

* Table I.

Degree of Accuracy attainable in reading Galvanometer Deflections.

43. If the galvanometer scale be so graduated that the number of *divisions* of deflection directly represent the proportionate strengths of the currents producing those deflections, then an error of, say $\frac{1}{m}$th of a division in d divisions will represent a percentage error, γ, in the strength of the current represented by d, which is given by the proportion

$$\gamma : \frac{1}{m} : : 100 : d,$$

or

$$\gamma = \frac{\frac{1}{m} \times 100}{d} \text{ per cent.} \qquad [A]$$

If, however, the instrument be a tangent galvanometer, and the deflection be read from the *degrees* scale, then an error of $\frac{1^\circ}{m}$ in d° will not represent an error of $\frac{\frac{1^\circ}{m} \times 100}{d^\circ}$ per cent., for in this case we must have the proportion

$$\gamma_\circ : \tan d\tfrac{1^\circ}{m} - \tan d^\circ : : 100 : \tan d^\circ,$$

or

$$\gamma_\circ = \frac{(\tan d\frac{1^\circ}{m} - \tan d)\,100}{\tan d^\circ} = \left(\frac{\tan d\frac{1^\circ}{m}}{\tan d^\circ} - 1\right) 100 \text{ per cent.} \qquad [B]$$

For example.

If the deflection d were 46 divisions, then $\frac{1}{4}$ of a division error $(\frac{1}{m})$ would be an error, γ, of

$$\gamma = \frac{\frac{1}{4} \times 100}{46} = \cdot 54 \text{ per cent.}$$

in the current strength represented by the deflection d; but if the deflection were 46°, then $\frac{1}{4}$° error would be an error, $\gamma_\circ$, of

$$\gamma_\circ = \left(\frac{\tan 46\frac{1}{4}^\circ}{\tan 46^\circ} - 1\right) 100 = \left(\frac{1\cdot 0446}{1\cdot 0355} - 1\right) 100 = \cdot 88 \text{ per cent.}$$

in the current strength.

44. In cases where we have two deflections to deal with, one of which, or the tangent of one of which, has to be $\frac{1}{n}$th (usually $\frac{1}{2}$) of the other, then after we have ascertained, as accurately as we can judge, the magnitude of the first deflection d, the latter (or the tangent of the latter) is divided by n, and then the resistances, &c., in the circuit of the galvanometer are adjusted until the deflection $\frac{d}{n}$ $\left(\text{or the deflection corresponding to } \frac{\tan d}{n}\right)$ is obtained as accurately as possible. Now, in adjusting to this latter deflection, we are liable to make a plus or minus error of $\frac{1}{m}$th of a division or degree as in the first case, and as $\frac{d}{n}$ $\left(\text{or } \tan \frac{d}{n}\right)$ may itself contain an error due to d being $\frac{1}{m}$th of a division or degree wrong in the first instance, the new deflection may be more than $\frac{1}{m}$th of a division or degree out. What then is the "total possible percentage of error which may exist in the second deflection"?

Now the *absolute* error which may be made in the two deflections must be the

same in both cases, viz. $\frac{1}{m}$, but the *percentage* value of the latter will be directly proportional to the value of the deflections; thus a $\frac{1}{4}$ division error in 50 divisions is a $\frac{1}{2}$ per cent. error, but a $\frac{1}{4}$ per cent. error in 25 divisions is a 1 per cent. error; in fact, if γ be the *percentage* error $\left(\text{corresponding to the } \textit{absolute} \text{ error } \frac{1}{m}\right)$ in d divisions, then $n\,\gamma$ will be the percentage error (corresponding to the absolute error $\frac{1}{m}$) in $\frac{d}{n}$ divisions. Now, if d contains a *percentage* error γ, then $\frac{d}{n}$ must also contain a *percentage* error γ; consequently, if we make a percentage error of $n\,\gamma$ in $\frac{d}{n}$ when d already contains a percentage error γ, then $\frac{d}{n}$ must contain a total percentage error, Γ, of

$$\Gamma = \gamma + n\,\gamma = \gamma\,(1 + n);\text{*}$$

or since

$$\gamma = \frac{\frac{1}{m} \times 100}{d},$$

we get

$$\Gamma = \frac{\frac{1}{m} \times 100}{d}(1 + n). \qquad \text{[C]}$$

(1) *For example.*

If d and $\frac{1}{m}$ were 58 divisions and $\frac{1}{2}$ division, respectively, and further, if the deflection d had to be halved, that is, if $n = 2$, then we should get

$$\Gamma = \frac{\frac{1}{2} \times 100}{58} \times 3 = 1 \cdot 3 \text{ per cent.}$$

If we have to deal with *degrees* of deflection instead of divisions, then in the case of a tangent galvanometer we should have

$$\Gamma_o = \left(\frac{\tan d \frac{1^\circ}{m}}{\tan d^\circ} - 1\right)100 + \left(\frac{\tan d_1 \frac{1^\circ}{m}}{\tan d_1{}^\circ} - 1\right)100 =$$

$$\left(\frac{\tan d \frac{1^\circ}{m}}{\tan d^\circ} + \frac{\tan d_1 \frac{1^\circ}{m}}{\tan d_1{}^\circ} - 2\right)100, \qquad \text{[D]}$$

where

$$\tan d_1{}^\circ = \frac{\tan d^\circ}{n}.$$

(2) *For example.*

If d°, $\frac{1^\circ}{m}$, and n, were 58°, $\frac{1}{2}$°, and 2, respectively, then we should have

$$\tan d_1{}^\circ = \frac{1 \cdot 6003}{2} = \cdot 8001 \; (= \tan 38\tfrac{2}{3}{}^\circ),$$

therefore

$$\Gamma_o = \left(\frac{1 \cdot 6160}{1 \cdot 6003} + \frac{\cdot 8026}{\cdot 7954} - 2\right)100 = 1 \cdot 7 \text{ per cent.}$$

---

* Strictly speaking, this is not absolutely correct, for it assumes that the second percentage should be calculated on $\frac{d}{n}$, whereas it ought to be calculated on $\frac{d + \frac{1}{m}}{n}$; but as $\frac{1}{m}$ is small compared with d, the consequent error is small also.

It may be pointed out that this last example shows the possible percentage of error which may occur when making a halved current test with the tangent galvanometer under the best possible conditions. Practically, therefore, we may say that under no possible conditions could the deflection error in a halved current test be regarded as being less than 1⅔ per cent. As will be seen when we come to consider such tests, other sources of error are met with which still further reduce the degree of accuracy with which the tests can be made.

45. Although in formulæ [B] and [D] the function of the deflections has been taken as the *tangent*, yet the formulæ apply equally well in cases where the current strengths are proportional to any other function of the deflections.

Calibration of Galvanometers.*

46. The deviations in *degrees* of the needle of a galvanometer which is not of the tangent form are not generally proportional to any simple function of those degrees, yet it is easy to determine the relative values of the deflections in terms of the currents which would produce them, that is, to *calibrate* the scale. In order to do this, it is simply necessary to join up in circuit with the galvanometer, a battery, a set of resistance coils, and also a galvanometer, the values of whose deflections are known (a *tangent* galvanometer, for example). This being done, and the galvanometers being set so that their needles are at zero, we insert sufficient resistance in the circuit to reduce the deflection in one of the instruments to 1°, and then by means of a "shunt" (Chapter IV.) we also reduce the deflection of the needle of the second galvanometer to 1°. We now reduce the resistance in the circuit step by step so as to produce deflections of 1°, 2°, 3°, 4°, &c., from the needle of the galvanometer whose scale is required to be calibrated. As each deflection is obtained we observe and note the corresponding deflection on the tangent instrument. When the whole range of the scale (or as much of it as is considered necessary) of the instrument under calibration has been gone through, we can construct a table for use with it by writing down opposite the various degrees of deflection the tangents of the deflections which were obtained on the tangent instrument, and which corresponded to the deflections in question. The table so constructed would be used precisely in the same way as would the table of tangents in the case of a tangent galvanometer, the use including, it may be remarked, the determination of the percentage value of an error in a deflection. It may also be remarked that the *angle of maximum* sensitiveness would be the deflection which was obtained when the needle of the tangent instrument pointed to 45°.

* See also Index.

THE THOMSON GALVANOMETER.

47. The accuracy with which measurements can be made depends chiefly upon the sensitiveness of the galvanometer employed in making those measurements. The Thomson mirror galvanometer supplies this requisite sensitiveness, and is the instrument which is almost invariably employed when great accuracy is required, and also when very high resistances have to be measured.

Principle.

48. The principle of the instrument is that of employing a very light and small magnetic needle, delicately suspended within a large coil of wire, and of magnifying its movements by means of a long index hand of light. This index hand is obtained by throwing a beam of light on a small mirror fixed to the suspended magnetic needle, the ray being reflected back on to a graduated scale. This scale being placed about 3 feet distant from the mirror, it is obvious that a very small angular movement of the mirror will cause the spot of light reflected on the scale to move a considerable distance across it.

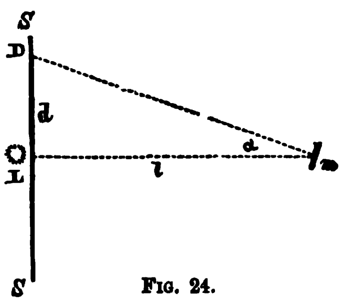


FIG. 24.

The needle being very small, and being placed in the centre of a large coil, the tangents of its deflections are approximately directly proportional to the strength of the currents producing them.

In Fig. 24, let L be a lamp which throws a beam upon the mirror *m*, which has turned through a small angle, and reflected the beam on the scale at D. Let *d* be the distance through which the beam has moved on the scale from the zero point at L, and let *l* be the distance between the scale and the mirror. Now the angle through which the beam of light turns will be twice the angle through which the mirror turns; this is clear if we suppose the mirror to have turned through 45°, when the reflected beam will be at 90°, or at right angles to the incident beam. If, then, we call $a°$ the angle through which the beam of light turns, $\frac{a°}{2}$ will be the angle through which the mirror will have turned.

Let C be the strength of current producing the deflection, then

$$C = \tan \frac{a^\circ}{2} \kappa,$$

where κ is a constant whose value depends on the make of the galvanometer, therefore

$$C = \frac{\sqrt{1 + \tan^2 a^\circ} - 1}{\tan a^\circ} \kappa,$$

$\sqrt{1 + \tan^2}$ being positive, as the angles are less than 90°.

l being the distance of the scale from the mirror, let d be the distance traversed on the scale by the beam of light, then

$$\tan a^\circ = \frac{d}{l},$$

therefore

$$C = \frac{\sqrt{1 + \frac{d^2}{l^2}} - 1}{\frac{d}{l}} \kappa,$$

or

$$C = \frac{1}{d}(\sqrt{l^2 + d^2} - l)\kappa. \qquad [A]$$

If we expand $\sqrt{l^2 + d^2}$ by the binomial theorem, we get

$$C = \frac{1}{d}\left(l + \frac{d^2}{2l} - \frac{d^4}{8l^3} + \ldots - l\right)\kappa$$

$$= \left(d - \frac{d^3}{4l^2} + \ldots\right)\frac{\kappa}{2l}. \qquad [B]$$

If l is large compared with d, all the terms beyond $\frac{d^3}{4l^2}$ may be neglected without introducing any sensible error; hence we see that C is proportional to d corrected by subtracting the quantity $\frac{d^3}{4l^2}$ from it.

For example.

A deflection of 300 divisions is obtained on the scale of a reflecting galvanometer, the distance of whose mirror from the centre of the scale is 1500 (l). What is the corrected deflection?

$$\text{Corrected deflection} = 300 - \frac{300^3}{4 \times 1500^2}$$

$$= 300 - 3 = 297 \text{ divisions.}$$

E

It should be stated that formula [B] ceases to be approximately accurate if the deflections are more than about $\frac{1}{5}$th of l. If this proportion is exceeded it is best to employ the accurate formula [A], which, to be in a similar form to [B], should be written

$$C = \frac{2\,l}{d}\left(\sqrt{l^2 + d^2} - l\right)\frac{\kappa}{2\,l};$$

that is to say, $\frac{2\,l}{d}(\sqrt{l^2 + d^2} - l)$ is the corrected deflection.

If the deflections to be compared are either both small, or if they are large and do not differ greatly, then we may take the deflections themselves as representative of the currents producing them, no corrections being necessary; this is what is usually done.

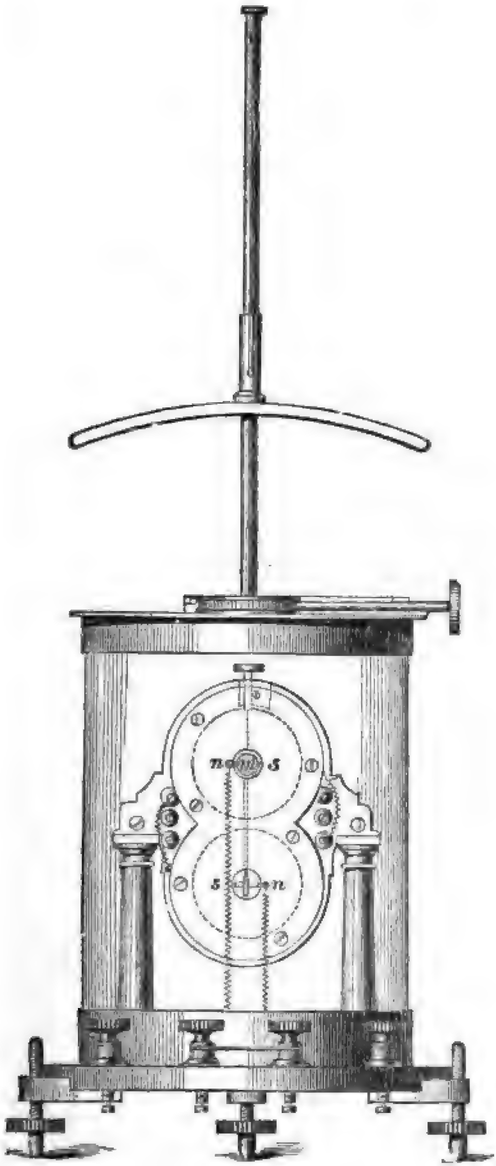
Description.

49. The galvanometer, as usually constructed, consists essentially of a very small magnetic needle, about three-eighths of an inch long, fixed to the back of a small circular mirror, whose diameter is about equal to the length of the magnet. This mirror, which is flat and is formed of microscopic glass, is suspended from its circumference by a cocoon fibre devoid of torsion, the magnetic needle being at right angles to the fibre. The mirror is placed in the axis of a large coil of wire, which completely surrounds it, so that the needle is always under the influence of the coil at whatever angle it is deflected to. A beam of light from a lamp placed behind a screen, about three feet distant from the coil, passes through a lens, falls on the mirror, and is reflected back on to a graduated scale placed just above the point where the beam emerges from the lamp. The scale is, as we have before said, straight, and is usually graduated to 360 divisions on either side of the zero point.

It is not absolutely necessary that the working zero be the middle or zero point of the scale, it is a very common practice to adjust the instrument so that the reflected beam of light normally falls near the end of the scale; by this adjustment an extreme range of 360 × 2, or 720 divisions, can be obtained.

50. The Thomson galvanometer is made in a variety of forms; Fig. 25 gives a front, and Fig. 26 a side elevation (with glass shade, &c., removed) of one very common pattern.

It consists of a base formed of a round plate of ebonite, provided with three levelling screws; two spirit-levels, at right



Front Elevation. ¼ real size.

Fig. 25.

angles to one another, are fixed on the top of this plate, so that the whole instrument can be accurately levelled: sometimes one circular level only is provided, but the double level is much the best arrangement.

From the base rise two brass columns, between which a brass plate is fixed, rounded off at the top and bottom. Against the faces of this plate are fixed the coils (c, c, c, c) of the instrument. The brass plate has shallow countersinks on its surface for the faces of the coils to fit into, so that they can be fitted in their correct places without trouble or danger of shifting. Round brass plates press against the outer surfaces of the coils by means

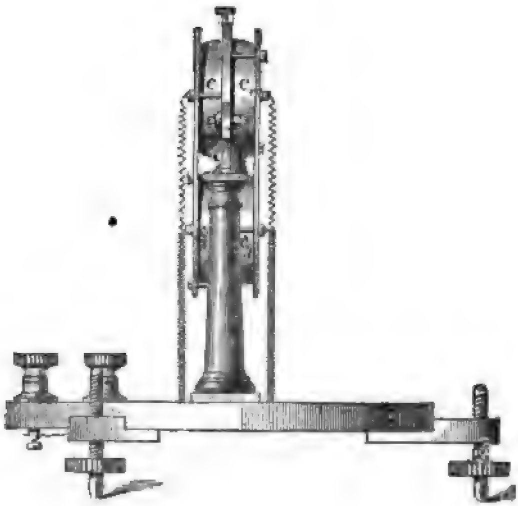


FIG. 26.—Side Elevation. (*Shade removed.*) ½ real size.

of screws, and keep them firmly in their places. There are two round holes in the brass plates coinciding with the centre holes in the coils.

The coils themselves, which are four in number, are wound on bobbins of thin insulating material, the wire being heaped up towards the cheek of the bobbin which bears against the brass plate. This heaping up is done in accordance with a formula worked out by Lord Kelvin, so as to obtain, as far as possible, a maximum effect out of a minimum quantity of wire; it is questionable, however, if the advantage of this heaping up is an appreciable one. The edges of the coils are covered with a protecting coating, so as to guard the wire from injury.

Within the holes in the brass plate are placed two small magnets, *n s* and *s n*,* formed of watch-spring highly magnetised; they are connected together by a piece of aluminium wire, so as to form an astatic pair of needles. A small groove is cut in the brass plate, between the upper and the lower hole, for the aluminium wire to hang freely in.

An aluminium fan is fixed at right angles to the lower needle; this fan acts as a damper, and tends to check the oscillations of the needles and to bring them to rest quickly.

Fig. 27.

Fig. 28.

In front of the top needle is fixed the mirror. It is suspended by a fibre attached at its upper end to a small stud which can be raised or lowered when required; when this stud is pressed down as far as it will go the needles rest on the coils, and the tension being taken off the fibre, there is no danger of breaking the latter by moving the instrument.

One end of each coil is connected to one of the four terminals in front of the base of the instrument, the other ends being con-

* In the more recent instruments it is usual to have several small magnets placed one above the other at a short distance apart, in the place of a single magnet.

nected to one another through the medium of the small terminals placed midway on either side of the coils.

The connections are so made, that when the two middle terminals on the base of the instrument are joined together the whole four coils are in the circuit of the two outer terminals, so that they all four act on the magnetic needles.

Referring again to Fig. 25; over the coils a glass shade is placed, from the middle of the top of which a brass rod rises. A short piece of brass tube slides over this rod, with a weak steel magnet, slightly curved, fixed at right angles to it. This magnet can be slid up or down the rod, or twisted round, as occasion may require. For fine adjustments a tangent screw is provided, which turns the brass rod round, and with it the magnet.

Figs. 27 and 28 show modified forms of the instrument, which, however, in general arrangement are similar to the pattern which has been described.

51. In the more recent galvanometers manufactured by Messrs. Elliott Brothers, the brass plates, which in the older instruments secured the coils in their places, are hinged to the frame, whilst the coils themselves are permanently fixed to the plates; by this arrangement the magnetic needles, with their mirror, fibre-suspension, &c., attachments, can be got at, if required, with the greatest facility. Altogether this improvement is one of the most convenient that has been made.

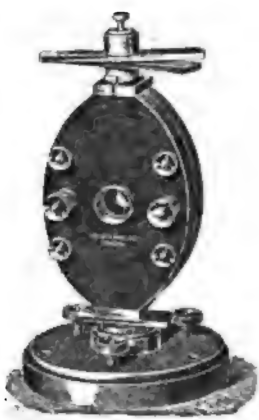


FIG. 29.

52. Fig. 29 shows a very convenient and cheap form of mirror galvanometer, manufactured by Messrs. Nalder. This instrument, without being elaborate, has a high "figure of merit." It is provided with a "scissors" control magnet, an arrangement which enables the "time of moving" of the needles to be adjusted with very great facility, and far more easily than with a single magnet.

53. About 5000 or 6000 ohms is usually the total resistance of the coils of these galvanometers, but in some cases as much as 350,000 ohms of very fine copper wire has been employed. An instrument of the latter resistance made for measuring insulation resistances (see Chapter XV.), and constructed by Messrs Nalder

Bros., is shown by Fig. 30. The arrangement is such that the very highest possible insulation of the whole instrument is obtained, the coils and terminals of the same being hung from corrugated ebonite columns, which are in turn supported by other columns. A similar form of instrument constructed by Messrs. Elliott is shown by Fig. 31.

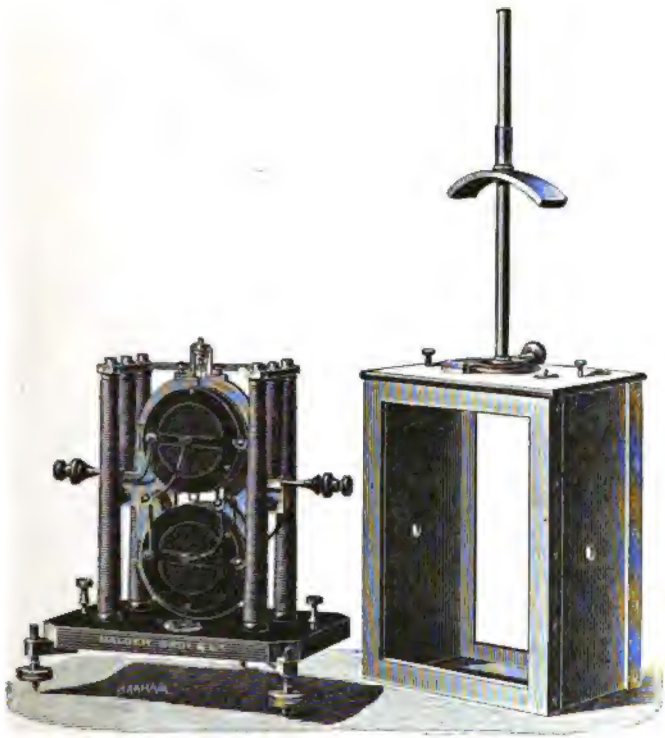


FIG. 30.

54. In factories where heavy machinery is continually at work it is often difficult to set up a mirror galvanometer so that it is unaffected by the vibration which takes place. The best method of neutralising the effects of such vibration is to sling the instrument by rubber piping. A stand specially adapted for this purpose is shown by Fig. 32. The galvanometer is placed on a platform which hangs from three rubber slings, adjustable by means of thumb-screws set on the upper part of the stand.

55. Fig. 33 shows a portable mirror galvanometer, which is very useful, especially for travelling purposes; the three legs are hinged at their junction with the lower part of the coil frame, so that they can be folded together, and thus made to occupy but little space. Owing to the instrument being provided with but

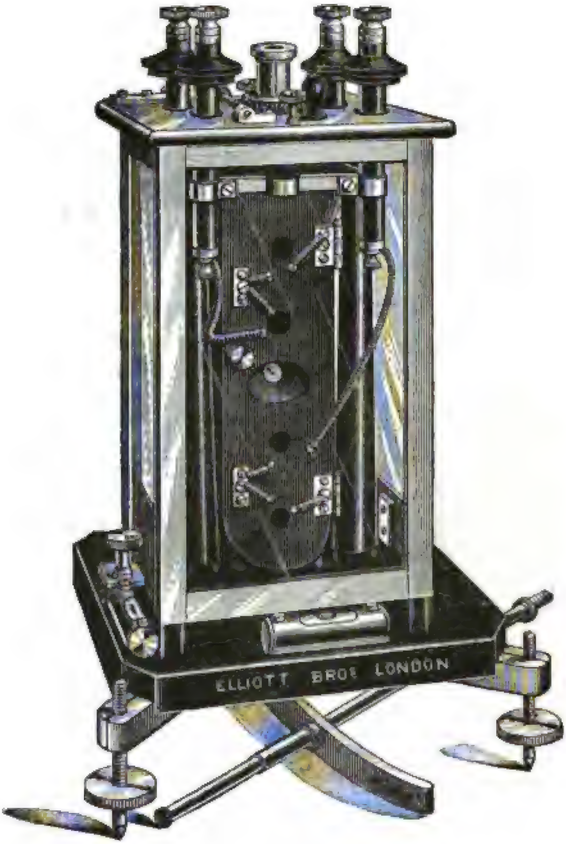


FIG. 31.

two coils (one in front of, and the other behind, the needle) its sensitiveness is not quite so great as that of the larger instruments with four coils, but for general purposes it is an excellent piece of apparatus.

56. The width of the spot of light on the screen can be regu-

lated by means of a brass slider fixed over the hole in the screen, through which the beam emerges from the lamp.

A much better arrangement than the spot of light is now provided with most instruments. The hole through which the

Fig. 32.

Fig. 33.

light emerges is made round, about the size of a sixpence, with a piece of fine platinum wire stretched vertically across its diameter. A lens is placed a little distance in front of this hole, between the scale and galvanometer, so that a round spot of light,

with a thin black line across it, is reflected on the scale. This enables readings to be made with great ease, as the figures on the scale can be very distinctly seen. When the spot of light only is

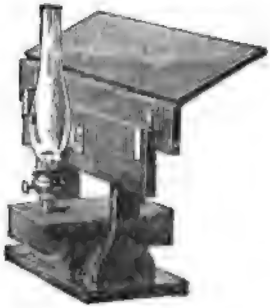


FIG. 34.

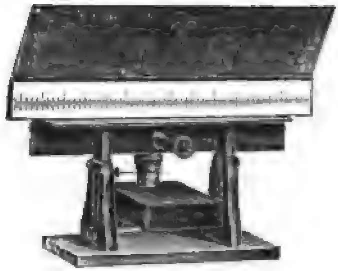


FIG. 35.

used, it is necessary to partially illuminate the scale with a second lamp. The general appearance of the scale frame with the lamp placed in position, is shown by Figs. 34 and 35.

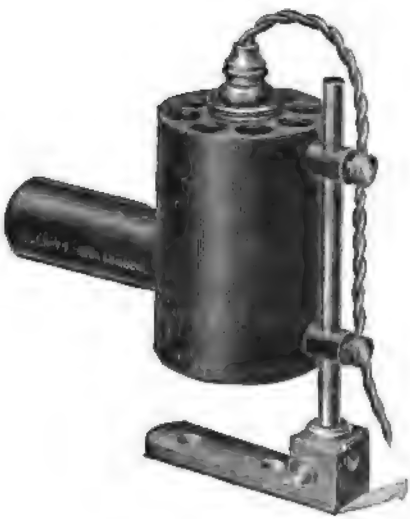


FIG. 36.

An enclosed incandescent electric lamp is sometimes used, with great advantage, in the place of the paraffin lamp. This arrangement is shown by Fig. 36.

Jacob's Transparent Scale.

57. The position of the ordinary form of scale for the Thomson's galvanometer is to a certain extent inconvenient, especially to near-sighted persons. Mr. F. Jacob has completely remedied this inconvenience by the arrangement shown in front view and cross section

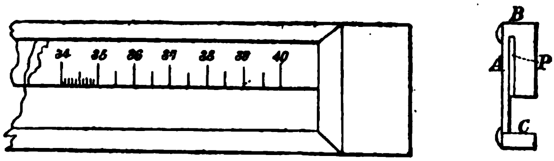


FIG. 37.

by Fig. 37. In this fig. B is a wooden scale-board with a longitudinal slot, as shown at C; P is the paper scale, cut so that all the division lines reach the lower edge; A is a slip of plane glass with its lower half finely ground from one end of the slip to the

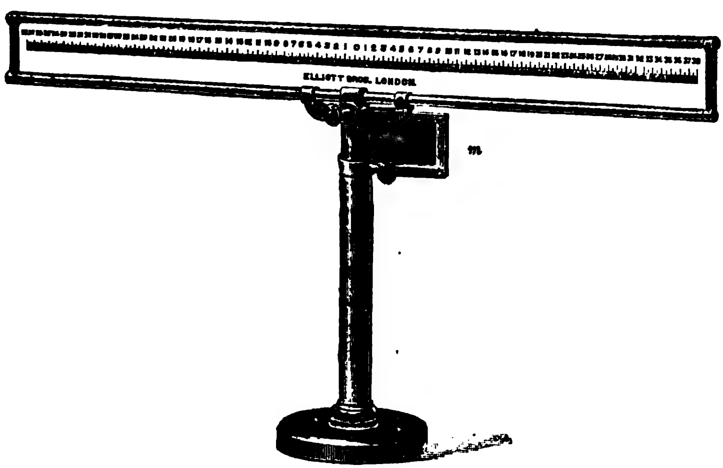


FIG. 38.

other, on the side towards C: the scale is so placed that the lower end of the division lines just touches the ground part of the glass slip. The image of the slit with a fine wire stretched across it is focussed in the ordinary manner on the ground part of the glass, and will of course be clearly seen by the observer on the opposite side of the scale; as the line and printed divisions are in the same

plane, there is no parallax, consequently a great increase in accuracy of reading the position of the air line is obtained, owing to the greater ease of observing that two lines coincide when end on to one another, than when superimposed; there is a further advantage from the fact that the room need not be darkened. The lamp and its slit is placed on one side, and reflects the beam of light on to the galvanometer by a mirror or total reflection prism, and by means of two long plane mirrors the actual distance between the galvanometer and scale is reduced, so as to have everything close to the observer's hand. The scale adopted is divided into half millimetres, and it is perfectly easy to read to a quarter of a division, and with a hand magnifying-glass still further. This arrangement has been adopted in the testing-rooms of Messrs. Siemens Brothers and Co., at Woolwich, and gives great satisfaction.

Fig. 39.

Fig. 38 shows a form of the transparent scale which is very convenient for general use. The arrangement is provided with a mirror *m* which receives the beam of light from the lamp, and reflects it on to the mirror of the galvanometer; this enables the lamp to be set at a convenient position and leaves the scale quite unincumbered. The lamp used with this scale is shown by Fig. 39, it has the advantage of being readily adjustable, and being provided with a metal chimney the annoying breakage of glasses which so often occurs in the ordinary lamp is avoided.

58. In the testing-rooms at the Silvertown Telegraph Works, the scales employed are of large dimensions, being about 5 feet long, and are set at a distance of several feet from the galvanometer. By this arrangement a greatly magnified image of the round spot of light with the black line across it is obtained, and the divisions on the scale being of correspondingly large dimensions, the readings can be made with great facility, and with very little fatigue to the eye. The only objection to the arrangement is the space which it necessarily occupies, but as it is not often that many instruments require to be set up in the same room, this need hardly be taken into account.

To Set up the Galvanometer.

59. It is essential, before proceeding to set up the instrument for use, to see that the ebonite base is thoroughly dry and clean, so that there may be no leakage from the wires to interfere with the tests taken. Indeed, it is as well to place the galvanometer and the other apparatus to be used on a large sheet of gutta-percha or ebonite, more especially if the room in which the tests are to be made is at all damp. Sometimes small ebonite cups are provided for the levelling screws of the instrument to stand in, which answers the purpose of insulating very thoroughly.

The instrument should be set up on a very firm table in a basement story. It is almost useless to test with it in an upper room, as the least vibration sends the spot of light dancing and vibrating to and fro. At all cable works the instrument is placed on a solid brick table built on the earth so that no vibration can possibly affect it.

A suitable table being chosen, set the galvanometer in any convenient position, and adjust the levelling screws until the bubbles of the level or levels show the instrument to be perfectly level.

Now remove the glass shade, and gently raise the stud at the top of the coils by squeezing the tips of the fingers between the head of the stud and the top of the brass plate in which it runs. If the stud is raised by a direct pull, there is almost a certainty of its coming up with a jerk and breaking the fibre. On no account must the stud be twisted round, except to get rid of any torsion which may exist in the fibre when it has been replaced after becoming broken.

The stud being raised sufficiently high to allow the mirror to swing clear of the coils, replace the glass shade, screw the brass rod with the magnet, on to its top, and set the magnet about half-way up the rod, the poles being placed so as to assist in keeping the magnetic needles north and south.

The scale lamp being lighted, place it in position on the scale stand, the *edge* of the wick being turned towards the brass slider which regulates the width of the beam of light. Having opened the slider to its full extent, the scale and lamp should be placed about 3 feet from the galvanometer, so that it stands parallel with the faces of the coils, and so that a line drawn at right angles to the scale from the lamp-hole will pass through the centre of the galvanometer. The reflected beam of light should then fall fairly on the scale. If too high, this may be remedied by propping up

the scale, and if too low, by screwing up the levelling screws of the galvanometer. Should the light be too high on the scale, it will be found an easier matter to prop up the scale than to lower the galvanometer by means of the levelling screws.

The spot of light should now be set at the zero point on the scale by turning the regulating magnet by means of the screw; the spot should next be focussed, by advancing or retreating the lamp and scale and by adjusting the lens-tube, until a sharply defined image is obtained on the scale. The width of the slit may then be diminished, by means of the brass slide, until a thin line of light only is obtained on the scale. If the round spot of light with the line across it is used, the focussing must be made so that the black line is sharply defined.

The position of the scale and galvanometer being once obtained, their positions on the table may be marked for future occasions, or, at least, the exact distance of the scale from the galvanometer noted, so that it can be placed right without trouble.

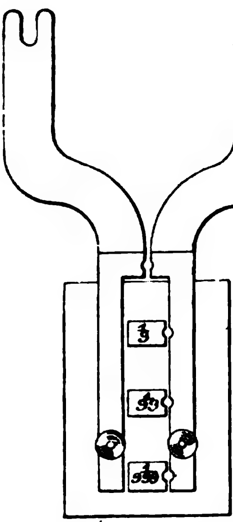
The instrument being now ready for use, if it is not required to be sensitive, place the regulating magnet low down; if, on the contrary, it is required to be sensitive, place it high up.

60. To obtain the *maximum sensitiveness*:—Raise the magnet to the top of the bar, and then turn it half round, so that its poles change places. The magnet will now be opposing the earth's magnetism, and consequently will tend to turn the magnetic needles round. If the magnet is at the top of the rod, the effect of the magnetism of the earth on the magnetic needles will be more powerful than the magnetism of the regulating magnet, and the needles will tend to keep north and south; but by placing the regulating magnet lower down, a point is reached where the earth's magnetism is just counteracted. Under these conditions the needles will stand indifferently in any position. By placing the regulating magnet about an inch higher than the position which gives this exact counteraction, the magnetism of the earth will be just sufficient to keep the magnets north and south, and consequently the spot of light at the zero on the scale, and at the same time leaves the magnets free to be moved by a very slight force. It will be noticed with the regulating magnet in this position, that in order to get the spot of light at the zero point, the magnet must be turned in the opposite direction to that in which it is required that the needles should move.

It is not advisable to adjust the instrument too sensitively, because it is difficult then to keep the spot exactly at zero, as any slight external action may throw it a degree or two out.

61. The presence of iron near the instrument is not prejudicial to its correct working, so long as the metal remains stationary. The experimenter should, however, remove any keys or knives he may have about him, as they very much affect the galvanometer if he moves about much. These precautions may seem too minute, but as the very object of the Thomson galvanometer is to enable measurements to be made with accuracy, all likely causes of disturbance should be avoided.

62. A resistance box, containing three *shunts*, is usually provided with the galvanometer of the values $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th of



½ real size

Fig. 40.

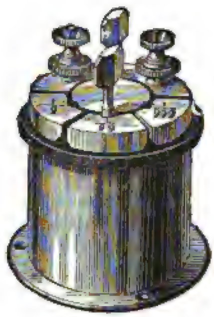


Fig. 41.

Fig. 42.

the resistance of its coils, which values, as will be shown in the next chapter, enable the sensitiveness of the instrument to be reduced to its $\frac{1}{10}$th, $\frac{1}{100}$th, and $\frac{1}{1000}$th part respectively.

Fig. 40 shows a form of this shunt. By inserting a plug into one or other of the holes, the required shunt is inserted.

The numbers are sometimes marked as $\frac{1}{10}$th, $\frac{1}{100}$th, and $\frac{1}{1000}$th, instead of $\frac{1}{9}$th, $\frac{1}{99}$th, $\frac{1}{999}$th, thereby indicating that the particular shunt reduces the deflections of the needle to that particular fraction, but they have really just the same adjustment in both cases.

The shunts are sometimes enclosed in a round brass box, as

shown by Fig. 41, which is perhaps a more portable and elegant form than that shown by Fig. 40.

For the galvanometer, shown by Fig. 30, p. 55, a special form of shunt box highly insulated has been designed. This is shown by Fig. 42.

The two broad strips of copper shown in Fig. 40 are used for the purpose of connecting the box with the galvanometer. The blank plug-hole is for the purpose of short-circuiting, which should always be done when the instrument is not actually in use.

Method of Fixing a Suspension Fibre.

63. The operation of replacing a broken suspension fibre is by no means an easy one, and unless attempted in a systematic way it is an operation which is exceedingly trying to the most even temperament; indeed it is not at all unusual for the attempt to be given up and the instrument maker to be resorted to in order to get the new fibre inserted. The following method, however, though perhaps it may not be one which is generally adopted (each maker having his own preference), answers very satisfactorily:—

Take a reel of copper wire (Fig. 43), the diameter of the latter being about 10 mils (·01 inch), and, if covered, remove the silk

FIG. 43.

covering for an inch or more, and then varnish the wire to about ¼ inch of the extremity with a little thick shellac varnish. Whilst the wire is still wet, wind the fine silk fibre with tweezers once or twice round the wet end, and then evaporate with a heated iron,

FIG. 44.

the shellac thus securing the silk firmly. With nippers, cut off about ½ inch of the wire; also treat the other end of the silk in a similar manner (Fig. 44).

Use now each of these wires as a sewing-needle would be used

Pass one of them through the eyelet of the astatic system and make a neat knot. In performing the operation have in each hand a pair of tweezers for the manipulation; never use the fingers. Next, in a similar manner fasten the other end of the fibre to the suspension pin (a horizontal suspension screw if the latter is used). *Shellac or shellac-varnish should never be used to fix the knots.* The background against which the operation is carried out should be black; a slab of ebonite answers well. It is important to remove any burr on the suspended eyelets or holes, to prevent cutting the fibre. With a steady hand and a little practice any one should be able to suspend with a single fibre.

The Thomson Dead-beat Galvanometer.

64. The needle of an ordinary form of galvanometer, when under the influence of a constant current, does not settle down at once to the angle of deflection which it eventually takes up, but oscillates to and fro several times before it finally comes to rest, and again it acts in the same way when the current is taken off and the needle returns towards the zero point. The oscillations often cause considerable inconvenience and loss of time in testing, and the object of the *dead-beat* galvanometer is to get rid of these inconvenient movements.

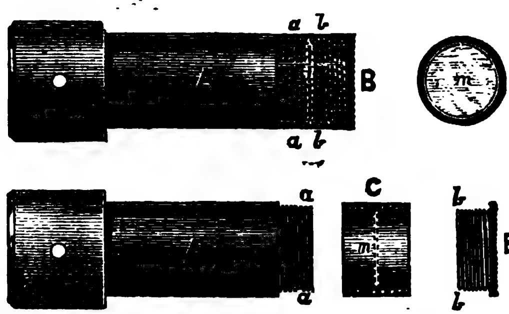


Fig. 45.

Fig. 45 shows the arrangement devised by Lord Kelvin for obtaining a dead-beat movement.

A is a brass tube, whose end *a a*, which is screwed, is closed by a piece of glass. B is a short piece of tube, which is screwed, and whose end *b b* is similarly closed by a piece of glass. C is a third short piece of tube, into which the ends of A and B screw. The length of this tube is such that when the whole arrangement

is united together there is a very small space between the ends *a a* and *b b*; a small air-tight cell, in fact, is formed.

Hanging midway inside C is a mirror *m*, with a magnetic needle fixed to it, as in the ordinary Thomson galvanometer. This mirror very nearly fits inside the tube, there being only just room for it to swing freely; it is suspended by a very short fibre.

The space between *a a* and *b b*, although very small, is just sufficient to enable the mirror to turn through an angle large enough to give a good deflection of the spot of light on the scale.

The complete arrangement is inserted in the centre of a single galvanometer coil, so that the mirror occupies the same position that it does in the ordinary galvanometer.

Owing to the air inside the cell being so closely confined, the violent movement of the mirror is checked when it is acted upon by a current passing through the coils, and the consequence is that the mirror, instead of overshooting the mark and then recoiling, turns with a gradually decreasing velocity towards its final deflection, and ceases to move when the latter is reached. The same thing takes place when the current is cut off; in this case the spot of light moves back to zero and ceases to move at that point.

The suspension fibre being very short, the mirror cannot turn so freely as the one in the ordinary galvanometer; its sensitiveness is therefore not quite so great, but it is sufficiently so for most purposes for which the latter would be used.

The fibre is very easily replaced when broken. One end being attached to the mirror, the other is passed through a small hole in the side of C, and is then drawn sufficiently tight to suspend the mirror inside the tube so that it does not touch the sides, a drop of shellac is then applied to the hole, which closes it and fixes the fibre. In some cases the cell is filled with paraffin oil, which still further tends to check the movement of the mirror.

The Thomson Marine Galvanometer.

65. This instrument is specially constructed for use on board ship, where the rolling of the vessel and the constant movement of masses of iron about would render an ordinary mirror galvanometer quite useless.

Fig. 46 shows a side view of this instrument, the upper part being drawn in section.

C C C C are the coils, which are similar in form to those employed in the ordinary Thomson galvanometer; there is, how-

ever, but one set, of two coils, instead of two sets as in the latter instrument.

The mirror, with the magnetic needle fixed to its back, is strung on a cocoon fibre in a brass frame. The fibre is fixed at

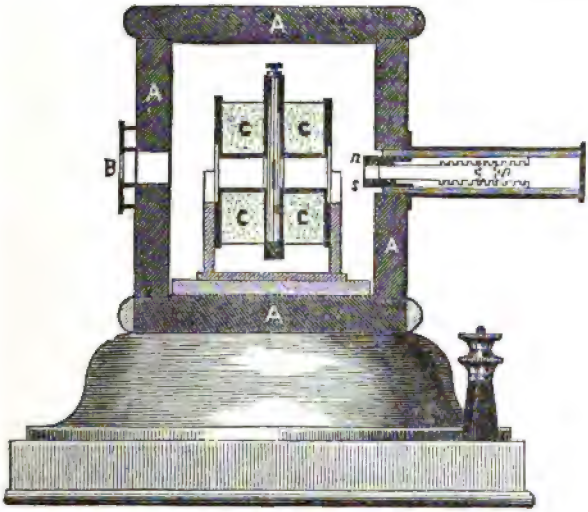


FIG. 46.

one end, and at the other is attached to a spring, which draws the fibre tight. The frame slides in a groove between the coils, so that it can be drawn out for the purpose of repairing the fibre. A powerful directing horse-shoe magnet (not shown in the figure) embraces the upper parts of the coils, and serves to overpower the directive effect of the earth's magnetism. This latter effect is still further rendered harmless by enclosing the whole system in a massive soft-iron case A A A A, a small window B being left, through which the rays of light reflected by the mirror enter and return.

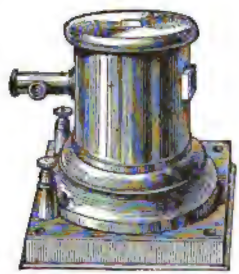


FIG. 47.

For obtaining exact adjustment of the spot of light to zero, two small magnets, *n* and *s*, as broad as the mirror magnet is long, are provided; by turning the pinion *p* these magnets can be made to advance or retreat, and so act on the mirror magnet to make it turn in one direction or the other, as required.

The resistance of this form of galvanometer (which is shown in general view by Fig. 47) is usually as high as 30,000 or 40,000 ohms.

The D'Arsonval-Deprez Dead-beat Galvanometer.

66. The main peculiarity of this instrument lies in the fact that, whereas in almost all galvanometers there is a fixed coil and a movable magnetic needle, in this galvanometer the coil is

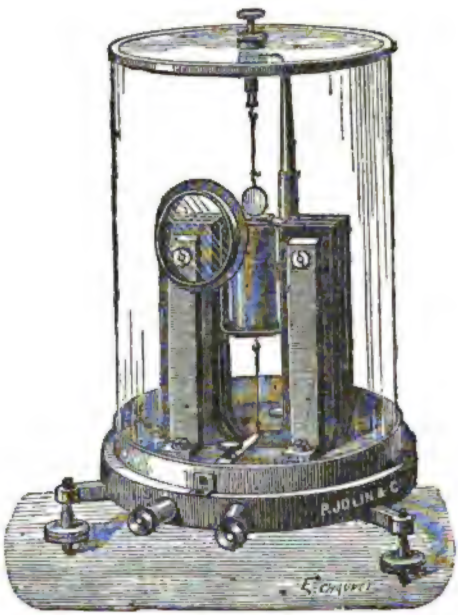


Fig. 48.

movable and the magnet—a massive compound horse-shoe of steel—is fixed. Fig. 48 represents the instrument itself, as manufactured by P. Jolin & Co., of Bristol. The steel magnet, made of three thin horse-shoes, each magnetised as strongly as possible, is firmly fixed to a metal base, with its poles upwards. Between the poles hangs the coil, which is rectangular in form, and weighs only a few grains; it is held in its place by a thin silver wire above and another thin silver wire below. The coil is made by winding the wire on a continuous rectangular frame made of copper or

silver as thin as possible; this frame, by the reactive effect of the induced currents which the movement of the coil sets up, causes the latter when deflected to come rapidly to rest.

To reinforce the magnetic field, a strong compound magnet of cylindrical shape is arranged so that the laminations are in a horizontal direction, and so that its north pole comes opposite the south pole of the horse-shoe magnet; it is placed in the hollow of the suspended rectangular coil without touching it, and is firmly fixed; the coil is then free to turn in the very narrow space between the compound magnet core and the external magnet-poles; and it need hardly be added that this contrivance produces a very intense magnet field. The current is led in by one of the silver sus-

FIG. 49.

pension wires, and leaves the coil by the other. A small mirror of 1 metre focus is fixed to the suspended coil; a brass spring at the bottom keeps the suspending wires stretched; and a screw-head at the top of the instrument serves both to regulate the tension in the wires and to let the coil down to a position of rest on the central iron cylinder, whenever the galvanometer is to be dismounted for removal to a distant place. The resistance of the coil is made from 150 to 750 ohms in the ordinary pattern of instrument.

As there is no suspended needle, no external magnetic forces affect the zero of the instrument; and, since the position of the coil is determined solely by the elasticity of the suspending wires and the magnetic action of the fixed magnet on the current in the coil, it can be used in any position, and is independent of the

earth's magnetic field. It can even be placed quite near to a dynamo-machine, and is therefore a most useful, and indeed an indispensable, type of instrument for working under such conditions as are met with in manufactories of electric-light plant.

The intensity of the magnetic field in which the coil is situated is such that whenever the galvanometer-circuit is closed—even through a considerable resistance—the motion of the needle is dead-beat. It takes less than *one second* to come to rest at its final position of deflection, and when it returns to zero it does so with the most complete absence of oscillations. Altogether, the form of instrument is an extremely satisfactory one.

A more recent pattern of this form of galvanometer, manufactured by Messrs. Nalder Brothers, is shown by Fig. 49.

The Sullivan Universal Galvanometer.

67. The special features which distinguish this most useful form of galvanometer, and which render it equally suitable for ship or shore work, are—

Independence of external fields.

It can be mechanically damped at will to any required degree.

Freedom from electrostatic effect with low or high voltages.

Equal applicability for measuring and signalling purposes.

The suspended coil system can be readily and accurately balanced so as to be unaffected by the rolling or pitching of a ship.

The damping device is a very fine camel's-hair brush, so adjustable that it can be made to more or less closely envelope a portion of the suspension, according to the degree of damping required. Freedom from the electrostatic effect (due to the fact that the insulated coil and the magnet and brasswork of the galvanometer form a Leyden or air condenser) is secured by so arranging that the coil and its surrounding magnet, &c., are charged to the same potential by the testing current, suitable means being taken to prevent surface leakage and loss. To obtain this Mr. Sullivan connects a fine wire between one end of the coil and the magnet, and carefully insulates the entire instrument from its wooden base, on a thick ebonite slab. This entirely remedies all static charge effect and ensures perfect insulation.

Used for speaking purposes, the instrument gives deadbeat signals on even a mile of cable or non-inductive line; this is a convenience in communicating through cables widely differing in length, a frequent necessity in cable operations.

The balancing arrangement consists of a fine leaden wire half

an inch long, soldered at one end to the coil system and projecting branchwise from the same. By bending this radial arm in the requisite direction, the centre of gravity is brought, and kept, exactly in line with the suspensions, steadiness being thus secured at any angle of roll or pitch of the ship.

By making the lower suspension arm of ebonite and fitting on to its lower side two platinum contact studs which are in connection with the two ends of the coil through its metallic suspensions and frame (and which when the frame is slid into position press upon two insulated springs in the hollow base of the instrument, and in permanent connection with the terminals), connecting wires

FIG. 50.

between the suspension frame and the terminals are dispensed with. By this arrangement the frame, in case of accident, can be changed in a few seconds.

The action of the camel's-hair brush is as follows:—The innumerable points of minute friction which it offers to the suspension, check the deflection of the coil until the received impulses have attained a potential sufficient to ensure a decided movement of the coil, thereby preventing on long cables the otherwise inevitable gliding or blending of successive signals of like sign into one another. In the case of shorter lengths the camel's-hair brush is not needed, and it may be drawn back clear of the suspension, the electro-magnetic damping of the coil-former or coil-frame being sufficient to make the instrument dead-beat on open circuit.

Fig. 50 gives a general view (with cover removed) of the latest

pattern of the instrument, the parts being shown in Figs. 51 and 52.

As regards the balancing device, referring to Fig. 52, it will be observed that the coil has *two* leaden arms attached to it—the one at the front and the other at the back—although if the coil be exactly wound and suspended, the weight of the mirror must of necessity over-weight it in front. The object is to secure a dynamic balance of the coil, and to allow of the final adjustments being more conveniently effected by means of the more accessible front wire (L W_1). It will be noted that, given exact equiponderance around the axis of rotation, L W must be heavy enough to counterbalance the mirror and L W_1 combined.

In adjusting the leaden arms, no shellac, tools, &c., are required, and the suspension frame need never be removed from the instru-

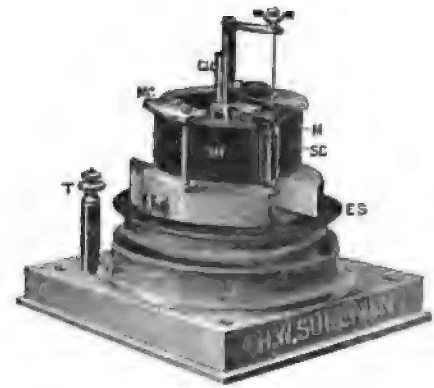


FIG. 51.

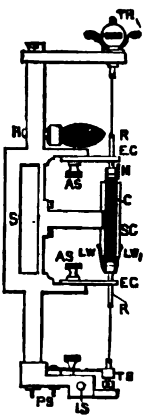


FIG. 52.

S M, C-shaped steel magnet. M C, magnet clamps. E S, ebonite slab. H B, hollow base. C U, cylindrical brass upright on which the suspension frame slides. C, rectangular coil wound on a metallic former. S C, fixed soft iron core. M, mirror. T, terminal. S, sleeve or tube part of suspension frame, which sleeve or tube fits over C U. H, Hole for the holder of camel's hair damping brush D B. P S, insulated platinum studs on lower arm of suspension frame. I S, ivory adjustment screw projecting on both sides of lower suspension arm; this screw fits smoothly between brass guide-plates fixed on H B, and is adjustable laterally for centering the coil system in the magnet gap. A S, adjustment screws for ebonite collars (E C, E C L) through which suspension rods R projecting above and below from the coil pass, without touching. L W, L W_1, leaden wire radial arms for balancing coil. T H, adjustable torsion heads for regulating the tension of the suspension wires.

ment; it will suffice to raise it a little on its cylindrical upright for each fresh adjustment so as to allow of the coil and its radial arms just clearing the magnet gap and being readily handled.

Owing to its perfect balance the instrument is unaffected not only by rolling and pitching, but also in a marked degree by severe vibratory influences, having been found to work well on a torpedo boat in a rough sea.*

With the comparatively stout and rigid suspensions required on board ship, the sensibility exceeds that of an ordinary marine galvanometer (§ 65), while for more delicate measurements on shore "figures of merit" (§ 78) up to that of the ordinary form of Thomson instrument (§ 47) are obtainable by suspending the coil more or less finely and from above only, connection with the bottom end being maintained by a fine non-directive wire spiral.

Its low resistance (the coil is wound ordinarily to about 800 ohms) and high sensibility render this form of galvanometer specially valuable in tests by the Wheatstone bridge or null method.†

Angle of Maximum Sensitiveness—Inferred Zero.

68. The *angle of maximum sensitiveness* (page 25) in a mirror galvanometer is the largest deflection which can be obtained, as the angle of deflection is but a very few *degrees* and, consequently, the true maximum angle can never be reached.

69. By turning the controlling magnet of the instrument so that the needle is turned through a large angle, the normal zero becomes at a considerable distance off the scale, and the sensitiveness of the galvanometer to changes in the current strength producing a deflection, can be made very great. Thus, supposing the needle to be normally at the ordinary zero, and suppose that a current caused it to deflect to 350 divisions, then an increase in the current of say 1 per cent. would increase the deflection of $\frac{350 \times 101}{100}$, or, 353·5; that is, would increase it 3·5 divisions. If now the working zero had been 350 divisions to, say, the left of the ordinary zero, and if the current had been strong enough to produce a deflection of 350 divisions to the right of the ordinary zero, then the deflection would be equivalent to 350 + 350, or 700 divisions, and an increase in the current of 1 per cent. would

* See 'Electrical Review,' April 29, 1898.

† See an article by Mr. W. J. Murphy, 'Electrician,' August 26, 1898.

increase the deflection to $\frac{700 \times 101}{100}$, or 707, that is to say, an increase of 7 divisions. If, lastly, the controlling magnet is turned so that the needle has a zero equivalent to, say, 2000 divisions to the left of the ordinary zero, that is an *inferred* zero, as it is called, of 2000, then if the needle were deflected to the right by a current sufficiently strong to bring the deflection on the scale, and to give it a value of 350 to the right of the ordinary zero, the deflection representing the current would be 350 + 2000, or 2350 divisions, and an increase in the current of 1 per cent. would increase the deflection to $\frac{2350 \times 101}{100}$, or, 2373·5, that is to say, an increase of 23·5 divisions. In actual practice it is often possible to use an inferred zero considerably greater than 2000, and with corresponding advantage.

The Ballistic Galvanometer.

70. The object of this instrument is the measurement of transient currents such as are produced by the discharge from a condenser, or currents set up by electro-magnetic induction.

The principle of the instrument depends upon the arrangement being such that the movement of the magnetic needle does not practically take place until the transient current has ceased. In the great majority of cases met with in ordinary testing this condition is satisfied approximately in the ordinary Thomson mirror galvanometer, since the discharge from a condenser of low capacity or from a moderate length of cable, such as would have to be measured in practice, is extremely brief. Of recent years, however, experimental work in connection with dynamo machinery has required the measurement of transient currents of comparatively long duration, hence the use of the ballistic galvanometer has become more general.

In the ballistic instrument the object required is effected either by enclosing the magnetic needle or needles in a hollow spherical bullet, or by having each needle formed of a cylindrical shape as shown (drawn to an enlarged scale) by Fig. 53.

The needle, it will be seen, is formed of a hollow steel thimble which is slit down its length, thus forming a horse-shoe with semi-cylindrical legs; the thimble is fixed on an axis *a b*, which axis may have several of these thimbles, i.e. two with their similar poles set opposite each other in the centre of each coil and others outside, but in close proximity to the coils (Fig. 55, page 77).

In whatever way the needles are arranged the object aimed at is that the motion of rotation shall be as little retarded by the air resistance as possible.

FIG. 53.

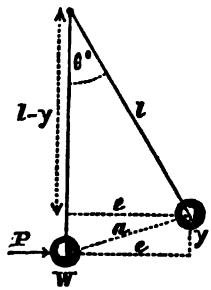


FIG. 54.

71. The principle of the galvanometer may be explained as shown by Fig. 54. Let W be a weight suspended from a string, and let this weight be pushed by a pressure P applied for a very short time t. The velocity v with which W will move at the end of the time t will be

$$v = f\,t;$$

but

$$f = \frac{P\,g}{W}$$

where g is the coefficient of terrestrial acceleration, therefore

$$v = \frac{P\,g}{W}\,t,$$

and

$$v^2 = \frac{P^2\,g^2}{W^2}\,t^2.$$

Now the work accumulated in W is

$$\frac{v^2\,W}{2\,g},$$

which equals

$$\frac{P^2\,g^2\,t^2}{W^2} \times \frac{W}{2\,g} = \frac{P^2\,g\,t^2}{2\,W}.$$

The work done when the weight has moved through the angle θ° is

$$W\,y, \qquad [A]$$

therefore

$$\frac{P^2\,g\,t^2}{2\,W} = W\,y,$$

or

$$\frac{P^2\,g\,t^2}{2\,W^2} = y.$$

Now

$$a^2 = y^2 + e^2 = y^2 + l^2 - (l - y)^2$$
$$= y^2 + l^2 - l^2 - y^2 + 2\,l\,y,$$

that is,

$$a^2 = 2\,l\,y,$$

or

$$y = \frac{a^2}{2\,l}; \qquad [B]$$

therefore

$$\frac{P^2\,g\,t^2}{2\,W} = \frac{a^2}{2\,l},$$

or

$$\frac{P^2\,t^2}{W^2} = \frac{a^2}{g\,l},$$

in which, since θ° is very small, a represents the number of divisions of deflection.

If, now, T be the time in seconds which the weight would take to make one *complete* oscillation, i.e. one backwards and forwards movement, when swinging freely, then

$$\frac{T}{2} = \pi\sqrt{\frac{l}{g}}, \quad \text{or} \quad \frac{1}{g} = \frac{T^2}{4\,\pi^2\,l},$$

therefore

$$\frac{P^2\,t^2}{W^2} = \frac{a^2\,T^2}{4\,\pi^2\,l^2},$$

or

$$P\,t = \frac{W\,T\,a}{2\,\pi\,l}.$$

Now if we push the weight by a continual force P′ to an angle β°, then

$$P' = W\tan\beta^\circ = W\frac{b}{l}$$

if β° is very small and b is the number of divisions of deflection. We have then

$$W = \frac{P' l}{b},$$

therefore

$$P t = \frac{\frac{P' l}{b} T a}{2 \pi l} = \frac{P' T a}{2 \pi b}.$$

Now $P t$ corresponds to a current kept on for a certain time, i.e. to a discharge of so many *coulombs* (Q) * of electricity, and P′ to a permanent current of, say, A ampères, we can therefore say

$$Q = \frac{A T a}{2 \pi b}.$$

The g, it should be remarked, in the mechanical problem corresponds to the deflective force of the earth's magnetism in the case of the galvanometer.

For example.

The discharge from a condenser gave on a ballistic galvanometer a throw of 120 divisions (a). The permanent deflection

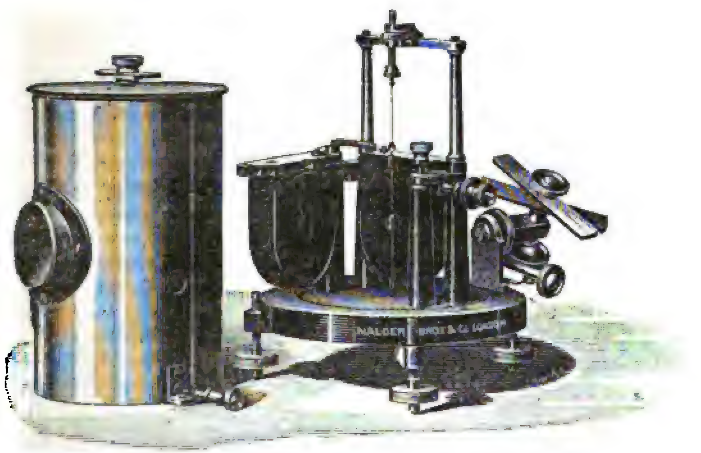


FIG. 55.

produced by a current of 50 milli-ampères (·05 ampères) was 240 divisions (b). The number of *complete* oscillations which the

* 1 coulomb is the quantity of electricity which passes during 1 second when a current of 1 ampère is flowing.

needle made in 60 seconds when swinging freely was 10, i.e. $T = \frac{60}{10} = 6$. How many coulombs did the condenser contain?

$$Q = \frac{\cdot 05 \times 6 \times 120}{2 \times 3\cdot 1416 \times 240} = \cdot 0239 \text{ coulombs.}$$

In order to determine the number of swings the needle makes in a given time the eye should be kept fixed on any point on the scale, and each time the spot of light passes that point *in the same direction* a count should be made.

72. A very convenient form of the instrument (designed by Messrs. Nalder) is that shown by Fig. 55. One of the coils is hinged, and when thrown back (as shown in the fig.) the needles can easily be got at if necessary. A clamping screw keeps the movable coil in its place when the coil is set in its normal position.

Throw Produced by a Permanent Current.

73. In some cases it is required to know to what permanent deflection the throw produced by a permanent current corresponds. This problem may be dealt with as follows:—

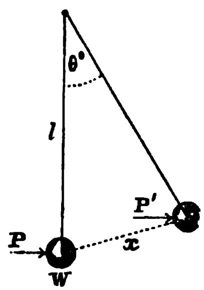


Fig. 56.

Let W (Fig. 56) be a weight urged forward by a constant pressure P, then the actual pressure P′ against W, when the latter has moved through the angle $\theta°$, will be the pressure P minus the resolved back pressure exerted by W; this back pressure will be W $\tan \theta°$, which equals $W\frac{x}{l}$ if $\theta°$ is very small, therefore

$$P' = P - W\frac{x}{l};$$

and the acceleration will be

$$f^1 = \frac{P'}{W_1} g = \left(\frac{P}{W} - \frac{x}{l}\right) g.$$

Now the movement of W is due to a continually decreasing pressure P′ acting against W, which pressure beyond the permanent deflection position becomes a retarding pressure which eventually stops the movement of W. This continually decreasing pressure is equivalent to a continually decreasing acceleration, so

that if f^1 is the acceleration after the weight has moved through a distance x, the velocity which W will have after it has moved through a distance b will be given by

$$V^2 = \int_0^b 2 f^1\, d\,x = \int_0^b 2\left(\frac{P}{W} - \frac{x}{l}\right) g\, d\,x$$

$$= 2\,g \left[{}_0^b\, \frac{P}{W} x - \frac{x^2}{2\,l}\right] = 2\,g \left[\frac{P\,b}{W} - \frac{b^2}{2\,l}\right].$$

If a be the permanent deflection when the needle has settled down to an angle a°, then

$$P = W \tan a^\circ = W \frac{a}{l}$$

if a° is small.

Therefore

$$V^2 = 2\,g \left[\frac{a}{l} b - \frac{b^2}{2\,l}\right] = \frac{g}{l}\left[2\,a\,b - b^2\right]. \qquad [A]$$

When the needle has moved to its fullest extent, then $V = 0$, hence

$$2\,a\,b - b^2 = 0,$$

or,

$$b = 2\,a,$$

that is to say, *the throw of the needle is twice the permanent deflection.*

Throw Produced by a Permanent Current Combined with a Sudden Impulse.

74. It sometimes happens that the throw of the needle is produced by a sudden discharge superimposed on a permanent current, as for instance when we have from a submarine cable a static discharge superimposed on an earth current; we may then require to know what portion of the throw is due to the discharge alone. In this case we have W urged at the moment the pressure P is applied, by an impulse which would give it, say, a velocity v. Now, if a body after moving from rest with an accelerated motion, is moving with a velocity V after it has passed through a certain distance, then if it starts off with a velocity v it will, after passing through the same distance, have a velocity V_1 such that

$$V_1{}^2 = V^2 + v^2.$$

Now, the velocity which W (Fig. 57) must have in order to move it through the angle θ_1, will be the velocity which it would acquire in falling through the distance y, that is

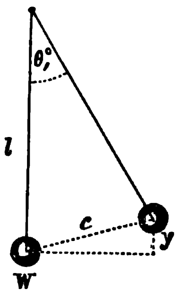


Fig. 57.

$$v^2 = 2gy;$$

but

$$y = \frac{c^2}{2l} \text{ (see [B] page 76).}$$

Therefore

$$v^2 = g\frac{c^2}{l};$$

therefore

$$V_1^2 = \frac{g}{l}\left[2ab - b^2\right] + g\frac{c^2}{l} = \frac{g}{l}\left[2ab - b^2 + c^2\right],$$

(see [A], page 79).

When the needle reaches its limiting position, then $V_1 = 0$, hence

$$2ab - b^2 + c^2 = 0,$$

or

$$c^2 = b^2 - 2ab,$$

that is

$$c = \sqrt{b(b - 2a)}. \qquad \text{[A]}$$

For example.

The discharge from a submarine cable gave a throw of 300 divisions (b), and after the needle had settled down it was observed that there was a permanent deflection of 100 divisions (a); what would the throw (c) have been had there been no current producing the permanent deflection?

$$c = \sqrt{300(300 - 2 \times 100)} = 173 \text{ divisions.}$$

(See also § 77, page 84.)

Correction for Damping.

75. The swing of the needle of a galvanometer, produced by a transient current, does not represent the actual throw due to the current, but the full vibration *damped* by the resistance of the air, &c.; if we wish to determine the actual throw which would take place provided no *damping* existed, we must make a correction. To obtain the true formula for this correction is a somewhat

difficult matter, involving an intricate calculation; the following, however, gives a closely approximate result.

Experiment has proved that all the vibrations of a damped needle are made in approximately the same time, i.e. although each vibration is less in magnitude than the one which precedes it, yet it takes the same *time* in its vibration. It is also known that the damping resistance is approximately directly proportional to the velocity with which the needle moves.

These two laws being admitted, we may deal with the problem in the following way:—

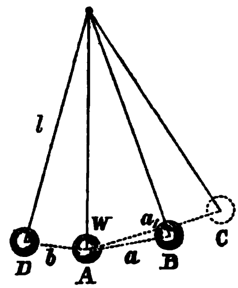


FIG. 58.

Let the transient current cause the weight W (Fig. 58) to swing from A to B, i.e. through a distance a, and let the distance through which it would have moved had there been no damping be a_1, i.e. the distance from A to C. Also let the weight, when it swings back from B, swing to D, i.e. a distance b to the left of A. Now we see from equations [A] and [B], page 76, that when there is no damping the work done in moving W through the distance a_1 is

$$\frac{W a_1^2}{2l} = K a_1^2, \text{ say,}$$

but owing to damping, W has only moved through a, hence we have

$$K a_1^2 = K a^2 + \text{resistance of damping.}$$

If t be the time taken by W in moving from A to B, then as the work done must be proportional to the resistance encountered, i.e. to

$$k \frac{a}{t},$$

where k is a constant and $\frac{a}{t}$ the velocity, and as it must also be proportional to the distance through which W moves, therefore

$$\text{resistance to damping} = k \frac{a^2}{t},$$

hence

$$K a_1^2 = K a^2 + k \frac{a^2}{t},$$

or

$$K(a_1^2 - a^2) = k\frac{a^2}{t}. \qquad [1]$$

Similarly the work done by W in moving from B to A must be

$$k\,a^2,$$

and the resistance encountered

$$k\frac{a^2}{t_1};$$

for W will not move back from B to A in the same time that it moves from A to B (actually it will move back slower); hence the energy left when W arrives at A will be

$$K\,a^2 - k\frac{a^2}{t_1}.$$

This remaining energy has to move W from A to D, that is to say, it has to do the work

$$K\,b^2 + k\frac{b^2}{t_2},$$

that is,

$$K\,a^2 - k\frac{a^2}{t_1} = K\,b^2 + k\frac{b^2}{t_2},$$

or

$$K(a^2 - b^2) = k\left(\frac{a^2}{t_1} + \frac{b^2}{t_2}\right). \qquad [2]$$

Dividing [2] by [1] we get

$$\frac{a^2 - b^2}{a_1^2 - a^2} = \frac{\frac{a^2}{t_1} + \frac{b^2}{t_2}}{\frac{a^2}{t}} = \frac{a^2\frac{t_2}{t_1} + b^2}{a^2\frac{t_2}{t}}.$$

Now, the time taken by W in moving from B to A will be to the time taken by it in moving from A to B, as a to b, hence we have

$$\frac{t_1}{t_2} = \frac{a}{b}, \quad \text{or} \quad \frac{t_2}{t_1} = \frac{b}{a}.$$

Further, the ratio of the magnitude of one vibration to the magnitude of the next is always a constant quantity, and it follows from this that the ratio of the two halves of one vibration on

either side of zero will be in the same proportion, i.e. if ρ be this ratio

$$\rho = \frac{a}{b}, \quad \text{or} \quad b^2 = \frac{a^2}{\rho^2}, \quad \text{and} \quad \frac{b}{a} = \frac{1}{\rho}.$$

Again, since the times of all the complete vibrations are the same, and since the ratio of the halves of the vibrations on either side of zero is the same, therefore the times of all the commencing halves of the vibrations must be the same (and also, of course, the times of all the concluding halves), hence we must have

$$t = t_2, \quad \text{or} \quad \frac{t_2}{t} = 1.$$

We therefore have

$$\frac{a^2 - b^2}{a_1^2 - a^2} = \frac{a^2 \frac{b}{a} + b^2}{a^2},$$

or

$$\frac{a^2 - \frac{a^2}{\rho^2}}{a_1^2 - a^2} = \frac{\frac{a^2}{\rho} + \frac{a^2}{\rho^2}}{a^2};$$

therefore

$$\frac{a^2 \rho^2 - a^2}{a_1^2 - a^2} = \frac{a^2 \rho + a^2}{a^2} = \rho + 1.$$

therefore

$$a^2 \rho^2 - a^2 = a_1^2 (\rho + 1) - a^2 \rho - a^2;$$

therefore

$$a_1^2 (\rho + 1) = a^2 \rho (\rho + 1);$$

therefore

$$a_1^2 = a^2 \rho,$$

or

$$a_1 = a \sqrt{\rho}.$$

76. If the damping is very decided, then if we note the amplitude of any two successive vibrations, the larger divided by the

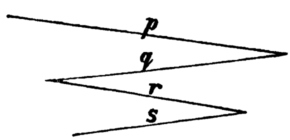


Fig. 59.

smaller gives us ρ. If, however, the damping is small, then it may be necessary to take a large number of vibrations, observing the

amplitude of the first and last, and also the number of vibrations made; from these we may calculate the value of ρ.

Let p, q, r, s (Fig. 59) represent a series of diminishing vibrations, then—

$$\begin{array}{llll} \text{1st vibration} & = & & p \\ \text{2nd} \quad ,, & = q = & & \dfrac{p}{\rho} \\ \text{3rd} \quad ,, & = r = & \dfrac{q}{\rho} = & \dfrac{p}{\rho^2} \\ \text{4th} \quad ,, & = s = & \dfrac{r}{\rho} = & \dfrac{p}{\rho^3} \\ n\text{th} \quad ,, & = z & = & \dfrac{p}{\rho^{n-1}}; \end{array}$$

therefore

$$\rho^{n-1} = \frac{p}{z},$$

or

$$\rho = \left(\frac{p}{z}\right)^{\frac{1}{n-1}};$$

that is,

$$\sqrt{\rho} = \left(\frac{p}{z}\right)^{\frac{1}{2(n-1)}}.$$

For example.

The first of a series of vibrations of the needle of a galvanometer was 280 divisions (p), and the 5th vibration was 240 divisions (z); what was the correction co-efficient, and what would be the throw on the instrument when undamped, supposing the throw when damped to be 310 divisions?

$$\text{Correction coefficient} = \left(\frac{280}{240}\right)^{\frac{1}{2(5-1)}} = 1 \cdot 0195;$$

hence the corrected throw is $310 \times 1 \cdot 0195 = 316$ divisions.

77. In the case of formula [A] page 80, the correction coefficient must be applied to b only and not to a, as the latter being a permanent deflection is of course not affected by the damping; the result worked out from the formula gives the undamped throw due to the discharge only.

Figure of Merit of Galvanometers.

78. The "figure of merit" of any galvanometer is the strength of current which will produce one division or degree of deflection. In order to find this current, we have simply to join up the galvanometer in circuit with a battery of a known electromotive force, and a resistance of a known value, and then note the deflection obtained; from this we can easily calculate the current required to produce 1 degree of deflection; thus, for example, if we had a tangent galvanometer which gave a deflection of 50° with a 10-cell Daniel battery, that is, with an electromotive force of 10 volts approximately, there being in circuit a total resistance of 1000 ohms, then the current producing this deflection would be

$$\frac{10}{1000} = \cdot 01 \text{ ampère.}$$

The current which would be required to produce a deflection of 1° would obviously be

$$\cdot 01 \times \frac{\tan 1^\circ}{\tan 50^\circ} = \cdot 01 \times \frac{\cdot 0175}{1 \cdot 198} = \cdot 000146 \text{ ampère;}$$

which is consequently the figure of merit of the instrument.

In the case of a Thomson galvanometer, we have simply to divide the current by the deflection obtained with the latter, since the deflections are approximately in direct proportion to the currents producing them.

If we require to determine the figure of merit of a galvanometer whose deflections throughout the scale are not proportionate to any ordinary function of the degrees of those deflections, then it is best to employ a sufficiently low electromotive force and high resistance in circuit to obtain a few degrees of deflection only, and then to divide the current by this number of degrees; for on every galvanometer the first few degrees of deflection are almost exactly proportional to the currents producing them, although the higher deflections are not so.

The "figure of merit" of a galvanometer has a considerable bearing upon the question of the degree of accuracy with which it is possible to make electrical measurements, as will be seen hereafter.

Sensitiveness of Galvanometers.

79. A galvanometer with a high "figure of merit," that is, a galvanometer whose needle will deflect from zero with a very

weak current, is not necessarily a highly *sensitive* instrument; by a *sensitive* galvanometer is meant one *whose needle when deflected under the influence of a current will change its deflection perceptibly with a very slight change in the current strength.*

In many tests it is far more important that the galvanometer used be one of great sensitiveness rather than one with a high figure of merit. As a rule it is rarely that an instrument whose needle has a *compass* suspension or is pivoted is highly sensitive, unless indeed the pivoting is exceptionally good. Practically, it may be taken that for high sensitiveness the needle must be suspended by a fine fibre so that its movements may be perfectly free.

80. In order to check the oscillations of the galvanometer needle when the latter is either deflected under the influence of a current, or when it recoils after the current is taken off, Mr. J. Gott suggests that a small coil of wire should be placed under the galvanometer in circuit with a small battery and a key, the coil being in such a position that when a current passes through it a deflection of the needle is produced; by a proper manipulation (easily acquired) of the key, it will be found that the oscillations of the needle can, with such an arrangement, be checked in a few seconds, and much time (an important item in some tests) saved.

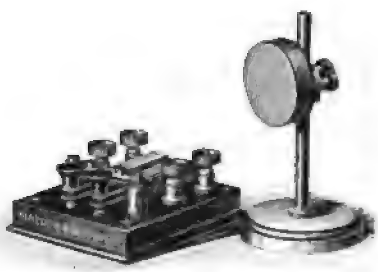


FIG. 60.

A very convenient contrivance of this kind is manufactured by Messrs. Nalder Bros.; it consists of a coil (Fig. 60), which can be placed in any convenient position near the galvanometer. A double key enables the current from a battery to be sent through the coil in either direction. The key is so arranged that when either lever is slightly depressed, the current from the battery has to pass through a resistance, and thus only a weak current flows; when, however, either lever is depressed firmly, then the resistance is cut out of circuit and the full current flows. This arrangement greatly facilitates the process of checking the oscillations, and altogether is a most useful device.

CHAPTER IV.

SHUNTS.

81. In making certain measurements we sometimes find that, owing to the sensitiveness of the galvanometer, we are unable to obtain a readable deflection, in consequence of the needle being deflected up to the stops. We may reduce this sensitiveness by the insertion of a *Shunt* between the terminals of the instrument. This arrangement is shown by Fig. 61.

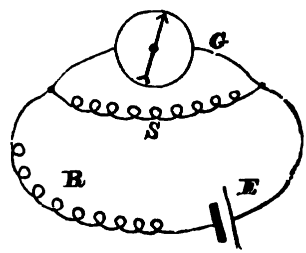


Fig. 61.

If it is required to reduce the strength of current which ordinarily passes through the galvanometer to any proportional part of that current, we must calculate, from the resistance of the galvanometer, what the resistance of the shunt should be to effect that purpose.

Now, if we call C the current passing through the galvanometer without a shunt, then on introducing the shunt, C will divide between the two resistances, the greater portion of the current going through the smaller resistance, and the smaller portion through the greater. Thus, if we suppose the total current, which passes from one terminal of the galvanometer to the other, to consist of G + S parts, then $\frac{G}{G+S}$ of these parts will go through the shunt, and $\frac{S}{G+S}$ parts through the galvanometer; that is to say, the current going through the shunt will be

$$C \frac{G}{G+S},$$

and the current going through the galvanometer,

$$C \frac{S}{G+S}.$$

If, in this last quantity, we put $S = G$, then current going through galvanometer will be

$$C \frac{G}{G + G} = \frac{C}{2}.$$

Again, if we make $S = \frac{G}{2}$, current going through galvanometer will be

$$C \frac{\frac{G}{2}}{G + \frac{G}{2}} = \frac{C}{3}.$$

Once more, if S be made equal to $\frac{G}{3}$, current going through galvanometer will be

$$C \frac{\frac{G}{3}}{G + \frac{G}{3}} = \frac{C}{4}.$$

Finally, if S be made equal to $\frac{G}{n-1}$, then current going through galvanometer will be

$$C \frac{\frac{G}{n-1}}{G + \frac{G}{n-1}} = \frac{C}{n}.$$

From this it is evident, that to reduce the current flowing through the galvanometer to its $\frac{1}{n}$th part, we must insert a shunt whose resistance is $\frac{1}{n-1}$th part of the resistance of the galvanometer.

82. In many galvanometers three shunts are provided,* which enable the strength of current flowing through the same to be reduced to its $\frac{1}{10}$th, $\frac{1}{100}$th, or $\frac{1}{1000}$th part. From what has been said, it will be evident that the resistances of the shunts necessary to produce these results will have to be respectively the $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th part of the resistance of the galvanometer.

* Page 63.

We are thus enabled to reduce the sensitiveness of the galvanometer to any one of these three proportions we wish.

Multiplying Power of Shunts.

83. Suppose now, in making a measurement, we placed a resistance box for a shunt between the terminals of the galvanometer, and then adjusted it until we obtained a convenient deflection for the purpose we required; what deflection should we obtain on removing the shunt? Let us call C, as before, the current which passes through the galvanometer when no shunt is inserted, and let C_1 be the current which flows through it when the shunt is inserted, then the current which flows through the shunt will be

$$C - C_1.$$

Now the two currents will flow through the shunt and galvanometer in the inverse proportion of their resistances, that is,

$$C_1 : C - C_1 :: S : G,$$

therefore

$$C = C_1 \times \frac{G + S}{S}.$$

Or expressed in words, we should say that the current which would flow through the galvanometer, when the shunt was removed, would be $\dfrac{\textit{Galvanometer} + \textit{Shunt}}{\textit{Shunt}}$ times the strength of the current which flows when the shunt is inserted. This proportion is called the *multiplying power* of the shunt.

Compensating Resistances.

84. It will be noticed in a circuit like that shown by Fig. 53, that when a shunt having a resistance equal to that of the galvanometer is introduced between the terminals of the latter, it will not exactly halve the current passing through the instrument. If we used a tangent galvanometer, we should find that if the deflection without the shunt were 40 divisions on the tangent scale, then the introduction of the shunt would not bring the deflection down to 20, but to some deflection greater than 20. The reason of this is that the introduction of the shunt reduces the total resistance in the battery circuit, and consequently increases the strength of the current passing out of the battery. It is this increased current

then, which splits between the galvanometer and shunt, and not the original current. If it is required to make up for this decreased resistance caused by the introduction of the shunt, it is necessary to add in the battery circuit a *compensating resistance* equal in value to the amount by which the original resistance has been reduced. In order to obtain this, we must first consider the law of

The Joint Resistance of two or more Parallel Circuits.

85. If we have several wires whose resistances are R_1, R_2, $R_3 \ldots$ respectively, then conductivity being the inverse or reciprocal of resistance, their conductivities may be represented by $\frac{1}{R_1}, \frac{1}{R_2}, \frac{1}{R_3} \ldots$ Now the joint conductivity of any number of wires is simply the sum of their respective conductivities. Thus, *two* wires of equal conductivities, when joined parallel to one another, will evidently conduct *twice* as well as one of them; and in like manner, *three* wires will conduct *three* times as well as one. Similarly, two wires, one of which has a conductivity of 2, will, when combined with one which has a conductivity of 1, produce a conductivity of 2 + 1 or 3, for this is simply the same as joining up three wires, each having a conductivity of 1; and so with any number of wires.

Therefore the joint *conductivity* of the several resistances, or of the multiple arc, as the combination is called, will be

$$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots,$$

and conductivity being, as we have said, the reciprocal of resistance, the resistance of the wires will be the reciprocal of this sum, or

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots}.$$

That is to say, *the joint resistance of any number of wires joined parallel to one another is equal to the reciprocal of the sum of the reciprocals of their respective resistances.*

A particular case of these combinations is that of a joint resistance of two resistances, thus

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2},$$

or, *the joint resistance of two resistances joined parallel to one another is equal to their product divided by their sum.*

86. Applying the foregoing law, the resistance between the terminals of the galvanometer before the introduction of the shunt being G, that on the introduction of the shunt will be $\frac{G\,S}{G+S}$. Or, as S is usually made some fractional value of G, say the $\frac{1}{n-1}$th part (which value would be used in reducing the sensitiveness of the galvanometer to $\frac{1}{n}$th), this combined resistance will be

$$\frac{G\frac{G}{n-1}}{G+\frac{G}{n-1}} = \frac{\frac{G}{n-1}}{1+\frac{1}{n-1}} = \frac{G}{n}. \qquad [1]$$

The resistance therefore to be added to the battery circuit will be

$$G - \frac{G}{n} = G\frac{n-1}{n}. \qquad [2]$$

For example.

It was required to reduce the sensitiveness of a galvanometer, whose resistance was 100 ohms (G), to $\frac{1}{5}$th. What should be the resistance of the shunt and of the compensating resistance?

Resistance of shunt equals

$$100 \times \frac{1}{5-1} = 25 \text{ ohms},$$

and compensating resistance equals

$$100 \times \frac{5-1}{5} = 80 \text{ ohms}.$$

Fig. 62 shows how a set of shunts and compensating resistances may be adapted to any galvanometer; we will consider how their values may be determined.

Let S, S_1, S_2, be the shunts which can be connected to the galvanometer by inserting plugs at A, B, or C.

Let r_1, r_2, r_l, be the compensating resistances, and let

$$r_1 + r_2 + r_l = R_1 \qquad [3]$$

$$r_2 + r_l = R_2. \qquad [4]$$

Fig. 62.

Now, what we have to do, is to find what values of S, S_1, S_2, and r_1, r_2, r_l, are necessary, so that when a plug is introduced either at A, B, or C, the resistance between D and E shall always be the same, whilst the necessary portion of the current is shunted off from the galvanometer.

Let us first consider the shunt S and the compensating resistance which, in this case, will be R_1.

When the shunts and compensating resistances are not in use, the resistance in circuit is of course G, and this value must always be preserved between D and E.

Let the value of the shunt S required be $\frac{1}{n}$th, then we know (page 88) that the resistance of S necessary to give this, is

$$S = \frac{G}{n-1},$$

and from [2] (page 91) that the value of R_1 must be

$$R_1 = G\frac{n-1}{n}. \qquad [5]$$

We next have to consider what value to give to S_1 and R_2.

Let it be required, by means of these resistances, to reduce the deflection by $\frac{1}{n_1}$th, then the value to be given to S_1 will be

$$S_1 = \frac{G + r_1}{n_1 - 1};$$

to solve which we must know the value of r_1.

Now the combined resistance of the shunt and $G + r_1$ we can see from [1] (page 91) is

$$\frac{G + r_1}{n_1};$$

therefore the value required to be given to R_2, in order to preserve the resistance between D and E, equal to G, when S_1 is connected, will be

$$R_2 = G - \frac{G + r_1}{n_1},$$

or

$$R_2 + \frac{r_1}{n_1} = G\frac{n_1 - 1}{n_1}; \qquad [6]$$

but from [3], [4], and [5] (page 92)

$$R_2 + r_1 = R_1 = G\frac{n-1}{n}; \qquad [7]$$

therefore, subtracting [7] from [6], we have

$$\frac{r_1}{n_1} - r_1 = G\left(\frac{n_1 - 1}{n_1} - \frac{n-1}{n}\right) = G\frac{n_1 - 1}{n\,n_1};$$

that is,

$$r_1\frac{1-n_1}{n_1} = G\frac{n_1 - n}{n\,n_1},$$

or

$$r_1 = G\frac{n-n_1}{n(n_1-1)};$$

consequently the value of S_1 will be

$$S_1 = G\frac{1 + \frac{n-n_1}{n(n_1-1)}}{n_1 - 1} = G\frac{(n-1)n_1}{n(n_1-1)^2}.$$

In like manner it could be shown that the resistance necessary to give to S_2 and $r_1 + r_2$ to reduce the deflection to its $\frac{1}{n_2}$th part would be

$$S_2 = G\frac{(n-1)n_2}{n(n_2-1)^2},$$

and

$$r_1 + r_2 = G\frac{n-n_2}{n(n_2-1)},$$

or

$$r_2 = G\frac{n-n_2}{n(n_2-1)} - r_1.$$

Finally we have from [3] and [5] (page 92)

$$r_t = R_1 - (r_1 + r_2) = G\frac{n-1}{n} - (r_1 + r_2).$$

To summarise, then,

$$S = G\frac{1}{n-1},$$

$$S_1 = G\frac{(n-1)n_1}{n(n_1-1)^2},$$

$$S_2 = G \frac{(n-1) n_2}{n (n_2 - 1)^2};$$

and for any other shunt S_p

$$S_p = G \frac{(n-1) n_p}{n (n_p - 1)^2}.$$

The compensating resistances *between* the shunts will be

$$r_1 = G \frac{n - n_1}{n (n_1 - 1)},$$

$$r_2 = G \frac{n - n_2}{n (n_2 - 1)} - r_1;$$

and also we have

$$r_1 + r_2 + \ldots + r_p = G \frac{n - n_p}{n_p - 1},$$

or

$$r_p = G \frac{n - n_p}{n (n_p - 1)} - (r_1 + r_2 + \ldots + r_{p-1}).$$

The *last* resistance r_l beyond the last shunt will be

$$r_l = G \frac{n-1}{n} - (r_1 + r_2 + \ldots + r_p).$$

For example.

It was required to provide a galvanometer with $\frac{1}{10}$th, $\frac{1}{100}$th, and $\frac{1}{1000}$th shunts, and with corresponding compensating resistances arranged according to Fig. 62 (page 91). What should be their values?

We have

$$n = 1000, \quad n_1 = 100, \quad n_2 = 10;$$

therefore

$$n - 1 = 999, \quad n_1 - 1 = 99, \quad n_2 - 1 = 9.$$

Then

$$S = G \frac{1}{999} = G \times \cdot 001001,$$

$$S_1 = G \frac{999 \times 100}{1000 \times 99 \times 99} = G \times \cdot 010193,$$

$$S_2 = G \frac{999 \times 10}{1000 \times 9 \times 9} = G \times \cdot 123333;$$

and

$$r_1 = G\frac{1000 - 100}{1000 \times 99} = G \times \cdot 0090909,$$

$$r_1 + r_2 = G\frac{1000 - 10}{1000 \times 9} = G \times \cdot 11\,;$$

from which

$$r_2 = G(\cdot 11 - \cdot 0090909) = G \times \cdot 1009091\,;$$

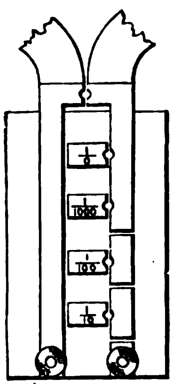
also

$$r_l = G\frac{999}{1000} - (r_1 + r_2) = G(\cdot 999 - \cdot 11) = G \times \cdot 889.$$

If the resistance of the galvanometer, for which these shunts and compensating resistances are to be provided, is 5000 ohms, then

$$\begin{aligned}
S &= 5000 \times \cdot 001001 &&= 5\cdot 005 \text{ ohms.}\\
S_1 &= 5000 \times \cdot 010193 &&= 50\cdot 965 \quad ,,\\
S_2 &= 5000 \times \cdot 123333 &&= 616\cdot 655 \quad ,,\\
r_1 &= 5000 \times \cdot 0090909 &&= 45\cdot 455 \quad ,,\\
r_2 &= 5000 \times \cdot 1009091 &&= 504\cdot 545 \quad ,,\\
r_l &= 5000 \times \cdot 889 &&= 4445\cdot 000 \quad ,,
\end{aligned}$$

Fig. 63 shows in plan, and Fig. 64 in general view, an ordinary



¼ real size

FIG. 63.

FIG. 64.

Thomson galvanometer shunt box arranged with compensating resistances, as manufactured by Messrs. Nalder Bros.

The plug hole $\frac{1}{0}$, when it has a plug inserted in it, connects the top left-hand brass block to the bottom left-hand block, and so leaves the galvanometer connected to the terminals of the shunt box without any additional resistance in its circuit. The connection between these brass blocks is shown by the dotted line in Fig. 62 (page 91).

Adjustment of Shunts.

87. The accurate adjustment of ordinary shunts is often a somewhat troublesome operation, in consequence of the numerical values of the resistances of which the shunts are composed not being whole numbers; thus, supposing the resistance of the galvanometer to be 5000 ohms, then the resistance of the $\frac{1}{10}$th shunt would have to be $5000 \div 9$, or $555 \cdot 56$; and, practically, this could not be adjusted to a greater degree of accuracy than one decimal place. Similarly, the $\frac{1}{100}$th shunt should have a resistance of $5000 \div 99$, or $50 \cdot 505$, and the $\frac{1}{1000}$th shunt a resistance of $5000 \div 999$, or $5 \cdot 005$, both of which numbers are somewhat awkward to adjust exactly.

Now on page 91 (equation [1]) we saw that the combined resistance of the galvanometer and its shunt was $\frac{G}{n}$, consequently to adjust the $\frac{1}{10}$th shunt we may connect it to its galvanometer coil, and adjust it until the joint resistance of the two becomes equal to $5000 \div 10$, or 500 ohms. Similarly, the $\frac{1}{100}$th shunt would be adjusted by connecting it to the galvanometer coil, and adjusting it until the joint resistance was found to be $5000 \div 100$, or 50 ohms; lastly, in like manner we should adjust the $\frac{1}{1000}$th shunt until the combined resistance of the two became $5000 \div 1000$, or 5 ohms.

AYRTON AND MATHER'S UNIVERSAL SHUNT.

88. As shown by Fig. 65, let the galvanometer be connected across its terminals by a resistance r, and suppose, first, lead 1 to be connected to the point p at the end of r, then if C be a current in lead 1, this current will divide at p, and the portion C_1, passing through the galvanometer, will be

$$C_1 = C \frac{r}{r + g}.$$

If now the end of lead 1 be connected to any point p, between

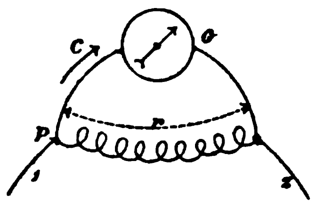


FIG. 65.

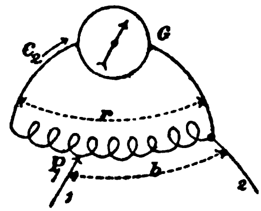


FIG. 66.

the ends of r (Fig. 66), then the current C_2, passing through the galvanometer, will be

$$C_2 = C\frac{b}{r+g},$$

so that

$$\frac{C_1}{C_2} = \frac{C\dfrac{r}{r+g}}{C\dfrac{b}{r+g}} = \frac{r}{b},$$

or

$$C_1 = C_2\frac{r}{b};$$

that is, the multiplying power (§ 83) of the shunt is

$$\frac{r}{b},$$

a ratio which as it does not contain g, the resistance of the galvanometer, is *independent of the resistance of the latter.* This fact, then, enables a shunt box to be constructed which can theoretically be used with any galvanometer.

To practically apply the principle, the shunt box would be arranged as shown by Fig. 67; the total resistance $a+b+c+d$ being any convenient value, and $b+c+d$ being $\frac{1}{10}$th, $c+d$ $\frac{1}{100}$th, and d $\frac{1}{1000}$th, of $a+b+c+d$; then, if $a+b+c+d$ were made, for example, 1000 ohms, a, b, c, and d would have the following values:—

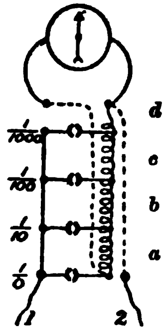


FIG. 67.

$d = 1$ (since $1 = \frac{1}{1000}$th of 1000)
$c = 9$ (since $9 + 1 = 10 = \frac{1}{100}$th of 1000)
$b = 90$ (since $90 + 9 + 1 = 100 = \frac{1}{10}$th of 1000)
$a = 900$ (since $900 + 90 + 9 + 1 = 1000$).

As stated, this arrangement of shunt box is one which, theoretically, enables the latter to be used for any galvanometer, that is to say, for example, it may be used either with a galvanometer of 1 ohm, or with a galvanometer of 10,000 ohms, resistance. The question then arises, is there any particular value which it is advisable to give to r? Now, the normal condition of the galvanometer is when the leads are connected to the two ends of the shunting resistance, and if this resistance is low it will obviously have the effect of reducing the figure of merit (page 85) of the instrument, which would probably be undesirable. On the other hand, if r is made high compared with g, then when lead 1 is shifted to an intermediate point on r, it introduces a high resistance into the circuit between the two leads. Thus, for example, if g were, say, 100 ohms, and r, say, 10,000 ohms; then, when the leads are connected at the ends of r, the 10,000 ohms will only reduce the figure of merit of the galvanometer by about one per cent., whilst the actual resistance between the leads will be rather less than 100 ohms (that is, 100 combined in multiple arc with 10,000); but when lead 1 is moved along r to the position at which g becomes shunted to $\frac{1}{10}$th, that is to say, to the position at which b (Fig. 66) is 1000, then the resistance between the leads becomes increased from rather less than 100 ohms, up to

$$\frac{1000\,(9000+100)}{1000+9000+100} = 901.$$

If r is 10 times g, then the resistance between the leads is the same when the latter are connected either at the ends of r, or at the intermediate point at which the galvanometer is shunted to $\frac{1}{10}$th. With such a shunt the figure of merit of the instrument would be reduced about 10 per cent., an insignificant amount. Lastly, if r has a resistance less than 10 times g, then the resistance between the leads when the latter are connected at the $\frac{1}{10}$th shunt point, will be less than the resistance when the leads are connected at the ends of r (as would be the case with an ordinary set of shunts), but the shunting effect of r commences to become apparent upon the figure of merit of the galvanometer.

Although, therefore, no definite rule can be laid down, it would probably be advisable to have the value of r about 10 times the average value of the resistances of the galvanometer with which the shunt is likely to be used; that is, assuming that $\frac{1}{10}$th is the highest shunt that the box is arranged for, the other shunts being as usual $\frac{1}{100}$th and $\frac{1}{1000}$th.

As manufactured by Messrs. Nalder Bros., the Ayrton-Mather shunt-boxes, Fig. 68, are arranged with $\frac{1}{3}$rd, $\frac{1}{10}$th, $\frac{1}{30}$th, $\frac{1}{100}$th, $\frac{1}{300}$th and $\frac{1}{1000}$th shunts, which would be most suitable for galvanometers having an average resistance 3 times the value of r.

The special advantage of the Ayrton-Mather shunt is its use for D'Arsonval (page 68), or ballistic (page 74) galvanometers, in which the error which occurs when ordinary shunts are employed, and which is referred to in Chapter XI. (Correction for Discharge Deflections), becomes greatly magnified owing to the heavy magnets employed. The use of a constant resistance r across the terminals of the instruments makes the path through which the induced current due to the movement of the needle circulates, always the same, and entirely gets rid of the error.

FIG. 68.

89. It has been shown in a previous chapter (page 48) that the deflections on the scale of a Thomson galvanometer, except when they are nearly equal, are not directly proportional to the current strengths which produce them, and that to compare them a formula must be used. If we wish to avoid the use of this formula we must adopt some method of avoiding widely different deflections. This we can do by using a variable shunt for the galvanometer, and with it obtaining either equal, or nearly equal, deflections for all measurements made in one set of tests.

The graduated scale of any galvanometer, it should be recollected, is not only for the purpose of enabling the strengths of two or more currents to be compared by different deflections, but is also for the purpose of enabling any deflection which may be obtained to be reproduced when required.

90. It is best to obtain as high a deflection as possible, for then not only will a slight variation from the correct resistance of the shunt produce a greater number of degrees of variation from the deflection required, than would be the case if a low deflection were used (§ 69, page 73), but also a higher resistance being required for the shunt, a greater range of adjustment is given to it.

Calibration of Galvanometers.

91. By the help of the points we have considered we can graduate or *calibrate* (page 47) the scale of a galvanometer. To do this, first calculate from the known resistance of the galvanometer, the resistance of shunts required to reduce the amount of current passing through the galvanometer when no shunt is inserted, to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., the amount passing when a shunt is inserted, then the resistance of the shunts necessary to reduce the current to

$$\tfrac{1}{2},\ \tfrac{1}{3},\ \tfrac{1}{4},\ \tfrac{1}{5},\ \ldots\ \tfrac{1}{n}\text{th}$$

will, as we have shown, be

$$1,\ \tfrac{1}{2},\ \tfrac{1}{3},\ \tfrac{1}{4},\ \ldots\ \tfrac{1}{n-1}\text{th}$$

of the resistance of the galvanometer. Now, as we have also shown, the insertion of shunts reduces the resistance of the circuit in which the galvanometer is placed; we must therefore also calculate the resistances necessary to be inserted in the circuit in order to compensate for the reduction of resistance which takes place when a shunt is inserted. These resistances will be respectively

$$\tfrac{1}{2},\ \tfrac{2}{3},\ \tfrac{3}{4},\ \tfrac{4}{5},\ \ldots\ \tfrac{n-1}{n}\text{th}$$

of the resistance of the galvanometer.

The shunts and their compensating resistances being calculated, to calibrate the galvanometer we proceed as follows:—

The galvanometer, a resistance box, and a battery are joined up in circuit. The $\frac{1}{2}$ shunt, that is, the shunt equal in resistance to the galvanometer, is then inserted, together with the corresponding compensating resistance in the resistance box. Sufficient resistance is now added in the latter to bring the deflection down to, say 1°; the shunt and compensating resistance are then removed, and as the resistance in circuit is the same as before, and also the whole of the current passing in the circuit now passes through the galvanometer, the strength of current affecting it is exactly double that which deflected the needle originally; the deflection of the needle, therefore, now represents a strength of current double that of the previous experiment. We next insert the $\frac{1}{3}$ shunt and its compensating resistance, and by again adjusting the resistance coils, obtain a deflection of 1°; on now removing the shunt and compensating resistance we get three times the strength of current passing

through the galvanometer; the deflection obtained therefore will represent that strength, and so by inserting

$$\tfrac{1}{4}, \tfrac{1}{5}, \ldots \tfrac{1}{n}\text{th}$$

shunts one after another, and repeating the process described, we can get the deflections corresponding to strengths of current equal to 1, 2, 3, 4, ... n, and the scale can be marked correspondingly; or these deflections and the corresponding currents producing them can be embodied in a table, so that by referring to the latter we can at once see the relative powers of various currents giving different deviations of the needle.

92. It is evident that if, in making a measurement, we want to reduce the deflection of our galvanometer to a readable value, we can do so, either by placing a large resistance in the circuit of the instrument, or by introducing a shunt between its terminals. It is possible also, in certain cases, to produce the same effect by connecting a shunt between the poles of the battery, but this is not always advisable, as it interferes with the constancy of the latter.

If the resistance of the battery and galvanometer in a simple circuit be very high it requires a very considerable increase of resistance in the circuit to produce an alteration in the deflection of the galvanometer needle, whereas just the reverse is the case if a shunt be used to produce that effect. This fact is an important one, as it has a considerable bearing upon the accuracy with which measurements can be made.

CHAPTER V.

MEASUREMENT OF GALVANOMETER RESISTANCE.

HALF DEFLECTION METHOD.

93. THE simplest method of determining the resistance of a galvanometer is perhaps the one we have already given on page 5 (§ 9). In this method it will be seen we joined up the galvanometer, whose resistance (G) was required, in circuit with a battery of very low resistance, and having obtained a certain deflection we added a resistance R, so that the current passing in the circuit became halved in strength; the resistance (G) of the galvanometer was then equal to the resistance R.

If we were measuring the resistance of a *tangent* galvanometer, the deflections obtained should be such that the tangent of one deflection is half the tangent of the other, the precaution against having the deflections too high or too low being duly taken (§ 31, page 25).

94. In measuring the resistance of an ordinary galvanometer by this method it would be necessary to know what ratio the deflections bore to the current strengths producing them, so that the resistance may be adjusted accordingly.

A convenient arrangement is to employ a tangent galvanometer of a known low resistance in circuit with the galvanometer whose resistance is required, and to take the readings from the tangent galvanometer, then the resistance R will evidently be the resistance of the two galvanometers together. If, then, we subtract from this result the known resistance of the tangent galvanometer, we get the resistance we are trying to obtain. If we have not a tangent galvanometer at hand, and if moreover we cannot tell what ratio the deflections bear to the current strengths producing them, we must of course employ a different method of testing.

95. Another method given on page 5 (§ 8) consisted in joining up the galvanometer, whose resistance (G) was required, in circuit with a resistance ρ, and a battery of very low resistance, and having obtained a certain deflection to increase ρ to R, so that the

current passing in the circuit became halved (as indicated by the galvanometer deflection); the resistance (G) of the galvanometer was then given by the formula

$$G = R - 2\rho.$$

For example.

With a tangent galvanometer whose resistance (G) was to be determined, and a battery whose resistance was very small, we obtained with a resistance in the resistance box (as the set of resistance coils is sometimes termed) of 10 ohms (ρ) a deflection of 58°, and by increasing the resistance to 120 ohms (R) the deflection was reduced to $38\frac{1}{2}°$ ($\tan 38\frac{1}{2}° = \frac{1}{2} \tan 58°$); what was the resistance of the galvanometer?

$$G = 120 - 2 \times 10 = 100 \text{ ohms}.$$

96. In the foregoing, and indeed in all tests, it is important to consider what resistances and battery power should be employed to make the measurements, so that the greatest possible accuracy may be ensured.

If we employ very high resistances to measure a low resistance, a considerable alteration in the former would produce but little alteration in the current flowing through the galvanometer, for the electromotive force being constant, this current, and consequently the galvanometer deflection, is dependent upon the total resistance in the circuit, and an alteration of several units in a large total practically leaves its value the same, but then a few units too much or too little inserted in a formula may make the result appear very much greater or less than its true value. Thus, in the test we have been considering, suppose the battery power had been such that we found it necessary to have the resistance $\rho = 2000$ ohms, and that to halve the deflection we found it necessary to increase ρ to 4100 ohms (R), this would make the resistance of the galvanometer to be, as we saw before,

$$G = 4100 - 2 \times 2000 = 100 \text{ ohms}.$$

Now, practically, if the resistance R had been made 4200 ohms the deflection would have been halved; whatever difference there was, would scarcely be appreciable.

If now we work the result out from the formula we get

$$G = 4200 - 2 \times 2000 = 200 \text{ ohms},$$

or double what it ought to be. It is possible indeed that the error

might be greater than this. The test, in fact, would be quite useless.

In order to have the best chance of accuracy we should make our resistances as low as possible, for then a small change or error in the latter produces the greatest increase or decrease in the current, and consequently also in the deflection of the galvanometer needle, and, on the other hand, it produces the smallest error in the value of G, when the latter is worked out from the formula.

In order to make R as low as possible it is evident that we must make ρ as low as possible.

97. What *degree of accuracy* is attainable in making the test? This is dependent upon the "total possible percentage of error which may exist in the second deflection" (§ 43, page 45). We have then to consider what error in the value of G the total error in the second deflection will cause.

The error in G must be occasioned by the value of R being obtained incorrectly, this wrong value of R being due to an error made in reading the magnitude of the second deflection. If in the formula

$$G = R - 2\rho$$

we make a mistake of, say, λ'_1 per cent. in R, then the resulting percentage error λ', in G will be $\lambda' = \lambda'_1 \frac{R}{G}$.

Now the accuracy with which we can adjust R is directly dependent upon the accuracy with which we can adjust (G + R), for the latter is the *total* resistance in the circuit of the galvanometer, and therefore any change or error made in the value of the galvanometer deflection (the second deflection) must be in direct proportion to the change or error made in (G + R); consequently if we are liable to make an error of γ' per cent. in the value of the second deflection, and an error of λ'_1 per cent in R, then we must have

$$\gamma' : \lambda'_1 :: R : G + R,$$

or

$$\lambda'_1 = \frac{(G + R)\gamma'}{R};$$

but

$$\lambda' = \lambda'_1 \frac{R}{G}, \quad \text{or,} \quad \lambda'_1 = \lambda' \frac{G}{R},$$

and

$$G = R - 2\rho, \quad \text{or,} \quad R = G + 2\rho,$$

therefore

$$\lambda' \frac{G}{R} = \frac{2(G + \rho)\gamma'}{R};$$

hence

$$\lambda' = 2\left(1 + \frac{\rho}{G}\right)\gamma'. \qquad [A]$$

For example.

In measuring the resistance of the galvanometer in the example given in § 95, page 103, it was known that the "total possible percentage of error (γ') which could exist in the second deflection" could not exceed 1·7 per cent. (Example 2,

page 46). What would be the percentage of accuracy (λ') with which the value of G could be determined?

$$\lambda' = 2\left(1 + \frac{10}{100}\right) 1 \cdot 7 = 3 \cdot 7 \text{ per cent.}$$

A single cell of a battery is the lowest electromotive force that can be practically employed in making the test, but we may find that this one cell gives too low a deflection with the lowest value we can give to ρ, that is 0, and two cells too high a deflection; we should have, therefore, to employ two cells and then increase ρ until the proper deflection is obtained. Now on pages 103 and 104 it was pointed out that it is best to make ρ of a low value, so that the deviation of the needle from its correct position, when R is not correctly adjusted, may be as great as possible; but equation [A] (page 104), which represents the relative values of the errors λ' and γ', although it shows that the error λ' is smallest when ρ is as small as possible, at the same time shows that we gain but little by making ρ *very* much smaller than G, for λ' is only twice as great when $\rho = 0$, as it is when $\rho = G$.

98. Practically we may say therefore that the

Best Conditions for making the Test

are to make ρ a fractional value of G; and in the case of a tangent galvanometer the two deflections obtained should be as nearly as possible 55° and $35\frac{1}{2}$° (§ 34, pages 28 and 29).

Also as regards the

Possible Degree of Accuracy attainable.

If we can determine the value or the deflection of the galvanometer needle to an accuracy of γ' per cent., then we can determine the value of G to an accuracy of $2\left(1 + \frac{\rho}{G}\right)\gamma'$ per cent.

If ρ is very small, then

$$\lambda' = 2\,\gamma';$$

so that even under the best conditions for making the test, the accuracy with which the value of G could be determined would be only one-half of the accuracy with which the deflections could be observed.

99. It must be understood that the resistance of the testing battery can only be neglected when it forms a small percentage of the total resistance in its circuit. If, then, the galvanometer to be measured has a low resistance, inasmuch as R will have to be proportionally small, the battery resistance can no longer be ignored without introducing an error; moreover, if R is made small, its range of adjustment becomes very limited. The test, therefore, is not suitable for measuring galvanometers whose resistance consists of a few units only.

Equal Deflection Method.

100. The theory of this method is as follows:—The galvanometer whose resistance G is required, a resistance ρ, and a battery E of very low resistance are joined up in circuit, as shown by Fig. 69, a shunt S being between the terminals of the galvanometer; a deflection of the galvanometer needle is produced. Let C be the current flowing out of the battery, then

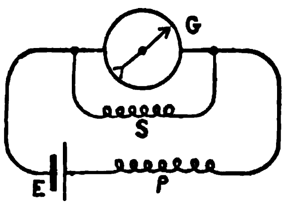


Fig. 69.

$$C = \frac{E}{\rho + \frac{G\,S}{G+S}}.$$

This current divides into two parts, one part going through S, and the other part through the galvanometer. It does this in the inverse proportion of the resistance of those circuits, the part, C_1, going through the galvanometer being

$$C_1 = \frac{E}{\rho + \frac{G\,S}{G+S}} \times \frac{S}{G+S} = \frac{E\,S}{S(G+\rho)+G\,\rho}.$$

The shunt S is now removed; this causes the deflection of the galvanometer needle to be increased. ρ is now increased to R, so that the deflection becomes the same as it was previous to the removal of the shunt, or in other words, so that the strength of the current passing through the galvanometer is C_1, then

$$C_1 = \frac{E}{R+G};$$

therefore

$$\frac{E\,S}{S(G+\rho)+G\,\rho} = \frac{E}{R+G}.$$

By dividing both sides by E and multiplying up, we get

$$S\,R + G\,S = G\,S + S\,\rho + G\,\rho;$$

therefore

$$G\,\rho = S\,R - S\,\rho,$$

from which

$$G = S\,\frac{R-\rho}{\rho}.$$

For example.

A galvanometer whose resistance (G) was required, was joined up in circuit with a resistance of 200 ohms (ρ), a shunt of 10 ohms (S) being between the terminals of the galvanometer.

On removing the shunt, it was necessary in order to reduce the increased deflection to what it was originally, to increase ρ to 2200 ohms (R). What was the resistance of the galvanometer?

$$G = 10 \frac{2200 - 200}{200} = 100 \text{ ohms}.$$

101. In making this test practically, we should proceed thus:— Join up the instruments, as shown by Fig. 70, taking care that the two infinity plugs are firm in their places. Plug up the

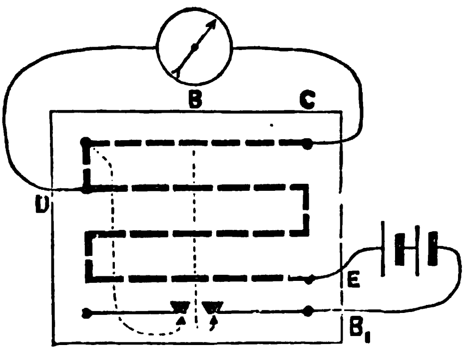


FIG. 70.

three holes between B and C, and remove the necessary plugs between D and B. Next remove plugs from between D and E, so as to introduce the resistance ρ. On the right-hand key being depressed the deflection of the galvanometer needle is obtained. The galvanometer should be gently tapped with the finger in order to see that the needle is properly deflected and is not sticking, as it is very liable to do, especially when a compass suspended needle is used.

The oscillations of the needle may be arrested by a skilful manipulation of the key; slightly raising it when the needle swings under the influence of the current and again depressing it when it recoils.

The needle being steadily deflected, and the precise resistance (ρ) in the box noted, the left-hand infinity plug must be removed, and the resistance between D and E increased until the deflection becomes the same as it was at first, and the resistance (R) being noted, the formula is worked out.

102. What are the best values of S and ρ to employ in making a test like this? Should we make S and ρ of low, high, or medium values?

The answer to these queries has an important bearing upon the accuracy with which the test can be made; and as we shall more than once have to consider questions of a similar kind, we shall in the present instance enter at some length into the problem.

There are two quantities whose values we have to determine, viz. S and ρ; let us first consider what S should be, supposing R to be a given quantity and ρ to vary along with S.

If we examine the formula we shall see that if we make S small, then an error of one or two units in the correct value of R will make a much greater difference in the formula than would be the case when there is the same number of units of difference with S large; thus to take a numerical example, suppose we had the following values in the formula:—

$$G = 5\,\frac{500 - 20}{20} = 120 \text{ ohms},$$

and suppose we made R 120 units too large, we should then have

$$G = 5\,\frac{620 - 20}{20} = 150 \text{ ohms};$$

or an error of $150 - 120 = 30$ ohms. Next let us suppose we had the following values:—

$$G = 480\,\frac{500 - 400}{400} = 120 \text{ ohms},$$

and as before let there be an error of 120 units in R, we then have

$$G = 480\,\frac{620 - 400}{400} = 264;$$

or an error of $264 - 120 = 144$ ohms, and if S and ρ had been higher still, we should have seen that the error would have been still greater.

To put the case in another way; in the last example let us suppose the error in R had been, not 120 units, but 25 units; that is, make $R = 500 + 25 = 525$; we then find that

$$G = 480 \frac{525 - 400}{400} = 150 \text{ ohms.}$$

The error in G, in fact, in the former case, where R was 120 units too large, was no greater than it was in the latter case, when the excess in the correct value of R was but 25 units. From this it must be evident that it is highly advantageous to make S as *small* as possible. Let us, however, put the matter in algebraical form; thus, let λ be the error in G, and let ϕ be the excess in the value of R which causes this error, then we have

$$G + \lambda = S \frac{R + \phi - \rho}{\rho} = S \frac{R - \rho}{\rho} + S \frac{\phi}{\rho},$$

and

$$G = S \frac{R - \rho}{\rho}, \quad \text{or} \quad \rho = \frac{S R}{G + S};$$

therefore by subtraction

$$\lambda = S \frac{\phi}{\rho} = S \phi \times \frac{G + S}{S R} = \frac{\phi (G + S)}{R}.$$

From this we see that with a constant error ϕ, made in R, the corresponding constant error λ, made in G, will be as small as possible when S is very small, as indeed we before proved; but we also see that we gain but little by making S a very small fractional value of G, for the error would be only twice as great with $S = G$ as it would be if S were very nearly $= 0$. It would not do, however, to make S greater than G, for $G + S$ increases very rapidly by increasing S. Practically, therefore, we may say:—make S a fractional value of G.

We have next to determine what is the best value to give to ρ, supposing S to be a fixed quantity.

Now if we put the equation

$$G = S \frac{R - \rho}{\rho}$$

in the form

$$G = S \left(\frac{R}{\rho} - 1\right),$$

we can see that whatever value ρ has, R will have an exactly proportional corresponding value; thus to take the example we first had, viz.:

$$G = 5\,\frac{500 - 20}{20} = 5\left(\frac{500}{20} - 1\right);$$

if in making the test we had made $\rho = 2 \times 20 = 40$, instead of 20, then the value to which R would have required to have been adjusted would have been $2 \times 500 = 1000$, instead of 500. Further, if R had had this value, then an error of 20 units in R would have produced the same error in G as would the 10 units in the first case, when R was 500. At first sight then it might appear that it would not matter what value we gave to ρ. Let us, however, consider in what way the adjustment of R is effected.

The means by which we adjust R is by observing the deflection of the galvanometer needle, and seeing whether we have brought it to the deflection it had when ρ and S were the resistances in the circuit; when this deflection is correct we know that R is correct. But the accuracy with which we can adjust R evidently depends upon the divergence of the needle from its correct position being as large as possible when R is not exactly adjusted, and if this divergence is greater when we alter R from 1000 to 1020 ohms than when we alter it from 500 to 510 ohms, then it is better so to arrange the value of ρ that R shall be 1000 ohms.

Or in other words, if the error in R, corresponding to a constant error in G, produces a greater divergence of the needle from its correct position when R is large that when it is small, then it is better to have R large than small.

Now the current C producing the deflection of the galvanometer needle is

$$C = \frac{E}{R + G},$$

and if we suppose there to be a diminution $-c$, in C, caused by an error ϕ, in R, then we have

$$C = c = \frac{E}{R + \phi + G};$$

or

$$c = C - \frac{E}{R + \phi + G};$$

but we know that

$$C = \frac{E}{R+G};$$

therefore

$$c = \frac{E}{R+G} - \frac{E}{R+\phi+G} = \frac{E\phi}{(R+\phi+G)(R+G)};$$

or, since ϕ is very small,

$$c = \frac{E\phi}{(R+G)^2};$$

c, however, represents the *absolute* change from the correct current, and as the latter is itself varied by the value of R, what we require to know is the *relative* change; this will be

$$\frac{c}{C},$$

which equals

$$\frac{E\phi}{(R+G)^2} \div \frac{E}{R+G} = \frac{\phi}{R+G}. \qquad [A]$$

But from page 109 we see that the constant error λ, caused in G by an error ϕ in R, is

$$\lambda = \frac{\phi(G+S)}{R},$$

or

$$\phi = \frac{\lambda R}{G+S};$$

substituting, then, this value of ϕ, we get

$$\frac{c}{C} = \frac{\lambda R}{(G+S)(R+G)} = \frac{\lambda}{(G+S)\left(1+\frac{G}{R}\right)}. \qquad [B]$$

From this equation we see that in order to make c as large as possible, we must make R as large as possible; but it is evident that we increase c very little by making R much larger than G, for the reason we gave when we determined the ratio which S should have to G.

We do not gain, then, anything as regards the sensitiveness of the arrangement by making R very large, but we gain as regards our power of adjusting R, for we can adjust a resistance with a much closer degree of accuracy when it consists of a large number than when it consists of a small number of units.

It is therefore advantageous to make R as large as possible.

Since when S, G, and ρ are given values, R must have a value dependent upon them; and since we have determined the value we must give to S, it follows that the value we should give to ρ must be such that R will be as large as possible.

As we cannot make R larger than the resistance we can insert in the resistance box, we must not make ρ so large that R will have to exceed that value.

From the equation

$$G = S\,\frac{R - \rho}{\rho}$$

we see that

$$\rho = \frac{S}{G + S}\,R.$$

Theoretically, therefore, we must not make ρ larger than the value we can give to $\frac{S}{G + S}\,R$.

The highest resistance we can practically give to R is 10,000 ohms; ρ, therefore, must not be larger than $\frac{S}{G + S} \times 10{,}000$ ohms. Thus, if we use a shunt whose resistance is $\frac{1}{9}$th the resistance of the galvanometer, we must not make ρ larger than $\frac{1}{10}$th of 10,000, that is, 1000 ohms.

Equation [A] shows that the value of c is dependent upon the value of S, and that to make c large we should make S small. We previously proved, however, that there was another reason why S should be small, consequently we have a double reason why S should have a low value.

103. What *degree of accuracy* is attainable in making the test? This, as in the last test, is dependent upon the value of the deflection error. We have, in fact, to consider what error in the value of G a definite error in reading the deflection of the galvanometer needle will cause.

This we can determine from equation [B] (page 111). Let us, then, in this equation substitute *percentages* for absolute values, that is to say, let us have

$$c = \frac{\gamma'}{100} \text{ of } C, \quad \text{or,} \quad \frac{c}{C} = \frac{\gamma'}{100}$$

and

$$\lambda = \frac{\lambda'}{100} \text{ of } G;$$

then we get

$$\frac{\gamma'}{100} = \frac{\lambda' G R}{100\,(G + S)\,(R + G)},$$

that is to say,

$$\lambda' = \left(1 + \frac{S}{G}\right)\left(1 + \frac{G}{R}\right)\gamma'. \qquad [C]$$

For example.

In measuring the resistance of the galvanometer in the example given on page 107, it was known that the possible error γ' in the current, due to the deflection being incorrect, would not exceed ·88 per cent. (Example, page 45.) What would be the percentage of accuracy (λ') with which the value of G could be determined?

$$\lambda' = \left(1 + \frac{10}{100}\right)\left(1 + \frac{100}{2200}\right) \cdot 88 = \cdot 96 \text{ per cent.}$$

104. The practical results, then, that we have arrived at through these investigations are, that to obtain the

Best Conditions for making the Test:

First make a rough test to ascertain approximately the value of G. Having done this, insert a shunt (S) between the terminals of G, of a fractional value of the resistance of G.

Next join up ρ in circuit with G and its shunt S, making ρ as large as possible, but not larger than $\frac{S}{G+S}$ R; R being the highest resistance that can be obtained.

Insert in the circuit sufficient battery power of low resistance to bring the deflection of the galvanometer needle as nearly as possible to the *angle of maximum sensitiveness* (page 25), adjusting ρ, if necessary, so that this angular deflection becomes exact, and note the exact value of ρ.

Now remove the shunt and increase ρ to R, so that the increased deflection becomes the same as it was at first. Note R, and then calculate G from the formula.

Possible Degree of Accuracy attainable.

If we can determine the value of the galvanometer deflection to an accuracy of γ' per cent., then we can determine the value of G to an accuracy (λ') of

$$\lambda' = \left(1 + \frac{S}{G}\right)\left(1 + \frac{G}{R}\right)\gamma' \text{ per cent.}$$

If S is very small, and R very large, then

$$\lambda' = \gamma',$$

so that under the best conditions for making the test, the accuracy with which the value of G could be determined would be the same as the accuracy with which the value of the deflection could be observed.

105. In the practical execution of the test, inasmuch as there are only three resistances between D and B (Fig. 70), our choice of a shunt is limited from this source, but these three will usually be sufficient for most purposes.

106. The method we have described of making the test may be modified by making S or ρ the adjustable resistances instead of R, but in either of these cases it can be shown, by an investigation precisely similar to the one we have made, that the proper values of the resistances should be those we have indicated.

The test could also be simplified by making $S = \rho$, in this case we get

$$G = S\,\frac{R - S}{S} = R - S;$$

such an arrangement, however, would not give the conditions for obtaining maximum accuracy.

Fahie's Method.

107. If in the last test we make S the adjustable resistance, and make $R = 2\,\rho$, we get

$$G = S\,\frac{R - \rho}{\rho} = S\,\frac{2\,\rho - \rho}{\rho} = S,$$

that is, the resistance of the shunt will be the resistance of the galvanometer.

108. The connections for making the test with the set of resistances shown by Fig. 70 would have to be so arranged that the resistances between D and E form the shunt, and those between D and C the resistances ρ and R. This arrangement, however, in consequence of there being so few plugs between D and C, is not a satisfactory one, as some difficulty would probably be found in adjusting the battery power and resistance R so as to obtain the deflection of maximum sensitiveness. With two sets of resistance coils, however, the test can easily be made.

As in the previous method, it is best to make the resistance R as high as possible, for then any small change in the value of S produces the greatest movement of the galvanometer needle.

The *possible degree of accuracy attainable* is the same as in the last test.

109. In order that satisfactory results may be obtained in the foregoing tests, it is necessary that the galvanometer be a *sensitive* one (page 85), otherwise even a moderate degree of accuracy

cannot be assured. It is also very advantageous to arrange the resistances in connection with a special form of key, K, as shown by Fig. 71; this key on being depressed comes in contact with a spring l, and then on being further depressed brings l into contact with a contact stop connected between the junction of the resistances ρ and a. Thus the action of the key is to insert the shunt S, and at the same time to reduce R to ρ by short-circuiting a; in practically making the test, therefore, what we have to do is to adjust S until the deflection remains the same whether the key is quite up or quite down.

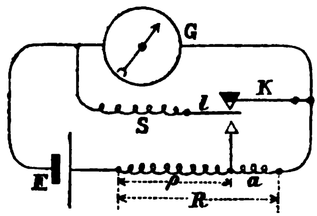


Fig. 71.

Thomson's Method.

110. Join up the galvanometer g with resistances a, b, and d, and a battery of electromotive force E and resistance r, as shown by Fig. 72, and let a key be inserted between the points E and B, so that by its depression these points can be connected together.

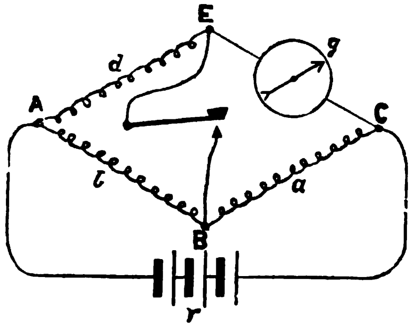


Fig. 72.

First, let us suppose the key to be up and the points consequently disconnected. The current C_1 flowing through the galvanometer will then be

$$C_1 = \frac{E}{r + \frac{(a+b)(d+g)}{a+b+d+g}} \times \frac{a+b}{a+b+d+g}$$

$$= \frac{E(a+b)}{r(a+b+d+g) + (a+b)(d+g)}. \qquad [1]$$

Next, suppose the key to be depressed, and the points E and B thereby to be connected together, then the current (C_2) flowing through the galvanometer will be

$$C_2 = \frac{E}{r + \frac{b\,d}{b+d} + \frac{a\,g}{a+g}} \times \frac{a}{a+g}$$

$$= \frac{E\,a\,(b+d)}{r\,(a+g)\,(b+d) + a\,g\,(b+d) + b\,d\,(a+g)}. \qquad [2]$$

Further, let us suppose the adjustment of the resistances to be such that

$$C_1 = C_2,$$

we then get

$$\frac{E\,(a+b)}{r\,(a+b+d+g) + (a+b)\,(d+g)}$$

$$= \frac{E\,a\,(b+d)}{r\,(a+g)\,(b+d) + a\,g\,(b+d) + b\,d\,(a+g)}; \qquad [3]$$

by multiplying up and arranging the quantities, we get

$$r\,[\,(a+b+g)\,(b+d)\,a + b\,g\,(b+d)\,] + b\,g\,(a+b)\,d + [d\,(b+g) + b\,g]\,(a+b)\,a = r\,[\,(a+b+g)\,(b+d)\,a + a\,d\,(b+d)\,] + a\,d\,(a+b)\,d + [d\,(b+g) + b\,g]\,(a+b)\,a;$$

therefore

$$b\,g\,[r\,(b+d) + (a+b)\,d] = a\,d\,[r\,(b+d) + (a+b)\,d];$$

that is,

$$a\,d = b\,g, \quad \text{or,} \quad g = \frac{a\,d}{b}.\text{*}$$

A great advantage of this test is the fact of its being entirely independent of the battery resistance. It is also very easily made, as must be evident.

* The truth of this proportion follows at once from the principle of the Wheatstone Bridge (Chapter VIII.). For if the points E and B on being connected together by a galvanometer show no deflection on the latter, then by the bridge principle

$$g = \frac{a\,d}{b}.$$

But if no deflection is produced on joining E and B by a galvanometer, i.e. if there is no tendency for a current to flow between E and B, then the joining of these points cannot affect the currents in the other branches, a, b, d, or g, that is to say, can produce no alteration in the movement of the needle of g, hence the absence of this movement, on depressing the key, indicates that there is no tendency for a current to flow between E and B, i.e. that the above proportion holds good.

In making the test practically, the connections would be made as shown by Fig. 73. The terminals E and B_1 would be joined by a short piece of thick wire. The other connections are obvious.

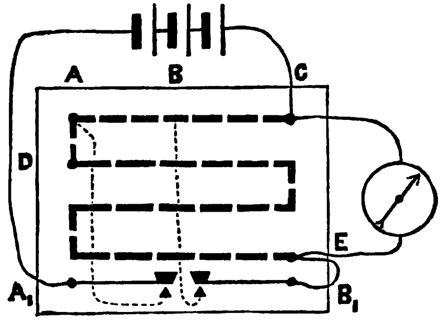


FIG. 73.

The left-hand key (which is not shown in the theoretical figure) being first depressed and then kept permanently down, the right-hand key must be alternately depressed and raised, the resistance *d*, that is, the resistance between A and E, being at the same time adjusted until the deflection of the galvanometer needle remains the same whether the key is up or down.

111. We will now determine the best arrangements of resistances for making the test. What we have to do is to suppose that in the equation

$$g = \frac{a\,d}{b}$$

there is a small but constant error in g, caused by a corresponding error in one of the other quantities, let us say d, and then find what values of d and say, a, will cause the alteration of the deflection of the galvanometer needle produced on raising and depressing the key, to be as large as possible.

Let λ be the difference between the exact value of g and the value given it by the formula when we have d too large, and let the increased value of d be d_1.

We then have

$$g + \lambda = \frac{a\,d_1}{b};$$

therefore

$$a\,d_1 = b\,g + b\,\lambda.$$

We next have to determine what the alteration in the strength of current passing through the galvanometer, produced by raising and depressing the key, is equal to.

If in either equation [1] or equation [2] (pp. 115, 116) we put $b\,g$ equal to $a\,d$, or b equal to $\frac{a\,d}{g}$, then the resulting equation will give the current C, which would

flow through the galvanometer when the adjustment is exact; by doing this we get

$$C = \frac{E\,a}{r(g+a) + a(d+g)}.$$

When the adjustment is not exact, the currents produced on raising and depressing the key will be obtained by equations [1] and [2] (pp. 115, 116), and the difference between these two currents relative to the current produced when exact equilibrium is obtained will give the relative current producing the alteration in the deflection of the galvanometer needle; hence we find

$$\frac{C_2 - C_1}{C} =$$

$$\frac{r(g+a) + a(d+g)}{a}\left\{\frac{a(d_1+b)}{r(d_1+b)(g+a) + d_1 b(g+a) + g\,a(d_1+b)}\right.$$

$$\left. - \frac{b+a}{r(d_1+g+b+a) + (d_1+g)(b+a)}\right\} =$$

$$\frac{(a\,d_1 - b\,g)\{r(d_1+b) + d_1(b+a)\}\{r(a+g) + a(d+g)\}}{a\{r(d_1+g+b+a)+(d_1+g)(b+a)\}\{r(d_1+b)(g+a) + d_1 b(g+a) + ga(d_1+b)\}};$$

but since $a\,d_1$ is very nearly equal to $b\,g$, we may without sensible error put $a\,d_1 = a\,d = b\,g$, or $b = \frac{a\,d}{g}$, except where differences are concerned; in which case we get

$$\frac{C_2 - C_1}{C} = \frac{g(a\,d_1 - b\,g)}{a(a+g)(d+g)};$$

and since $a\,d_1 = b\,g + b\,\lambda$, and $\frac{g}{a} = \frac{d}{b}$, we get

$$\frac{C_2 - C_1}{C} = \frac{\lambda\,d}{(a+g)(d+g)} = \frac{\lambda}{(a+g)\left(1 + \frac{g}{d}\right)}. \qquad \text{[A]}$$

From this it is evident that, in order to make $\frac{C_2 - C_1}{C}$ as large as possible, we must make d as large, and a as small, as possible. It is evident also that, as regards increasing $\frac{C_2 - C_1}{C}$, it is useless making d very much larger, or a very much smaller, than g. If we make d about ten times as large, and a ten times as small, as g, we shall have good conditions for ensuring accuracy, though as regards our power of adjustment, it would be advantageous to make d larger still, if possible.

From the equation

$$b\,g = a\,d$$

we see that g being a fixed quantity, and a as small as possible, we can make d as large as we like by making b as large as possible.

112. It may be pointed out, that when a is small and d and b large, we have the battery connecting the junction of the two greater with the junction of the two lesser resistances.

113. What *degree of accuracy* is attainable in making the test? This we can

determine from equation [A]. Let us then, in the latter, substitute *percentages* for *absolute* values, that is, let

$$C_2 - C_1 = \frac{\gamma'}{100} \text{ of } C, \quad \text{or,} \quad \frac{C_2 - C_1}{C} = \frac{\gamma'}{100},$$

and let

$$\lambda = \frac{\lambda'}{100} \text{ of } g;$$

then we get

$$\frac{\gamma'}{100} = \frac{\lambda' g}{100\,(a+g)\left(1+\frac{g}{d}\right)},$$

that is,

$$\lambda' = \left(1+\frac{a}{g}\right)\left(1+\frac{g}{d}\right)\gamma'.$$

For example.

In measuring the resistance of a galvanometer by the foregoing method, the values of a, b, and d were 10, 100, and 300 ohms respectively. What was the resistance of the galvanometer, and what was the possible degree of accuracy attainable? The smallest change in the value of the galvanometer deflection which it was possible to observe* was ·88 per cent. (§ 43, page 45).

$$g = \frac{10 \times 300}{100} = 30 \text{ ohms.}$$

$$\lambda' = \left(1+\frac{10}{30}\right)\left(1+\frac{30}{300}\right) \cdot 88 = 1 \cdot 3 \text{ per cent.}$$

To sum up, we have

Best Conditions for making the Test.

114. Make a not greater than $\frac{1}{10}$th of g, and make b not less than ten times as great as g, and preferably as much higher than g as possible, but not of such a high value that d, when exactly adjusted, has to exceed all the resistance we can insert between D and E (Fig. 73, page 117).

Adjust d approximately, and then if necessary adjust the battery power, so that the final deflection is as nearly as possible that of maximum sensitiveness, and then, having exactly adjusted d, calculate g from the formula.

* This is synonymous with "the degree of accuracy with which the value of the galvanometer deflection can be read" (page 45).

Possible Degree of Accuracy attainable.

If we can read the galvanometer deflection to an accuracy of γ' per cent., then we can determine the value of g to an accuracy (λ') of

$$\lambda' = \left(1 + \frac{a}{g}\right)\left(1 + \frac{g}{d}\right)\gamma' \text{ per cent.}$$

If a is small and d large, then we get

$$\lambda' = \gamma',$$

so that under the best conditions for making the test, the accuracy with which the value of G could be determined would be the same accuracy with which the value of a change in the deflection could be observed.

115. In the practical execution of the test with the set of resistance coils shown by Fig. 73 (page 117), the lowest value we could give to a would be 10 units, unless we improvised a resistance of less value, which it might be necessary to do.

THOMSON'S METHOD WITH A SLIDE WIRE RESISTANCE.

116. The foregoing test is sometimes made by having $a + b$, a slide wire resistance (§ 20, page 17) d being a fixed resistance; in this case the slide would be moved along between A and C, until the point is found at which the depression and raising of the key makes no alteration in the permanent deflection of the galvanometer needle.

As in the equation

$$r = \frac{a\,d}{b},$$

$\frac{a}{b}$ is merely the ratio of the resistances into which the total resistance $a + b$ is divided, and as the resistances are directly proportional to the lengths of the wire on either side of the slide, it is sufficient for a and b to be expressed in terms of the divisions into which the length of wire is divided.

Now as the total length, k, of the slide wire is constant, that is, as

$$a + b = k, \quad \text{or,} \quad b = k - a,$$

therefore we must have

$$g = d\left(\frac{a}{k - a}\right).$$

k is usually divided into 1000 divisions, hence

$$g = d\left(\frac{a}{1000 - a}\right).$$

For example.

In the foregoing test, equilibrium was produced when d was 1 ohm, and a, 450 divisions; what was the resistance of the galvanometer?

$$g = 1\left(\frac{450}{1000 - 450}\right) = \frac{450}{550} = \cdot 85 \text{ ohm}.$$

117. *The best conditions for making the test* in the case where a slide wire is used are generally similar to those in the previous case, that is to say, we should require to have a small and d large.

Now, the total resistance of $a + b$ in the case of a slide wire would, under most conditions met with in actual practice, be small compared with g; consequently a would be small also. For this reason, therefore, it would not signify what were the relative values of a and b in making the test. But in order to make d large, it is obvious that a must be small compared with b; thus, if d is to be 10 times g, then a must be 10 times b. In the first case, when the test was made by adjusting d, it was pointed out that although there is an advantage in making d as large as possible, in so far that by so doing the range of adjustment is made large, yet as regards the general sensitiveness of the whole arrangement, there is little, if any, advantage in making d greater than about 10 times g. In the case of the slide wire, where d is not the adjustable resistance, there is an actual disadvantage in making d excessively large, for the reason that, if we do so, we make a correspondingly smaller than b, and, when this is the case, the error in g (when worked out by the formula $g = \frac{a\,d}{b}$) produced by the value of a being, say, 1 scale division out, becomes comparatively large. Thus, if the slide wire scale were graduated into 1000 divisions (which is usually the case), it is clear that if the slider stood at, say, the "10" division mark on the scale, then an alteration or a mistake of 1 division would mean a change of 10 per cent. in the value of a, whilst if the slider stood at "100," then a change of 1 division would only mean a 1 per cent. change in the value of a. The change in a corresponding to a movement of 1 division would obviously be less if the slider were near the centre of the scale, that is, near the "500" division mark, but in this case the increase in the range of adjustment would be more than compensated for by the reduced sensitiveness of the arrangement.

The *possible degree of accuracy attainable* in making the test would be as follows:—

Let there be an error λ in g, caused by the slider being δ divisions out of correct adjustment, then we have

$$g + \lambda = d\left(\frac{a + \delta}{1000 - (a + \delta)}\right)$$

$$\text{or,}\quad \lambda = d\left(\frac{a + \delta}{1000 - (a + \delta)}\right) - g = d\left[\frac{a + \delta}{1000 - (a + \delta)} - \frac{a}{1000 - a}\right]$$

$$= \frac{d\,1000\,\delta}{(1000 - a)^2}$$

since δ is very small.

If we put percentages instead of absolute values, that is to say, if we have

$$\lambda = \frac{\lambda'}{100} \text{ of } g = \frac{\lambda'}{100} \times d \left(\frac{a}{1000 - a}\right),$$

then we get

$$\lambda' = \frac{100000\ \delta}{a\,(1000 - a)} \text{ per cent.}$$

If the galvanometer is sufficiently sensitive to enable the position of the slider to be determined to an accuracy of 1 division, then $\delta = 1$.

For example.

In the last example, what would be the degree of accuracy, λ', with which the value of g could be obtained, supposing that the position of the slider could be determined to an accuracy of 1 division (δ)?

$$\lambda' = \frac{100000 \times 1}{450\,(1000 - 450)} = \cdot 40 \text{ per cent.}$$

118. The facility and accuracy with which all the foregoing tests (except the half deflection test) can be made, may be greatly increased by the following device:—Instead of making the test with the galvanometer needle brought to the "angle of maximum sensitiveness" (page 25), make it with the needle brought approximately to zero by means of a powerful permanent magnet set near the instrument. Under these conditions the galvanometer needle will be highly sensitive to any small change in the current strength.

119. In the case of Thomson's test with the slide wire, if the test is made by using a permanent magnet in the manner described, it is best to make d of a higher value than would otherwise be the case; for then, since the slider would have to be set near the centre of the wire, a greater range of adjustment is given to it, for 5 divisions near the centre portion of the wire (500 division mark) is equivalent to only 1 division near the 100 division mark. It is true that the arrangement is not quite so sensitive as when the slider has to be set towards the end of the scale; but still if *sufficient* sensitiveness be obtained, the small loss is more than compensated for by the advantage gained in having an increased range on the scale.

120. In order that satisfactory results may be obtained in the foregoing tests, it is necessary that the galvanometer be "sensitive" (page 85), otherwise even a moderate degree of accuracy cannot be assured.

DIMINISHED DEFLECTION DIRECT METHOD.

121. This method, which has been generally described in Chapter I. (§ 5, page 4), is as follows:—The galvanometer G, a battery of low resistance, and a resistance ρ, are joined up in simple circuit; the deflection obtained is noted. Let this deflection be due to a current C_1, then calling E the electromotive force of the battery, we have

$$C_1 = \frac{E}{G + \rho}, \quad \text{or,} \quad G\,C_1 + \rho\,C_1 = E.$$

The resistance ρ is now increased to R, so that a new deflection due to a current C_2 is produced; then we have

$$C_2 = \frac{E}{G + R}, \quad \text{or,} \quad G\,C_2 + R\,C_2 = E;$$

hence

$$G\,C_1 + \rho\,C_1 = G\,C_2 + R\,C_2,$$

or

$$G\,(C_1 - C_2) = R\,C_2 - \rho\,C_1,$$

therefore

$$G = \frac{R\,C_2 - \rho\,C_1}{C_1 - C_2}. \qquad [A]$$

In the case of a tangent galvanometer, if the deflections, D and *d*, are read from the *tangent* scale, then those deflections can be directly substituted for the quantities C_1, C_2, for

$$D : d :: C_1 : C_2;$$

in this case, then we have

$$G = \frac{R\,d - \rho\,D}{D - d}. \qquad [B]$$

(1.) *For example.*

With a tangent galvanometer whose resistance G was required, and a battery of very small resistance, we obtained with a resistance of 10 ohms (ρ) in the circuit, a deflection of 60 divisions (D) on the tangent scale of the instrument; when the resistance was increased to 230 ohms (R) the deflection was reduced to 20 divisions (*d*); what was the resistance of the galvanometer?

$$G = \frac{230 \times 20 - 10 \times 60}{60 - 20} = 100 \text{ ohms}.$$

If the readings are made from the *degrees* scale, then we must

substitute the tangents of the deflections for the deflections themselves; the formula then becomes

$$G = \frac{R \tan d^\circ - \rho \tan D^\circ}{\tan D^\circ - \tan d^\circ}. \qquad [C]$$

(2.) *For example.*

In a measurement similar to the foregoing, the readings were made from the *degrees* scale of the instrument, and deflections of 50° (D°) and $21\frac{3}{4}$° (d°) respectively were obtained with resistances of 10 ohms (ρ) and 229 ohms (R) in the circuit. What was the resistance of the galvanometer?

$$\tan 50^\circ = 1 \cdot 1918, \qquad \tan 21\tfrac{3}{4}^\circ = \cdot 3990,$$

therefore

$$G = \frac{229 \times \cdot 3990 - 10 \times 1 \cdot 1918}{1 \cdot 1918 - \cdot 3990} = 100 \text{ ohms.}$$

122. If in equations [B] (page 123) and [C] we have $\rho = 0$, that is to say, if we make the best by having at first no resistance in the circuit except that of the galvanometer itself, then we get

$$G = R \frac{d}{D - d} \qquad [D]$$

and

$$G = R \frac{\tan d^\circ}{\tan D^\circ - \tan d^\circ}. \qquad [E]$$

123. What are the "Best conditions for making the test?" and, what is the "Possible degree of accuracy attainable?" There are two points to be considered in the first question; one is—what value should ρ have? and the other—what should be the relative values of C_1 and C_2?

Now we are liable to make an error in reading the value of C_1, or an error in reading the value of C_2, or again we may make errors both in C_1 and C_2, but inasmuch as the result of two errors would, of course, be greater than one only, it is advisable to make the test under conditions which ensure the result of the double error being as small as possible. Let us, therefore, in equation [A] (page 123) suppose that there is a small error, c_2, in C_2, and a small error, c_1, in C_1, the error c_2 being plus and c_1 minus, so that the resulting total error in G is as great as possible; also let λ be this total error, that is, let us have

$$G + \lambda = \frac{R(C_2 + c_2) - \rho(C_1 - c_1)}{(C_1 - c_1) - (C_2 + c_2)},$$

or

$$\lambda = \frac{R(C_2 + c_2) - \rho(C_1 - c_1)}{(C_1 - c_1) - (C_2 + c_2)} - G;$$

but

$$G = \frac{R\,C_2 - \rho\,C_1}{C_1 - C_2}, \quad \text{or,} \quad R = \frac{(C_1 - C_2)G + \rho\,C_1}{C_2}.$$

If we insert this value of R in the above equation, and multiply up, cancel, &c., then we get

$$\lambda = \frac{(C_1 c_2 + C_2 c_1)}{C_2[(C_1 - c_1) - (C_2 + c_2)]}(G + \rho);$$

or, since c_1 and c_2 are very small, we may say

$$\lambda = \frac{C_1 c_2 + C_2 c_1}{C_2(C_1 - C_2)}(G + \rho). \qquad [F]$$

From this equation we can see that if C_1 and C_2 have fixed values, then λ varies directly as $G + \rho$, consequently in order to make λ as small as possible, we must make ρ as small as possible; but we can also see that there is no great advantage in making ρ *very* much smaller than G.

We have next to consider what the relative values of C_1 and C_2 should be, ρ being taken as constant. In order to do this, we must assume C_1 to be constant, and then determine what value C_2 should have. We have then in equation [F] to find what value of C_2 makes λ as small as possible; to do this we require to make

$$\frac{C_1 c_2 + C_2 c_1}{C_2(C_1 - C_2)}$$

as small as possible by variation of C_2.

Now

$$\frac{C_1 c_2 + C_2 c_1}{C_2(C_1 - C_2)} = \frac{c_2}{C_1}\left[\frac{C_1 - C_2}{C_2} + \frac{C_2(\kappa + 1)}{C_1 - C_2} + \kappa + 2\right]$$

where $\kappa = \frac{c_1}{c_2}$; and since $\frac{c_2}{C_1}$ is constant, what we have to do is to make

$$\left[\frac{C_1 - C_2}{C_2} + \frac{C_2(\kappa + 1)}{C_1 - C_2} + \kappa + 2\right]$$

as small as possible.

Now

$$\left[\frac{C_1 - C_2}{C_2} + \frac{C_2(\kappa + 1)}{C_1 - C_2} + \kappa + 2\right] = \frac{C_1 - C_2}{C_2}\left[1 - \frac{C_2\sqrt{\kappa + 1}}{C_1 - C_2}\right]^2 + 2\sqrt{\kappa + 1} + \kappa + 2,$$

and in order to make the latter as small as possible we must make $1 - \frac{C_2\sqrt{\kappa + 1}}{C_1 - C_2}$ as small as possible, that is to say, we must make it equal to 0, therefore

$$1 - \frac{C_2\sqrt{\kappa + 1}}{C_1 - C_2} = 0, \quad \text{or,} \quad C_1 - C_2 = C_2\sqrt{\kappa + 1},$$

from which we get

$$C_2(\sqrt{\kappa + 1} + 1) = C_1, \quad \text{or,} \quad C_2 = \frac{C_1}{\sqrt{\kappa + 1} + 1}. \qquad [G]$$

The greatest possible value which λ could have would be that which would result when both the errors c_1 and c_2 existed, these two errors being of equal value, or rather c_2 being as large as c_1. If the deflections are read in *divisions*, then c_1 and c_2 would be equal; but if the deflections are read in *degrees*, then c_1 will be larger than c_2 in proportion as C_2 is smaller than C_1. In the case where

the greatest possible error can exist, that is, when $c_2 = c_1$, or $\kappa = 1$, then we have

$$C_2 = \frac{C_1}{\sqrt{2} + 1} = \frac{C_1}{2 \cdot 4142}.$$

Practically we may make

$$C_2 = \frac{C_1}{3};$$

for although this does not give the exact minimum value to λ, yet the difference between it and the actual minimum is very small; thus if

$$C_2 = \frac{C_1}{2 \cdot 4142},$$

then from equation [F] we get

$$\lambda = c_1 \frac{C_1 + \frac{C_1}{2 \cdot 4142}}{\frac{C_1}{2 \cdot 4142}\left(C_1 - \frac{C_1}{2 \cdot 4142}\right)} (G + \rho) = \frac{c_1}{C_1} 5 \cdot 828 (G + \rho);$$

but if

$$C_2 = \frac{C_1}{3},$$

then

$$\lambda = c_1 \frac{C_1 + \frac{C_1}{3}}{\frac{C_1}{3}\left(C_1 - \frac{C_1}{3}\right)} (G + \rho) = \frac{c_1}{C_1} 6 \cdot 000 (G + \rho);$$

that is to say, the errors would be as

6·000 to 5·828,

a difference which is of no practical importance.

If the readings were made from the *degrees* scale of a tangent galvanometer, then the error c_1 would be larger than the error c_2, in which case it would be actually an advantage to make C_2 equal to $\frac{C_1}{3}$ in preference to making it equal to $\frac{C_1}{2 \cdot 4142}$; thus, if c_1 were, say, 3 times as large as c_2, then the best value to give to C_2 would be

$$C_2 = \frac{C_1}{\sqrt{3 + 1} + 1\frac{1}{2}} = \frac{C_1}{3}.$$

The rule that C_2 should approximately equal $\frac{C_1}{3}$ may therefore be taken as the one which would enable satisfactory results to be obtained under all conditions.

If the deflections, D d, are read in *divisions*, then we must have

$$d = \frac{D}{3}$$

approximately. But if the deflections are in *degrees*, and we read from a tangent

galvanometer, then we must have

$$\tan d^\circ = \frac{\tan D^\circ}{3}$$

approximately.

124. We have next to consider what is the "Possible degree of accuracy attainable" when ρ and G have any particular values; this we can ascertain from equation [F] (page 125). Let us then, in this equation, put percentages for absolute values, that is to say, let us have

$$\lambda = \frac{\lambda'}{100} \text{ of } G, \quad \text{or,} \quad \lambda' = \frac{100\,\lambda}{G},$$

then we get

$$\lambda' = \frac{(C_1\,c_2 + C_2\,c_1)\,100}{C_2\,(C_1 - C_2)}\left(1 + \frac{\rho}{G}\right). \qquad [H]$$

If the deflections are read in *divisions*, then the errors in both must be of the same absolute values; let each of these values be $\frac{1}{m}$th of a division, then we must have

$$\lambda' = \frac{\frac{1}{m}\,(D + d)\,100}{d\,(D - d)}\left(1 + \frac{\rho}{G}\right). \qquad [I]$$

For example.

In example (1) (page 123) what would be the degree of accuracy with which the test could be made? The deflections could be read to an accuracy of $\frac{1}{4}$ of a division.

$$\lambda' = \frac{\frac{1}{4}\,(60 + 20)\,100}{20\,(60 - 20)}\left(1 + \frac{10}{100}\right) = 2 \cdot 8 \text{ per cent.}$$

If the deflections are read in *degrees* from a tangent galvanometer, then we must have

$$\lambda' = \frac{(\tan D^\circ\,\delta_2 + \tan d^\circ\,\delta_1)\,100}{\tan d^\circ\,(\tan D^\circ - \tan d^\circ)}\left(1 + \frac{\rho}{G}\right) \text{ per cent.}$$

where δ_1 and δ_2 are of the respective values

$$\delta_1 = \tan D\,\tfrac{1}{m}^\circ - \tan D, \quad \text{and,} \quad \delta_2 = \tan d\,\tfrac{1}{m}^\circ - \tan d^\circ,$$

$\frac{1}{m}$ being the possible error in the deflections.

For example.

In example (2) (page 124) what would be the degree of accuracy with which the test could be made? The deflections could be read to an accuracy of $\frac{1}{4}^\circ$.

$$\delta_1 = \tan 50\tfrac{1}{4}^\circ - \tan 50^\circ = \cdot 0106,$$

and

$$\delta_1 = \tan 22^\circ - \tan 21\tfrac{3}{4}^\circ = \cdot 0050;$$

therefore

$$\lambda' = \frac{(1 \cdot 1918 \times \cdot 0050 + \cdot 3990 \times \cdot 0106)\,100}{\cdot 3990\,(1 \cdot 1918 - \cdot 3990)}\left(1 + \frac{10}{100}\right) = 3 \cdot 6 \text{ per cent.}$$

To sum up, then, we have

Best Conditions for making the Test.

125. Make ρ as small as possible.

Make R of such a value that when the deflections, D, d, are in *divisions*, then

$$d = \frac{D}{3}$$

approximately; and when the deflections are in *degrees* on a tangent galvanometer, then

$$\tan d^\circ = \frac{\tan D^\circ}{3}$$

approximately.

Possible Degree of Accuracy attainable.

If the deflections are in *divisions*, and if we can read their value to an accuracy of $\frac{1}{m}$th of a division, then we can determine the value of G to an accuracy, λ', of

$$\lambda' = \frac{\frac{1}{m}(D + d)\,100}{d\,(D - d)}\left(1 + \frac{\rho}{G}\right) \text{ per cent.}$$

If the deflections are in *degrees* on a tangent galvanometer, then if we can read their value to an accuracy of $\frac{1}{m}$th of a degree, we can determine the value of G to an accuracy, λ', of

$$\lambda' = \frac{(\tan D^\circ\,\delta_2 + \tan d^\circ\,\delta_1)\,100}{\tan d^\circ\,(\tan D^\circ - \tan d^\circ)}\left(1 + \frac{\rho}{G}\right) \text{ per cent.}$$

where

$$\delta_1 = \tan D\,\tfrac{1}{m}^\circ - \tan D^\circ, \text{ and, } \delta_2 = \tan \tfrac{1}{m}^\circ - \tan d^\circ.$$

Diminished Deflection Shunt Method.

126. Referring to Fig. 74, this method is as follows:—

The galvanometer G, whose resistance is to be determined, is joined up with a resistance R, a battery E, and a shunt S_1; the deflection obtained is noted; let this deflection be due to a current C_1, then (page 106) we have

$$C_2 = \frac{E\,S_1}{G\,(S_1 + R) + S_1\,R},$$

or

$$\frac{C_1 G (S_1 + R) + C_1 S_1 R}{S_1} = E.$$

The resistance of the shunt is now reduced to S_2, so that the galvanometer deflection is also reduced; let this new deflection be due to a current C_2, then we must have

$$\frac{C_2 G (S_2 + R) + C_2 S_2 R}{S_2} = E;$$

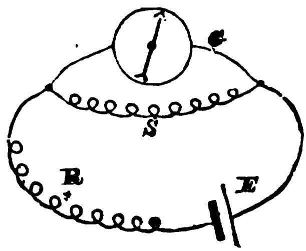


FIG. 74.

therefore

$$\frac{C_2 G (S_2 + R) + C_2 S_2 R}{S_2} = \frac{C_1 G (S_1 + R) + C_1 S_1 R}{S_1},$$

that is

$$G [C_2 S_1 (S_2 + R) - C_1 S_2 (S_1 + R)] = S_1 S_2 R (C_1 - C_2),$$

from which we get

$$G = \frac{S_1 S_2 R (C_1 - C_2)}{C_2 S_1 (S_2 + R) - C_1 S_2 (S_1 + R)},$$

or

$$G = \frac{C_1 - C_2}{C_2\left(\frac{1}{S_2} + \frac{1}{R}\right) - C_1\left(\frac{1}{S_1} + \frac{1}{R}\right)}. \qquad [A]$$

In the case of a tangent galvanometer, if the deflections, D and d, are read from the *tangent* scale, then we should have

$$G = \frac{D - d}{d\left(\frac{1}{S_2} + \frac{1}{R}\right) - D\left(\frac{1}{S_1} + \frac{1}{R}\right)}. \qquad [B]$$

(1) *For example.*

With a tangent galvanometer whose resistance, G, was required and a battery of very small resistance, we obtained with a shunt of 200 ohms (S_1), a deflection of 60 divisions (D) on the tangent scale of the instrument; when the shunt was reduced to 25 ohms (S_2), the deflection was reduced to 20 divisions (d). The resistance, R, was 400 ohms. What was the resistance of the galvanometer?

$$G = \frac{60 - 20}{20\left(\frac{1}{25} + \frac{1}{400}\right) - 60\left(\frac{1}{200} + \frac{1}{400}\right)} = 100 \text{ ohms.}$$

If the deflections are read in *degrees*, then in equation [B] we must substitute $\tan D^\circ$ and $\tan d^\circ$ for D and d respectively; we then get

$$G = \frac{\tan D^\circ - \tan d^\circ}{\tan d^\circ\left(\frac{1}{S_2} + \frac{1}{R}\right) + \tan D^\circ\left(\frac{1}{S_1} + \frac{1}{R}\right)}. \qquad [C]$$

(2) *For example.*

In a measurement similar to the foregoing the readings were made from the *degrees* scale of the instrument, and deflections of 50° (D°) and $21\frac{3}{4}°$ (d°) respectively were obtained. The values of S_1, S_2, and R were 200, 25, and 380 ohms respectively. What was the resistance, G, of the galvanometer?

$$\tan 50^\circ = 1 \cdot 1918, \quad \tan 21\tfrac{3}{4}^\circ = \cdot 3990,$$

therefore

$$G = \frac{1 \cdot 1918 - \cdot 3990}{\cdot 3990\left(\frac{1}{25} + \frac{1}{380}\right) - 1 \cdot 1918\left(\frac{1}{200} + \frac{1}{380}\right)} = 100 \text{ ohms.}$$

127. If we make the test by having no shunt inserted when the first deflection is observed, that is to say, if we have $S_1 = \infty$, or, $\frac{1}{S_1} = 0$, then equation [B] (page 129) becomes

$$G = d\,\frac{D - d}{\left(\frac{1}{S_2} + \frac{1}{R}\right) - \frac{D}{R}}, \qquad [D]$$

and equation [C]

$$G = \frac{\tan D^\circ - \tan d^\circ}{\tan d^\circ\left(\frac{1}{S_2} + \frac{1}{R}\right) - \frac{\tan D^\circ}{R}}. \qquad [E]$$

Further still, if we make R a very high resistance, that is, if in equations [D] and [E] we make $\frac{1}{R} = 0$, then we get the simplifications

$$G = S_2\left(\frac{D}{d} - 1\right) \qquad [F]$$

and

$$G = S_2\left(\frac{\tan D^\circ}{\tan d^\circ} - 1\right). \qquad [G]$$

128. In order to determine the "Best conditions for making the test," and also the "Possible degree of accuracy attainable," let us write equation [A] (page 129) in the form,

$$\frac{1}{G} = \frac{C_2\left(\frac{1}{S_2} + \frac{1}{R}\right) - C_1\left(\frac{1}{S_1} + \frac{1}{R}\right)}{C_1 - C_2}.$$

Now this equation is similar in form to equation [B] (page 123) in the last test (Diminished deflection direct method), the only difference being that we have $\frac{1}{G}$ instead of G, and $\left(\frac{1}{S_2} + \frac{1}{R}\right)$ and $\left(\frac{1}{S_1} + \frac{1}{R}\right)$ instead of R and ρ, respectively; and inasmuch as an λ' per cent. error in $\frac{1}{G}$ is an λ' per cent. error in G (though of the opposite sign), we can see that the value of λ' must be expressed by an equation of the same form as equation [H] (page 127), that is to say, we must have

$$\lambda' = \frac{(C_1 c_2 + C_2 c_1)\,100}{C_2(C_1 - C_2)}\left[1 + G\left(\frac{1}{S_1} + \frac{1}{R}\right)\right] \text{ per cent.} \qquad [H]$$

We can see, therefore, from the investigations in the last test that we must have

Best Conditions for making the Test.

129. Make S_1 and R as large as possible * (§ 125, page 128).

Make S_2 of such a value that when the deflections, D and *d*, are in *divisions*, then

$$d = \frac{D}{3}$$

* The investigations in the case of the last test prove that we should make $\left(\frac{1}{S_1} + \frac{1}{R}\right)$ as *small* as possible; this, of course, is equivalent to making S_1 and R as *large* as possible.

approximately; and when the deflections are in *degrees* on a tangent galvanometer, then

$$\tan d^\circ = \frac{\tan D^\circ}{3}$$

approximately.

Possible Degree of Accuracy attainable.

If the deflections are in *divisions*, and if we can read their value to an accuracy of $\frac{1}{m}$th of a division, then we can determine the value of G to an accuracy, λ', of

$$\lambda' = \frac{\frac{1}{m}(D + d)\,100}{d\,(D - d)}\left[1 + G\left(\frac{1}{S_1} + \frac{1}{R}\right)\right] \text{ per cent.}$$

If the deflections are in *degrees* on a tangent galvanometer, then if we can read their value to an accuracy of $\frac{1}{m}$th of a degree, we can determine the value of G to an accuracy, λ', of

$$\lambda' = \frac{(\tan D^\circ\,\delta_2 + \tan d^\circ\,\delta_1)\,100}{\tan d^\circ\,(\tan D^\circ - \tan d^\circ)}\left[1 + G\left(\frac{1}{S_1} + \frac{1}{R}\right)\right] \text{ per cent.}$$

where

$$\delta_1 = \tan D_{\frac{1}{m}}^\circ - \tan D^\circ, \quad \text{and,} \quad \delta_2 = \tan d_{\frac{1}{m}}^\circ - \tan d^\circ.$$

130. It may be remarked, that in the foregoing methods unless the galvanometer under measurement has a high degree of "sensitiveness" (page 85), then even a moderate degree of accuracy in making the tests cannot be assured.

CHAPTER VI.

MEASUREMENT OF THE INTERNAL RESISTANCE OF BATTERIES.

Half Deflection Method.

131. On page 5 a formula is given for determining the resistance r of a battery, viz.:—

$$r = R - (2\rho + G),$$

where G is the resistance of the galvanometer employed to make the test, ρ a resistance which gave a certain current through the galvanometer, and R a larger resistance which caused the strength of this current to be halved.

As this, though a simple, is a very good test, and is one which is very frequently made use of, a numerical example may prove of value.

For example.

With a galvanometer whose resistance was 100 ohms (G), and a battery whose resistance (r) was to be determined, we obtained with a resistance in the resistance box of 150 ohms (ρ), a deflection representing a current of a certain strength, and on increasing ρ to 600 ohms (R), we obtained a deflection which showed the current strength to be halved. What was the resistance of the battery?

$$r = 600 - (2 \times 150 + 100) = 200 \text{ ohms}.$$

To avoid mistakes, it should be carefully observed that in working out the formula we "*First double the small resistance*; *to the result add the resistance of the galvanometer, and deduct this total from the greater resistance.*"

132. A very common method of making this test is to employ a galvanometer of practically no resistance, and to take the first deflection with no resistance in the circuit except that of the battery itself. In this case $(2\rho + G) = 0$, so that

$$r = R$$

or the added resistance is the resistance of the battery.

133. If we compare the first method (§ 131) with the test for determining the resistance of a galvanometer described on page 102 (§ 93), we can see that the two are almost identical. In the one case we determine the resistance of the galvanometer, and in the other we determine the resistance of the battery plus the galvanometer, and then from the result deduct the value of the galvanometer. This being so, we can see that the

Best Conditions for making the Test

are obtained by making $\rho + G$ a fractional value of r; to do which we should require a galvanometer of low resistance.

As regards the *possible degree of accuracy attainable*, we can see from the galvanometer test referred to, that

$$\lambda' = 2\left(1 + \frac{G}{r}\right)\gamma';$$

that is to say:—

Possible Degree of Accuracy attainable.

If we can be certain of the value of the galvanometer deflection to an accuracy of γ' per cent., then we can be certain of the accuracy of the value of r within $2\left(1 + \frac{G}{r}\right)\gamma'$; per cent.

Or if we employ a galvanometer of low resistance, then we can be certain of the accuracy of the value of r within $2\,\gamma'$ per cent.

If the galvanometer deflection be too high, i. e. above about 55° (page 30, § 35), with the lowest value we can give to ρ, then the galvanometer must be reduced in sensitiveness by being shunted, and the value of G in the formula will then be the combined resistance of the galvanometer and shunt, that is, the product of the two divided by their sum (page 90).

Thomson's Method.

134. Fig. 75 shows the theoretical, and Fig. 76 the practical methods of making this test.

The theory of the method is as follows: The galvanometer G, a resistance ρ, and the battery whose resistance r is required, are joined up in simple circuit with a shunt S between the poles of the battery; a deflection of the galvanometer needle is produced with a resistance ρ in the resistance box. The shunt is now removed; this causes the deflection to become larger; ρ is

then increased until the deflection becomes the same as it was at first. Let the new resistance be R, and let E be the electromotive force of the battery and C the current passing through the galvanometer.

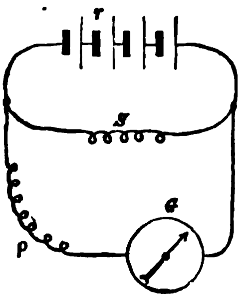


FIG. 75.

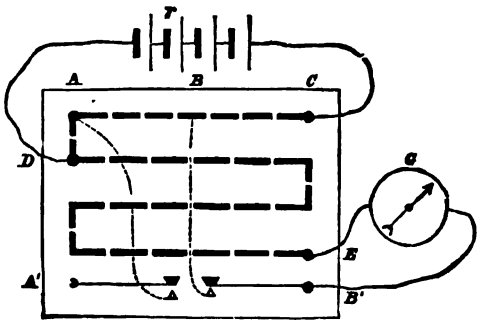


FIG. 76.

In the first case we have

$$C = \frac{E}{r + \dfrac{S(\rho + G)}{S + \rho + G}} \times \frac{S}{S + \rho + G}$$

$$= \frac{ES}{r(S + \rho + G) + S(\rho + G)},$$

and in the second case

$$C = \frac{E}{r + R + G};$$

therefore

$$\frac{E}{r + R + G} = \frac{ES}{r(S + \rho + G) + S(\rho + G)}.$$

By multiplying up and cancelling,

$$r(\rho + G) = S(R - \rho),$$

or

$$r = S\frac{R - \rho}{\rho + G}.$$

For example.

A battery whose resistance (r) was required, was joined up in circuit with a resistance of 200 ohms (ρ) and a galvanometer of

100 ohms (G), a shunt of 10 ohms (S) being between the poles of the battery.

On removing the shunt it was necessary, in order to reduce the increased deflection to what it was originally, to increase ρ to 3200 ohms (R). What was the resistance of the battery?

$$r = 10 \frac{3200 - 200}{200 + 100} = 100 \text{ ohms.}$$

135. The investigation for determining the best resistances to employ in making this test would be conducted in precisely the same manner as that given on page 108 *et seq.* For the equation

$$r = S \frac{R - \rho}{\rho + G}$$

is the same as

$$r = S \frac{(R + G) - (\rho + G)}{\rho + G},$$

which is the same kind of equation as the one in the test we have referred to, viz.:—

$$G = S \frac{R - \rho}{\rho};$$

and as in this case we proved that S was to be as small and R as large as possible, so from the preceding equation we should prove that S should be as small, and R + G as large, as possible. In order, therefore, to obtain the

Best Conditions for making the Test,

136. First make a rough test to ascertain approximately what is the value of r. Having done this, insert a shunt (S) between the poles of the battery, of less resistance than r.

Next joint up ρ in circuit with G, with the battery, and with its shunt S, making $\rho + G$ not larger than $\frac{S}{G}(G + R)$; R being the highest resistance that can be inserted in the circuit.

The galvanometer needle being obtained at the angle of maximum sensitiveness, note the value of ρ.

Now remove the shunt and increase ρ to R, so that the increased deflection becomes the same as it was at first. Note R and calculate r from the formula.

Possible Degree of Accuracy attainable.

From the galvanometer test referred to, we can see that if we can determine the value of the galvanometer deflection to an accuracy of γ' per cent., then we can determine the accuracy of r to an accuracy of

$$\left(1 + \frac{S}{r}\right)\left(1 + \frac{r'}{R + G}\right)\gamma' \text{ per cent.}$$

137. As we cannot in this test vary the resistance of the galvanometer so as to obtain the deflection at the angle of maximum sensitiveness, we must, if the deflection be too high with the highest resistances we can put in the circuit, reduce its sensitiveness by means of a shunt between its terminals; the value of G in the formula will then be the combined resistance of the galvanometer and its shunt.

The constancy of a battery being likely to be impaired by its being on a circuit of low resistance, it is not advisable to reduce the deflection of the galvanometer by making S very small. In fact S, although it should be lower than the resistance of the battery, should not, in this test, be made lower than we can help. Thus, if the resistance of the battery were about 200 ohms, it would be preferable to make S 100 rather than 10 ohms. Should the deflection of the galvanometer needle be too low, the only thing to be done is to use another which has a higher figure of merit.

138. A Thomson galvanometer (page 48) answers very well for tests like this, as its figure of merit can always be made sufficiently low by placing a shunt made of a short piece of wire between its terminals.

139. If we adjust ρ in the first place so that together with G it equals S, we get the simplified formula

$$r = S\frac{R - \rho}{S} = R - \rho;$$

that is, the added resistance is the resistance of the battery.*

Again, if we commence with no other resistance in the galvanometer circuit beyond that of the galvanometer itself, we get the simplification

$$r = S\frac{R}{G};$$

* Sabine's 'The Electric Telegraph,' p. 314.

Lastly, if we make $S = G$, then we get

$$r = R.$$

If we arrange the tests, however, so as to use these simplified formulæ, we are obliged to employ an arrangement of resistances which would not be at all advisable if we wish for accuracy, and it is very questionable whether any advantage is gained by adopting a simplification of a formula, in itself simple, at the expense of accurate testing.

The arrangement of key described in § 109, page 114, may obviously be applied to the foregoing tests with advantage.

Siemens' Method.

140. Fig. 77 shows the arrangement of resistances, &c., for determining the resistance of a battery by Siemens' method.

A C is a resistance on the slide principle (§ 19, page 17), R a resistance connected to the junction of the galvanometer G and the battery whose resistance r is required. The other end of R is connected to the slider B.

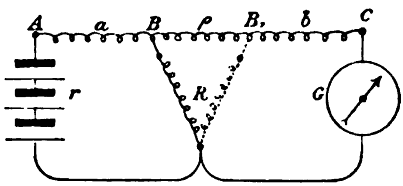


Fig. 77.

Now it will be found that if B be moved towards A or towards C from a certain point midway between A and C, the current flowing through the galvanometer will be increased.

It follows from this that if we put B near A and obtain a certain deflection, we can also obtain this same deflection by sliding B to a point near C.

Let B and B_1 be these points, and let a be the resistance between A and B, b the resistance between B_1 and C, and ρ the resistance between B and B_1. Also let E be the electromotive force of the battery, and r its resistance, and let C be the current deflecting the galvanometer needle.

Now when the slider is at B

$$C = \frac{E}{r + a + \dfrac{R(\rho + b + G)}{R + \rho + b + G}} \times \frac{R}{R + \rho + b + G}$$

$$= \frac{E\,R}{(r + a)(R + \rho + b + G) + R(\rho + b + G)},$$

and when the slider is at B_1

$$C = \frac{E\,R}{(r + a + \rho)(R + b + G) + R(b + G)};$$

therefore

$$(r + a)(R + \rho + b + G) + R(\rho + b + G)$$
$$= (r + a + \rho)(R + b + G) + R(b + G);$$

therefore

$$(r + a)\,\rho + R\,\rho = \rho\,(R + b + G);$$

from which

$$r + a = b + G$$

or

$$r = G + b - a.$$

In making this test, then, what we have to do is to note what are the values of A B (a) and B_1 C (b) when the same deflections are obtained on the galvanometer, then from these values and the resistance of the galvanometer we can determine the resistance of the battery.

141. Another way of making the test is to find the point between A and C which gives the least deflection; then a and b will be the resistances on either side of this point.

142. Let us now consider what are the "Best conditions for making the test." The points to be considered are, what are the best resistances to make R and A C, and also, at what point should we place the slider to commence with, that is, should we place it near one of the ends of A C, or at some point nearer the middle of the latter?

From the equation

$$r = G + b - a$$

it is clear that any error made in b or a will make an exactly corresponding error in r; in considering the problem, therefore, we have simply to determine what arrangement of resistances, &c., will cause any slight error in a or b, that is any slight movement of the slider, to make the greatest possible alteration in the current, that is in the deflection of the galvanometer needle.

Let us suppose the slider was at B for the first observation, and let us suppose that when the slider was at that point, a current C flowed though the galvanometer, and that when the slider was moved to B_1, the current was also C. Further, when the slider was moved a distance λ beyond B towards, say, A, let us suppose the current was increased to $C + c$.

We have then to determine what arrangement of resistances, &c., will make $\frac{c}{C}$ as large as possible.

Now

$$C = \frac{E\,R}{(r+a)(R+\rho+b+G)+R(\rho+b+G)},$$

and we know that

$$r+a=b+G;$$

consequently

$$C = \frac{E\,R}{(r+a)(R+\rho+r+a)+R(\rho+r+a)},$$

and by putting $a-\lambda$ for a, and $\rho+\lambda$ for ρ, we get

$$C+c = \frac{E\,R}{(r+a-\lambda)(R+\rho+r+a)+R(\rho+r+a)} = C_1,$$

or

$$c = C_1 - C;$$

therefore

$$\frac{c}{C} = \frac{C_1}{C} - 1;$$

therefore

$$\frac{c}{C} = \frac{\lambda(R+\rho+r+a)}{(r+a-\lambda)(R+\rho+r+a)+R(\rho+r+a)},$$

or, since λ is a very small quantity, we may say

$$\frac{c}{C} = \frac{\lambda(R+\rho+r+a)}{(r+a)(R+\rho+r+a)+R(\rho+r+a)}, \qquad [A]$$

or

$$\frac{c}{C} = \frac{\lambda}{r+a+\dfrac{R(\rho+r+a)}{R+(\rho+r+a)}}. \qquad [B]$$

We will first determine at what point the slider should be placed to commence with.

Now if we show at what point it should be placed near A, we determine the point at which it should be placed near C, for $r+a$ must equal $G+b$. What we have to do then is to determine the best value to give to a.

To do this we must suppose the resistance A C to be constant, or since r and G are naturally constants, we must have

$$r+a+\rho+b+G;$$

that is,

$$r+a+\rho+r+a,$$

equal to a constant, say, K; therefore

$$\rho+r+a = K-(r+a),$$

therefore, by equation [A], we get

$$\frac{c}{C} = \frac{\lambda(R+K-(r+a))}{(r+a)(R+K-(r+a))+R(K-(r+a))}$$

$$= \frac{\lambda(R+K-(r+a))}{(r+a)(K-(r+a))+R\,K}.$$

From this we see that the smaller we make $(r+a)$ the larger will be the numerator of the fraction. Also if $r+a$ be less than $\frac{K}{2}$ (which it must be in

the test), the smaller we make it the smaller will be the denominator of the fraction. This may be proved as follows:—

$$(r+a)\left(K-(r+a)\right)=(r+a)\,K-(r+a)^2=\frac{K^2}{4}-\left((r+a)-\frac{K}{2}\right)^2.$$

If in the latter expression we make

$$r+a=\frac{K}{2},$$

then

$$\left((r+a)-\frac{K}{2}\right)^2=0,$$

which makes the expression as small as possible.

But if we make $r+a$ either larger *or* smaller than $\frac{K}{2}$, then $\left((r+a)-\frac{K}{2}\right)^2$ does not equal 0, but it has a plus value which increases in proportion as we make either $(r+a)$ larger than $\frac{K}{2}$, or $\frac{K}{2}$ larger than $(r+a)$; for although $\left((r+a)-\frac{K}{2}\right)$ in one case will have a positive, and in the other case a negative value, still $\left((r+a)\,\frac{K}{2}\right)^2$ is positive in both cases.

If, therefore, we make $(r+a)$ smaller than $\frac{K}{2}$, the value of the expression referred to, and consequently the value $(r+a)\left(K-(r+a)\right)$, will increase in proportion.

Consequently the smaller we make $(r+a)$, and therefore a, the larger will $\frac{C}{c}$ be.

It is best, therefore, to place the slider to commence with as near to one end of A C as possible.

Next we have to determine what value we should give to A C. This we shall do if we determine what value ρ should have. If we write equation [B] (page 140) in the form

$$\frac{c}{C}=\frac{\lambda}{r+a+\cfrac{1}{\cfrac{1}{R}+\cfrac{1}{\rho+r+a}}},$$

we can see that r, a, and R being constant, $\frac{c}{C}$ is made as large as possible by making ρ as small as possible; but we can also see that there is but little use in making ρ much smaller than $r+a$, or, as a ought to be small, in making it much smaller than r.

Lastly, we have to find what value it is best to give to R; this we can also determine from the last equation. We can see from the latter that, r, a, and ρ, being constant quantities, $\frac{c}{C}$ is made as large as possible by making R as small as possible; but we can also see that we gain but very little by making R much smaller than $r+a$, or, as a ought to be small, by making it smaller than r. Actually, of course, we could not make R extremely small, for the reason that the

battery and galvanometer would then be practically short-circuited, and a readable deflection could not be obtained.

Since

$$r + a = G + b,$$

a can only be made small by having G small; it is therefore best to have a galvanometer of as low a resistance as possible, or rather of a resistance not exceeding r.

We proved that the slider should be as near one end of A C as possible. The end we can place it nearest to must evidently be the end to which the greatest resistance is connected; therefore, whichever value of r or G happens to be the greatest, at the end to which that larger value is connected should the slider be placed to commence with.

In order to determine the "percentage of accuracy attainable" we must in equation [B] (page 140) put percentages λ' and γ' for the absolute values λ and c, that is to say, we must have

$$\lambda = \frac{\lambda'}{100} \text{ of } r, \quad \text{and,} \quad c = \frac{\gamma'}{100} \text{ of C,}$$

in which case we get

$$\lambda' = \left[r + a + \frac{R(\rho + r + a)}{R + \rho + r + a} \right] \frac{\gamma'}{r} \text{ per cent.}$$

To summarise the results, then, we have

Best Conditions for making the Test.

143. The slider at commencing should be as near as possible to that end of A C to which is connected the greatest of the values r and G. The value of A C should not be less than the value of the greater of the two quantities r and G. R should be lower than the greater of the two quantities r and G.

The galvanometer resistance should not exceed r, and the deflection should be obtained at the angle of maximum sensitiveness. This can be done by varying R; but inasmuch as the latter should be lower than r, it is desirable to use a galvanometer of such sensitiveness that R can be made sufficiently small without reducing the deflection too low.

Possible Degree of Accuracy attainable.

If we can be certain of the galvanometer deflection to an accuracy of γ' per cent., then we can be certain of the value of r to an accuracy, λ', of

$$\lambda' = \left[r + a + \frac{R(\rho + r + a)}{R + \rho + r + a} \right] \frac{\gamma'}{r} \text{ per cent.}$$

If R, a, and ρ are very small compared with r, then we get

$$\lambda' = \gamma'.$$

144. As in previous tests, we should first determine the value of r roughly, and then more exactly with the resistances properly arranged.

145. We have hitherto supposed A C to be a *slide resistance*, but it is not absolutely necessary that it should be so; the test can very well be made in the following manner :—

Referring to Fig. 77, and supposing r to be greater than G, let the resistances ρ and b be ordinary ones and both capable of variation, and let the resistance a be done away with.

Having connected R to B, that is, to the pole A of the battery, plug up all the resistance in b and adjust ρ and R till the deflection of maximum sensitiveness is obtained on the galvanometer. Care must be taken that the adjustment of ρ and R is so made that R is less and ρ greater than G. If the galvanometer has a sufficiently high figure of merit, there will be no difficulty in doing this.

Next shift the connection of R from B to B_1, and proceed to adjust a nd ρ until the original deflection is reproduced, the adjustment being made in such a manner that the same resistance is plugged up in ρ that is unplugged in b; then

$$r = G + b.$$

It must be noted that of the two quantities G and r the one which has the greatest resistance must be connected to ρ at B. In the case we have considered we have supposed that r was the larger quantity, but if G had been the larger of the two the position of G and r would have had to have been reversed, and the resistance of r would have been given by the formula

$$r = G - b.$$

The *modus operandi* of the test would, however, be precisely the same in the two cases.

Two sets of resistance coils are evidently necessary to make this test, as it cannot be made with a single set of the ordinary kind (Fig. 6, page 14).

Mance's Method.

146. This test is of a very similar nature to Thomson's method of determining the resistance of a galvanometer given on page 115. Fig. 78 (page 144) shows the theoretical method of making the test.

In the theoretical figure, a, b, and d are resistances, g a galvanometer, and E a battery whose resistance r is required.

A key is inserted between the junctions of a with b and d with r. By depressing this key the junctions are connected together.

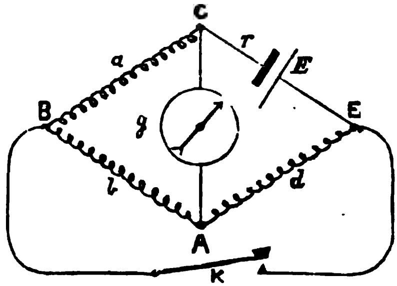


FIG. 78.

Let us first suppose the key to be up, then the current C_1 flowing through the galvanometer will be

$$C_1 = \frac{E}{r + d + \frac{(a+b)g}{a+b+g}} \times \frac{a+b}{a+b+g}$$

$$= \frac{E(a+b)}{g(a+b+d+r) + (a+b)(d+r)}. \qquad [1]$$

Next, suppose the key to be pressed down; then the current C_2 flowing through the galvanometer will be

$$C_2 = \frac{E}{r + \frac{\left(\frac{bd}{b+d} + g\right)a}{\frac{bd}{b+d} + g + a}} \times \frac{a}{\frac{bd}{b+d} + g + a}$$

$$= \frac{E(b+d)a}{g(a+r)(b+d) + bd(a+r) + ar(b+d)}. \qquad [2]$$

Now if the resistances be adjusted so that the deflection of the galvanometer needle remains the same whether the key is depressed or not, then equations [1] and [2] are equal; that is

$$\frac{E(a+b)}{g(a+b+d+r) + (a+b)(d+r)}$$

$$= \frac{E(b+d)a}{g(a+r)(b+d) + bd(a+r) + ar(b+d)}.$$

If we refer to "Thomson's galvanometer resistance test" on page 116, we can see that this equation is similar to equation [3] on that page with the exception that r and g are interchanged. It must therefore be obvious, by the same development of the equation as that given on the page referred to, that

$$r = \frac{a\,d}{b}.*$$

147. The great advantage of this test is that the electromotive force of the battery need only be constant during the very short interval of time occupied in depressing and raising the key.

148. In making the test practically the connections would be made as shown by Fig. 78. Terminals E and B′ would be joined by a short piece of thick wire; the other connections are obvious.

The left-hand key puts the galvanometer on; this key must be depressed and held permanently down, and the right-hand key then alternately depressed and raised, and the resistance d, that is, the resistance between A and E, at the same time adjusted until the deflection of the galvanometer needle remains the same whether the key is up or down.

* The truth of this proportion also follows from the principle of the Wheatstone Bridge (Chapter VIII.). For, supposing to commence with, we have no electromotive force in the arm C E, and that we have a battery in circuit with the key K—the ordinary Wheatstone Bridge arrangement, in fact—then, when

$$r = \frac{a\,d}{b}$$

we have no movement of the galvanometer needle on opening or closing K. Let us next suppose the electromotive force E to exist, then the result of E is in no way to affect the values of the resistances, it simply causes currents to flow through the latter, and through g amongst others, these currents being superimposed on those caused by the battery which we have supposed to be in circuit with the key K. But inasmuch as there is no current through g due to the battery in circuit with K, and as all the other conditions remain unchanged, the movement of K cannot affect the current which is now flowing through g, though it would do so if the proportion $r = \frac{a\,d}{b}$ were not satisfied, for in that case a current due to the battery in circuit with K would flow through g when K is depressed, this current being superimposed on the current due to E. Lastly, if no effect is produced on the galvanometer needle when there is a battery in circuit with K, and when K is raised and depressed, then it is obvious that no effect can be produced if there is no battery in the circuit of K. If, however, under the latter condition the raising and depression of K *does* cause the galvanometer needle to move, then it is clear that the proportion $r = \frac{a\,d}{b}$ is not satisfied, that is to say, the absence of any movement shows that $r = \frac{a\,d}{b}$.

149. Again referring to Thomson's galvanometer resistance test; it must be clear, by substituting r for g in the equations, that to obtain the

Best Conditions for making the Test,

Make a as low as possible and b as high as possible, but not so high that d when exactly adjusted would exceed all the resistance we could insert between D and E (see Fig. 79).

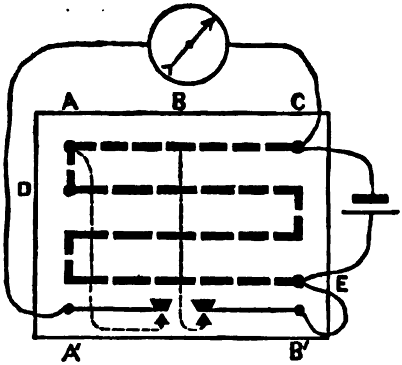


FIG. 79.

Adjust d approximately, and then, if necessary, adjust the resistance of the galvanometer shunt (which it will be necessary to employ) so that the final deflection is as nearly as possible that of maximum sensitiveness, and then, having exactly adjusted d, calculate r from the formula.

Possible Degree of Accuracy attainable.

If we can determine the value of the galvanometer deflection to an accuracy of γ per cent., then we can be certain of the value of r to an accuracy of $\left(1+\frac{a}{r}\right)\left(1+\frac{r}{d}\right)\gamma$ per cent.

150. In the practical execution of the test with the set of resistance coils shown by Fig. 79, the lowest value we could give to a would be 10 units, unless we improvised a resistance of less value, which it might be necessary to do.

MANCE'S METHOD WITH THE SLIDE WIRE BRIDGE.

151. Mance's test is sometimes made by having $a + b$ a slide wire resistance, d being a fixed resistance; in this case the slider

would be moved along between A and C until the point is found at which the depression or raising of the key makes no alteration in the deflection of the galvanometer needle.

For practically executing the test the apparatus known as the "Slide Wire" or "Metre Bridge" may be used. This apparatus, which is shown by Fig. 80, is described in Chapter VIII.

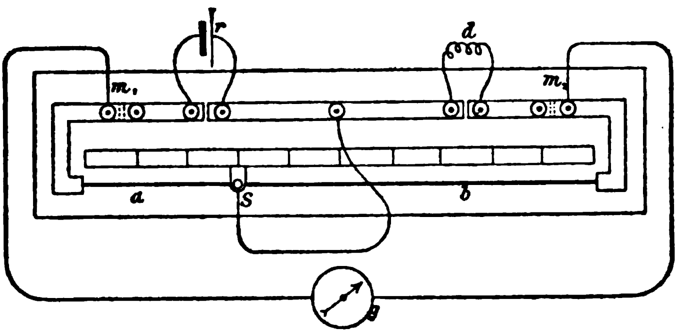


FIG. 80.

(The Wheatstone Bridge). The slide wire, $a + b$, which is 1 metre long, is stretched upon an oblong board (forming the base of the instrument) parallel to a metre scale divided throughout its whole length into millimetres, and so placed that its two ends are as nearly as possible opposite to divisions 0 and 1000 respectively of the scale. The ends of the wire are soldered to a broad, thick copper band, which passes round each end of the graduated scale, and runs parallel to it on the side opposite to the wire. This band is interrupted by four gaps, at m_1, r, d, and m_2. On each side of these gaps are terminals. In making the test under consideration, the gaps, m_1 and m_2, are closed by thick copper straps. The slider S makes contact with the slide wire by the depression of a knob on S.

The battery, r, a resistance, d, and a galvanometer, g, being joined up as shown, the slider S is moved along the scale, the knob being depressed at intervals, until the point is reached at which the depression makes no change in the permanent deflection of the galvanometer needle. When this is the case, then as in Thomson's galvanometer test (page 115), we have

$$r = d\left(\frac{a}{1000 - a}\right).$$

For example.

In the foregoing test, equilibrium was produced when d was 1 ohm, and a, 450 divisions; what was the resistance, r, of the battery?

$$r = 1 \frac{450}{1000 - 450} = \frac{450}{550} = \cdot 85 \text{ ohm.}$$

152. The *best conditions for making the test* are similar to those required for "Thomson's galvanometer test" (page 115), namely, we should make d larger than r, but not greater than about ten times r.

As a rule the complete slide wire bridge is furnished with but four resistance coils of 1 ohm each, so that the choice of a resistance to insert in d is limited, and it may not be possible to follow out the rule of "making d about 10 times as large as r." In this case the possibility of an accurate measurement becomes proportionately reduced below the highest possible standard, so that on the one hand a cell whose resistance is much less than one-tenth of an ohm, or, on the other hand, a cell whose resistance exceeds 4 ohms, cannot be measured with the highest possible accuracy.

Strictly speaking (as has been pointed out), in order to ensure accuracy it is necessary that the resistance of the portion of the slide wire, a, be less than the resistance of the battery to be measured; but as the resistance of the whole length of the wire will not exceed one-tenth of an ohm, the resistance of the length a will practically be less than the resistance of the battery, unless, of course, this resistance is extremely low.

The *possible degree of accuracy attainable* we can see from Thomson's galvanometer test (page 115) must be given by the equation

$$\lambda' = \frac{100000\,\delta}{a\,(1000 - a)} \text{ per cent.}$$

where δ is the degree of accuracy in divisions to which the slider, S, can be adjusted. If we can adjust to an accuracy of 1 division, then $\delta = 1$.

For example.

In the last example, what would be the degree of accuracy, λ', with which the value of r could be obtained, supposing that the position of the slider could be determined to an accuracy of 1 division (δ)?

$$\lambda' = \frac{100000 \times 1}{450\,(1000 - 450)} = \cdot 40 \text{ per cent.}$$

153. The sensitiveness of Mance's test can be considerably increased by including a battery in the key circuit; this must be

evident from the note on page 145. In making the test in this manner the additional battery should preferably be connected up so as to act in the same direction as the battery being measured, i. e. in the case shown by Fig. 78, the zinc of the additional battery must be connected to E.

154. The facility and accuracy with which all the foregoing tests (except the half-deflection test) can be made may be greatly increased by the following arrangement: Use a galvanometer with a high "figure of merit" (page 85), and instead of making the test with the needle brought to the "angle of maximum sensitiveness" (page 25), make it with the needle brought approximately to zero by means of a powerful permanent magnet set near the instrument; under these conditions the galvanometer needle will be highly sensitive to any small change in the current strength.

Another arrangement which may be very conveniently adopted is to employ a galvanometer with a high "figure of merit," and wound with two wires. One of these wires would be joined in circuit with the battery under test, &c., in the usual way; the other would be connected in circuit with a small battery and a set of resistance coils, the connections being so made that the currents through the two coils oppose one another. When the deflection due to the battery under test is obtained, the second battery and resistance coils are connected up, and then this battery is adjusted until the needle is brought to zero as nearly as possible. The test is then made as in the case where a permanent magnet is used.

155. In the case of Mance's test with the slide-wire bridge, if the test is made either by using a permanent magnet in the way described, or by using a galvanometer wound with a double wire, it is best to make d as nearly equal to the resistance of the battery as possible (it should not be made less), as in this case, since the slider, S, will have to be set near the centre of the scale, a greater range of adjustment is given to it, for 5 divisions near the centre portion of the scale (500 division mark) are equivalent to only 1 division near the 100 division mark. It is true the arrangement is not quite so sensitive as it would be if the slider were set towards the end of the scale; but still, if we can employ a galvanometer with a high figure of merit, this small loss of sensitiveness is more than compensated for by the increased range which can be obtained on the scale.

Diminished Deflection Direct Method.

156. This method, which has been generally described in Chapter I. (§ 6, page 4), is as follows:—

The battery whose resistance, r, is required, a galvanometer of resistance, G, and a resistance, ρ, are joined up in simple circuit; the deflection obtained is noted. Let this deflection be due to a current, C_1, then calling E the electromotive force of the battery, we have

$$C_1 = \frac{E}{r + G + \rho}, \quad \text{or,} \quad C_1 (r + G) + C_1 \rho = E.$$

The resistance, ρ, is now increased to R, so that a new deflection due to a current, C_2, is produced, then we have

$$C_2 = \frac{E}{r + G + R}, \quad \text{or,} \quad C_2 (r + G) + C_2 R = E;$$

hence

$$C_1 (r + G) + C_1 \rho = C_2 (r + G) + C_2 R,$$

or

$$(r + G)(C_1 - C_2) = C_2 R - C_1 \rho;$$

therefore

$$r + G = \frac{C_2 R - C_1 \rho}{C_1 - C_2},$$

that is

$$r = \frac{C_2 R - C_1 \rho}{C_1 - C_2} - G. \qquad [A]$$

If a tangent galvanometer is employed for making the test, then if the deflections, D and d, are read from the *tangent* scale of the instrument, those deflections can be directly substituted for the quantities, C_1, C_2, for

$$D : d :: C_1 : C_2;$$

in this case, then, we have

$$r = \frac{d R - D \rho}{D - d} - G. \qquad [B]$$

(1) *For example.*

With a tangent galvanometer whose resistance was 10 ohms (G), and a battery whose resistance, r, was required, a deflection

of 60 divisions (D) on the tangent scale of the instrument was obtained, when a resistance of 10 ohms (ρ) was in circuit; when the latter resistance was increased to 230 ohms (R) the deflection was reduced to 20 divisions (d). What was the resistance of the battery?

$$r = \frac{20 \times 230 - 60 \times 10}{60 - 20} - 10 = 90 \text{ ohms.}$$

If the readings are made from the *degrees* scale, then we must substitute the tangents of the deflections for the deflections themselves; the formula then becomes

$$r = \frac{\tan d^\circ \, R - \tan D^\circ \, \rho}{\tan D^\circ - \tan d^\circ} - G. \qquad [C]$$

(2) *For example.*

In a measurement similar to the foregoing the readings were made from the *degrees* scale of the galvanometer, and deflections of 50° (D°) and $21\frac{3}{4}$° (d°) respectively were obtained with resistances of 10 ohms (ρ) and 229 ohms (R) in the circuit. The resistance of the galvanometer was 10 ohms (G). What was the resistance, r, of the battery?

$$\tan 50^\circ = 1 \cdot 1918, \qquad \tan 21\tfrac{3}{4} = \cdot 3990,$$

therefore

$$r = \frac{\cdot 3990 \times 229 - 1 \cdot 1918 \times 10}{1 \cdot 1918 - \cdot 3990} - 10 = 90 \text{ ohms.}$$

157. If in equations [B] and [C] we have $\rho = 0$, that is to say, if we make the test by having at first no resistance in the circuit except that of the galvanometer and the battery itself, then we get

$$r = R \frac{d}{D - d} - G \qquad [D]$$

and

$$r = R \frac{\tan d}{\tan D - \tan d} - G. \qquad [E]$$

158. In order to determine the "Best conditions for making the test," and also the "Possible degree of accuracy attainable," let us write equation [A] in the form

$$r = \frac{C_2 (R + G) - C_1 (\rho + G)}{C_1 - C_2}.$$

Now this equation is similar to equation [B] (page 123) in the "Diminished Deflection Direct Method" of determining the resistance of a galvanometer, except that in the latter method we have the quantities R and ρ in the place of the quantities $(R + G)$ and $(\rho + G)$; consequently we can at once see from the investigation in the test referred to that we must have—

Best Conditions for making the Test.

159. Make ρ as small as possible.

Make R of such a value that when the deflections, D, d, are in *divisions*, then

$$d = \frac{D}{3}$$

approximately; and when the deflections are in *degrees* on a tangent galvanometer, then

$$\tan d^\circ = \frac{\tan D^\circ}{3}$$

approximately.

Possible Degree of Accuracy attainable.

If the deflections are in *divisions*, and if we can read their value to an accuracy of $\frac{1}{m}$th of a division, then we can determine the value of r to an accuracy, λ', of

$$\lambda' = \frac{\frac{1}{m}(D + d)\,100}{d\,(D - d)}\left(1 + \frac{\rho + G}{r}\right) \text{ per cent.}$$

If the deflections are in *degrees* on the tangent galvanometer, then if we can read their value to an accuracy of $\frac{1}{m}$th of a degree, we can determine the value of G to an accuracy, λ', of

$$\lambda' = \frac{(\tan D^\circ\,\delta_2 - \tan d^\circ\,\delta_1)\,100}{\tan d^\circ\,(\tan D^\circ - \tan d^\circ)}\left(1 + \frac{\rho + G}{r}\right) \text{ per cent.}$$

where

$$\delta_1 = \tan D\tfrac{1}{m}^\circ - \tan D^\circ, \quad \text{and,} \quad \delta_2 = \tan d\tfrac{1}{m}^\circ - \tan d^\circ.$$

Diminished Deflection Shunt Method.

160. This method is shown by Fig. 81. The battery, r, whose resistance is to be determined, is joined up in circuit with a resistance, R, a galvanometer, G, and a shunt, S_1; the deflection obtained is noted; let this deflection be due to a current C_1, then calling E the electromotive force of the battery, we have (page 135)

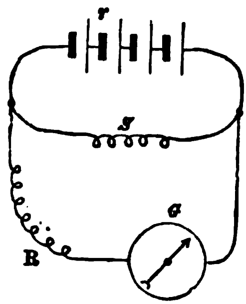


Fig. 81.

$$C_1 = \frac{E\, S_1}{r(S_1 + R + G) + S_1(R + G)},$$

or

$$\frac{C_1 r(S_1 + R + G) + C_1 S_1 (R + G)}{S_1} = E.$$

The resistance of the shunt is now reduced to S_2, so that the galvanometer deflection is also reduced; let this new deflection be due to a current C_2, then we have

$$\frac{C_2 r(S_2 + R + G) + C_2 S_2 (R + G)}{S_2} = E;$$

therefore

$$\frac{C_2 r(S_2+R+G)+C_2 S_2(R+G)}{S_2} = \frac{C_1 r(S_1+R+G)+C_1 S_1(R+G)}{S_1},$$

that is,

$$r[C_2 S_1(S_2+R+G) - C_1 S_2(S_1+R+G)] = S_1 S_2 (R+G)(C_1 - C_2),$$

from which we get

$$r = \frac{S_1 S_2 (R + G)(C_1 - C_2)}{C_2 S_1 (S_2 + R + G) - C_1 S_2 (S_1 + R + G)},$$

or

$$r = \frac{C_1 - C_2}{C_2\left(\frac{1}{S_2} + \frac{1}{R + G}\right) - C_1\left(\frac{1}{S_1} + \frac{1}{R + G}\right)}. \qquad [A]$$

In the case of a tangent galvanometer, if the deflections, D and d, are read from the *tangent* scale, then we should have

$$r = \frac{D - d}{d\left(\frac{1}{S_2} + \frac{1}{R + G}\right) - D\left(\frac{1}{S_1} + \frac{1}{R + G}\right)}. \qquad [B]$$

(1) *For example.*

With a tangent galvanometer whose resistance was 10 ohms (G), and a battery whose resistance, r, was required, we obtained with a shunt of 200 ohms (S_1), a deflection of 60 divisions (D) on the tangent scale of the instrument; when the shunt was reduced to 25 ohms (S_2) the deflection was reduced to 20 divisions (d). The resistance, R, was 710 ohms. What was the resistance of the battery?

$$r = \frac{60 - 20}{20\left(\frac{1}{25} + \frac{1}{710 + 10}\right) - 60\left(\frac{1}{200} + \frac{1}{710 + 10}\right)} = 90 \text{ ohms.}$$

If the deflections are read in *degrees*, then in equation [B] (page 153) we must substitute tan D° and tan d° for D and d respectively, we then get

$$r = \frac{\tan D^\circ - \tan d^\circ}{\tan d^\circ\left(\frac{1}{S_2} + \frac{1}{R + G}\right) - \tan D^\circ\left(\frac{1}{S_1} + \frac{1}{R + G}\right)}. \quad [C]$$

(2) *For example.*

In a measurement similar to the foregoing the readings were made from the *degrees* scale of the galvanometer, and deflections of 50° (D°) and $21\frac{3}{4}$° (d°), respectively, were obtained. The values of S_1, S_2, R, and G were 200, 25, 655, and 10 ohms, respectively. What was the resistance r, of the battery?

$$\tan 50^\circ = 1{\cdot}1918, \qquad \tan 21\tfrac{3}{4}^\circ = {\cdot}3990.$$

Therefore

$$r = \frac{1{\cdot}1918 - {\cdot}3990}{{\cdot}3990\left(\frac{1}{25} + \frac{1}{655 + 10}\right) - 1{\cdot}1918\left(\frac{1}{200} + \frac{1}{655 + 10}\right)} = 90 \text{ ohms.}$$

161. If we make the test by having no shunt inserted when the first deflection is observed, that is to say, if we have $S_1 = \infty$, or, $\frac{1}{S_1} = 0$, then equation [B] becomes

$$r = \frac{D - d}{d\left(\frac{1}{S_2} + \frac{1}{R + G}\right) - \frac{D}{R + G}} \quad [D]$$

[and equation C]

$$r = \frac{\tan D^\circ - \tan d^\circ}{\tan d^\circ\left(\frac{1}{S_2} + \frac{1}{R + G}\right) - \frac{\tan D^\circ}{R + G}}. \quad [E]$$

Further still, if we make R a very high resistance, that is, if in equations [D] and [E] we make $\frac{1}{R + G} = 0$, then we get the simplifications

$$r = S_2\left(\frac{D}{d} - 1\right) \qquad [F]$$

and

$$r = S_2\left(\frac{\tan D^\circ}{\tan d^\circ} - 1\right). \qquad [G]$$

162. If we refer to the "Diminished deflection shunt method" (page 128) of determining the resistance of a "galvanometer" we can see that equation (A) (page 129) in that test is almost precisely similar to equation (A) (page 153) of the present test, the only difference being that in the latter we have $\frac{1}{R + G}$ in the place of $\frac{1}{R}$, consequently we must have—

Best Conditions for making the Test.

Make S_1 and R as large as possible.

Make S_2 of such a value that when the deflections, D, d, are in *divisions*, then

$$d = \frac{D}{3}$$

approximately; and when the deflections are in *degrees* on a tangent galvanometer, then

$$\tan d^\circ = \frac{\tan D^\circ}{3}$$

approximately.

Possible Degree of Accuracy attainable.

If the deflections are in *divisions*, and if we can read their value to an accuracy of $\frac{1}{m}$th of a division, then we can determine the value of r to an accuracy, λ', of

$$\lambda' = \frac{\frac{1}{m}(D + d)\,100}{d\,(D - d)}\left[1 + r\left(\frac{1}{S_1} + \frac{1}{R + G}\right)\right] \text{ per cent.}$$

If the deflections are in *degrees* on a tangent galvanometer, then

if we can read their value to an accuracy of $\frac{1}{m}$th of a *degree*, we can determine the value of r to an accuracy, λ, of

$$\lambda' = \frac{(\tan D^\circ\, \delta_2 + \tan d^\circ\, \delta_1)\, 100}{\tan d^\circ (\tan D^\circ - \tan d^\circ)} \left[1 + r\left(\frac{1}{S_1} + \frac{1}{R + G}\right)\right] \text{ per cent.}$$

where

$$\delta_1 = \tan D_{\frac{1}{m}}^\circ - \tan D^\circ, \quad \text{and,} \quad \delta_2 = \tan d_{\frac{1}{m}}^\circ - \tan d^\circ.$$

163. In all the foregoing tests it is very necessary that the galvanometer used be a highly sensitive one (page 85), otherwise even a moderate degree of accuracy cannot be obtained.

164. Other methods of measuring the resistance of batteries will be referred to hereafter (see Index); these methods involve principles which can be more conveniently discussed later on.

CHAPTER VII.

MEASUREMENT OF THE ELECTROMOTIVE FORCE OF BATTERIES.

165. The methods of measuring or comparing the electromotive forces of batteries are perhaps more numerous than any other class of measurements.

Although no absolute standard of the unit of electromotive force (the *volt*) exists, yet there are several standards of known value with which comparisons may be made.

166. For rough measurements any form of Daniell cell may be used as a standard, the electromotive force being taken as 1 volt; this value, however, cannot be regarded as being correct to a closer degree of accuracy than about 10 per cent., the value of a Daniell cell being actually more than 1 volt.

Standard Cells.

POST OFFICE STANDARD.

167. Originally the Post Office used as a standard a particular form of Daniell cell; this, however, has been entirely abandoned in favour of an ordinary "dry" cell, which is from time to time compared with a Clark standard (§ 169) and its correct value registered. These dry cells have an electromotive force of 1·5 volts approximately. A cell of this kind is enclosed in a box, the cover of which is kept sealed with an official seal. Included in the box is a resistance of 1144 ohms, whose use is explained in the Appendix, and which also prevents the cell from being accidentally short-circuited.

FLEMING'S STANDARD CELL.

168. This cell, devised by Dr. J. A. Fleming, is thus arranged:—

A large U-tube, about ¾ inch in diameter and 8 inches long in the limb, has four side tubes (Fig. 82). The two top ones, A and B, lead to two reservoirs Z and C, and the bottom ones C and D

are drainage-tubes. The side tubes are closed by glass taps. The whole is mounted on a vertical board, with a pair of test-tubes between the limbs. The left-hand reservoir S Z is filled with a solution of sulphate of zinc, and the right-hand reservoir S C with a solution of sulphate of copper. The electrodes are zinc and copper rods, Zn and Cu, passed through vulcanised-rubber corks, P and Q, fitting air-tight, into the ends of the U-tube.

The operation of filling is as follows:—Open the tap A and fill the whole U-tube with the denser zinc sulphate solution; then insert the zinc rod and fit it tightly by the rubber cork P. Now, on opening the tap C the level of the liquid will begin to fall in the right-hand limb but be retained in the closed one. As the level commences to sink in the right-hand limb, by opening the tap B copper sulphate solution can be allowed to flow in gently to replace it; and this operation can be so conducted that the level of demarcation of the two liquids remains quite sharp, and gradually sinks to the level of the tap C. When this is the case, all taps are closed and the copper rod inserted in the right-hand limb.

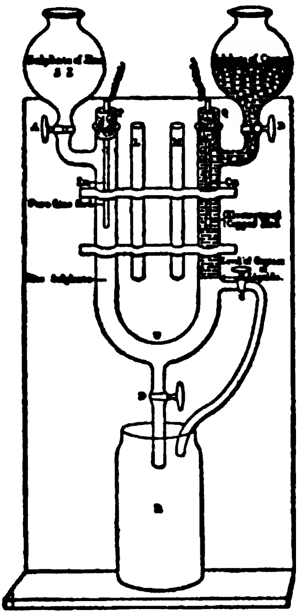


Fig. 82.

Now it is impossible to stop diffusion from gradually mixing the liquids at the surface of contact; but whenever the surface of contact ceases to be sharply defined, the mixed liquid at the level of the tap C can be drawn off, and fresh solutions supplied from the reservoirs above.

In this way it is possible to maintain the solution pure and unmixed round the two electrodes with very little trouble; and the electrodes, when not in use, can be kept in the idle cells or test-tubes L and M, each in its own solution.

The electrodes are made of rods of the purest zinc and copper, about 4 inches long and $\frac{1}{4}$ inch diameter. The zinc found most suitable is made from zinc twice distilled and cast into rods; the copper is prepared by electro-depositing on a very fine copper wire, until a cylinder of the required thickness is obtained.

The value of the electromotive force of the cell depends, to a considerable extent, upon the density of the solutions used. The latter should be as follows:—

For the zinc solution, dissolve 555 grammes of chemically pure sulphate of zinc in 445 grammes of distilled water. This solution should have a specific gravity of 1·4 at 15° C.

For the copper solution, dissolve 83 grammes of chemically pure sulphate of copper in 417 grammes of distilled water. This solution should have a specific gravity of 1·1 at 15° C.

Especial care must also be taken to lightly electrotype the copper rod with a fresh pure surface of new copper the instant before using. This is done in the small copper voltameter which the tube M forms, using a single Leclanché cell for the purpose. The pure zinc rod should be cleaned with new glass-paper. If these precautions are carried out the electromotive force of the cell will be 1·086 B.A. volts, which value will be correct within the ordinary ranges of temperature.

CLARK'S STANDARD CELL.

169. In this cell the electromotive force is that due to pure zinc opposed to pure mercury, the two metals being in a saturated solution of mercurous sulphate and zinc sulphate in water. Actually, the mercury is covered with a paste formed of mercurous sulphate in a thoroughly saturated (not *super*saturated) solution of zinc sulphate, and the zinc rests in the paste.

The best method of forming the paste is, according to Lord Rayleigh, as follows:—Rub up in a mortar 150 grammes of mercurous sulphate, 5 grammes of zinc carbonate, and use sufficient zinc sulphate solution (not *super*saturated) to make a thick paste; leave the whole in the mortar for two or three days, occasionally pounding it up in order to allow the carbonic anhydride which forms to escape. Dr. A. Muirhead, who has had a very lengthened experience with the Clark cells, prefers to make the paste as follows:—A saturated solution of zinc and mercurous sulphates is prepared by heating *gently* in the saturated solution of zinc sulphate a portion of the mercurous sulphate, adding thereto a little free mercury to preserve the basicity of the mercurous salt; mercurous sulphate is then mixed into a paste with the solution so prepared. The mercurous sulphate can be obtained commercially; but it may be prepared by dissolving pure mercury in excess in hot sulphuric acid at a temperature below boiling-point. The salt, which is a nearly insoluble white powder, should be well

washed in distilled water, and care should be taken to obtain it free from the mercuric sulphate (persulphate), the presence of which may be known by the mixture turning yellowish on the addition of water. The careful washing of the salt is a matter of essential importance, as the presence of any free acid, or of persulphate, produces an irregularity in the electromotive force of the cell for some time after charging. The paste (of the consistence of cream) is poured on to the surface of the mercury (which should have been distilled in vacuo); a piece of pure zinc is then suspended in the paste, and the vessel sealed up with marine glue (*not* paraffin wax). Contact with the mercury may be made by means of a platinum wire passing down a glass tube and dipping below the surface of the mercury.

Board of Trade Specification for the Preparation of the Clark Cell.

Definition of the Cell.

170. The cell consists of zinc and mercury in a neutral saturated solution of zinc sulphate and mercurous sulphate in water, prepared with mercurous sulphate in excess, and is conveniently contained in a cylindrical glass vessel.

Preparation of the Materials.

(1) *The Mercury.*—To secure purity it should be first treated with acid in the usual manner, and subsequently distilled in vacuo.

(2) *The Zinc.*—Take a portion of a rod of pure redistilled zinc, solder to one end a piece of copper wire, clean the whole with glass paper or a steel burnisher, carefully removing any loose pieces of the zinc. Just before making up the cell dip the zinc into dilute sulphuric acid, wash with distilled water, and dry with a clean cloth or filter paper.

(3) *The Mercurous Sulphate.*—Take mercurous sulphate, purchased as pure, mix with it a small quantity of pure mercury, and wash the whole thoroughly with cold distilled water by agitation in a bottle; drain off the water, and repeat the process at least twice.* After the last washing, drain off as much of the water as possible.

(4) *The Zinc Sulphate Solution.*—Prepare a *neutral* saturated solution of pure ("pure re-crystallised") zinc sulphate by mixing in a flask distilled water with nearly twice its weight of crystals of pure zinc sulphate, and adding zinc oxide in the proportion of about 2 per cent. by weight of the zinc sulphate crystals to neutralise any free acid.* The crystals should be dissolved with the aid of gentle heat, but the temperature to which the solution is raised should not exceed 30° C. Mercurous sulphate treated as described in (3) should be added in the proportion of about 12 per cent. by weight of the zinc sulphate crystals to neutralise any free zinc oxide remaining, and the solution filtered, while still warm, into a stock bottle. Crystals should form as it cools.

(5) *The Mercurous Sulphate and Zinc Sulphate Paste.*—Mix the washed mercurous sulphate with the zinc sulphate solution, adding sufficient crystals of

* *See* Notes, page 161 *et seq.*

zinc sulphate from the stock-bottle to ensure saturation, and a small quantity of pure mercury. Shake these up well together to form a paste of the consistence of cream. Heat the paste, but not above a temperature of 30° C. Keep the paste for an hour at this temperature, agitating it from time to time, then allow it to cool; continue to shake it occasionally while it is cooling. Crystals of zinc sulphate should then be distinctly visible, and should be distributed throughout the mass; if this is not the case add more crystals from the stock-bottle, and repeat the whole process.

This method ensures the formation of a saturated solution of zinc and mercurous sulphates in water.

Contact is made with the mercury by means of a platinum wire about No. 22 gauge. This is protected from contact with the other materials of the cell by being sealed into a glass tube. The ends of the wire project from the ends of the tube; one end forms the terminal, the other end and a portion of the glass tube dip into the mercury.

To Set Up the Cell.

The cell may conveniently be set up in a small test-tube of about 2 cm. diameter and 4 or 5 cm. deep. Place the mercury in the bottom of this tube, filling it to a depth of say ·5 cm. Cut a cork about ·5 cm. thick to fit the tube; at one side of the cork bore a hole through which the zinc rod can pass tightly; at the other side bore another hole for the glass tube which covers the platinum wire; at the edge of the cork cut a nick through which the air can pass when the cork is pushed into the tube. Wash the cork thoroughly with warm water, and leave it to soak in water for some hours before use. Pass the zinc rod about 1 cm. through the cork.

Clean the glass tube and platinum wire carefully, then heat the exposed end of the platinum red hot, and insert it in the mercury in the test-tube, taking care that the whole of the exposed platinum is covered.

Shake up the paste and introduce it without contact with the upper part of the walls of the test-tube, filling the tube above the mercury to a depth of rather more than 1 cm.

Then insert the cork and zinc rod, passing the glass tube through the hole prepared for it. Push the cork gently down until its lower surface is nearly in contact with the liquid. The air will thus be nearly all expelled, and the cell should be left in this condition for at least 24 hours before sealing, which should be done as follows:—

Melt some marine glue until it is fluid enough to pour by its own weight, and pour it into the test-tube above the cork, using sufficient to cover completely the zinc and soldering. The glass tube containing the platinum wire should project some way above the top of the marine glue.

The cell thus set up may be mounted in any desirable manner. It is convenient to arrange the mounting so that the cell may be immersed in a water-bath up to the level of, say, the upper surface of the cork. Its temperature can then be determined more accurately than is possible when the cell is in air.

In using the cell sudden variations of temperature should as far as possible be avoided.

Notes.

The Zinc Sulphate Solution.—The object to be attained is the preparation of a neutral solution of pure zinc sulphate saturated with $ZnSO_4.7H_2O$.

At temperatures above 30° C. the zinc sulphate may crystallise out in another form; to avoid this 30° C. should be the upper limit of temperature. At this temperature water will dissolve about 1·9 times its weight of the crystals. If any of the crystals put in remain undissolved they will be removed by the filtration.

The zinc sulphate should be free from iron and should be tested before use with sulphocyanide of potassium to ascertain that this condition is satisfied. If an appreciable amount of iron is present it should be removed by the method given in the Instructions for setting up Clark's Cells issued from the Physical Technical Institute of Berlin, *Zeitschrift für Instrumentenkunde*, 1893, Heft 5.

The amount of zinc oxide required depends on the acidity of the solution, but 2 per cent. will, in all cases which will arise in practice with reasonably good zinc sulphate, be ample. Another rule would be to add the zinc oxide gradually until the solution became slightly milky. The solution when put into the cell should not contain any free zinc oxide; if it does then, when mixed with the mercurous sulphate, zinc sulphate and mercurous oxide are formed; the latter may be deposited on the zinc and affect the electromotive force of the cell. The difficulty is avoided by adding as described about 12 per cent. of mercurous sulphate before filtration; this is more than sufficient to combine with the whole of the zinc oxide originally put in, if it all remains free: the mercurous oxide formed together with any undissolved mercurous sulphate is removed by the filtration.

The Mercurous Sulphate.—The treatment of the mercurous sulphate has for its object the removal of any mercuric sulphate which is often present as an impurity.

Mercuric sulphate decomposes in the presence of water into an acid and a basic sulphate. The latter is a yellow substance—turpeth mineral—practically insoluble in water; its presence, at any rate, in moderate quantities, has no effect on the cell. If, however, it is formed, the acid sulphate is formed also. This is soluble in water, and the acid produced affects the electromotive force. The object of the washings is to dissolve and remove this acid sulphate, and for this purpose the three washings described in the Specification will in nearly all cases suffice. If, however, a great deal of the turpeth mineral is formed, it shows that there is a great deal of the acid sulphate present, and it will then be wiser to obtain a fresh sample of mercurous sulphate rather than to try by repeated washings to get rid of all the acid.

The free mercury helps in the process of removing the acid, for the acid mercuric sulphate attacks it, forming mercurous sulphate and acid which is washed away.

Pure mercurous sulphate when quite free from acid shows on repeated washing a faint primrose tinge, which is due to the formation of a basic mercurous salt, and is distinct from the turpeth mineral or basic mercuric sulphate. The appearance of this primrose tint may be taken as an indication of the fact that all the acid has been removed, and the washing may with advantage be continued until this primrose tint appears. Should large quantities of this basic mercurous salt be formed, the sulphate should be treated as described in the directions already quoted, *Zeitschrift für Instrumentenkunde*, 1893, Heft 5.

The cell may be sealed in a more permanent manner by coating the marine glue, when it is set, with a solution of sodium silicate, and leaving it to harden.

If the sides of the test-tube above the cork be soiled by the introduction of the paste, the marine glue does not adhere to the glass; the liquid in the cell rises by capillary action between the glue and the glass, and may damage the cell.

The form of the vessel containing the cell may be varied. In the H form devised by Lord Rayleigh, and modified by Dr. Kahle, the zinc is replaced by an amalgam of 10 parts by weight of zinc to 90 of mercury. The other materials should be prepared as already described. Contact is made with the amalgam in one leg of the cell, and with the mercury in the other, by means of platinum wires sealed through the glass.

MUIRHEAD'S IMPROVED CLARK STANDARD CELL.

171. The usual forms of the Clark cell, especially when newly set up, are unsuitable for transport; the mercury, being free, is apt to leave the platinum-wire contact when the cell is inverted or upset, and to fall through the paste into contact with the zinc rod, thereby either short-circuiting the cell altogether or destroying the value of its electromotive force. To remedy this defect Dr. A. Muirhead constructs the cell as shown by Fig. 83. A is a flat closely-wound spiral of platinum wire (shown in plan in the lower part of the figure), which has been coated or

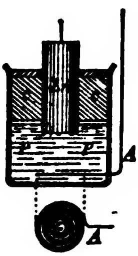


Fig. 83.

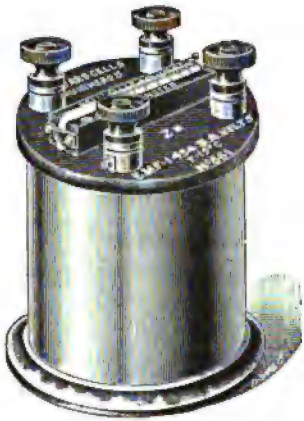


Fig. 84.

amalgamated with pure mercury either by boiling it in the latter or by dipping the spiral, when heated red-hot, into mercury; the continuation of the wire is sealed into the glass cell, forming the outer connection. Zn is a rod of pure zinc supported by a cork, *c*, covered with cement. Inside the cell is placed the paste, *p*, composed of pure mercurous sulphate and a saturated solution of pure zinc sulphate.

Fig. 84 shows a very compact and useful form of this standard.

The four terminals belong to two entirely distinct cells, the advantage being that the two cells may be used as a check one upon the other. A thermometer is contained within the box, and the stem, being bent at right angles, lies in a groove across the top of the case. By this thermometer the temperature at the time of the reading can be ascertained.

172. The electromotive force of the Clark standard cells is, according to determinations by Lord Rayleigh, 1·434 S * volts at 15° C. The effect of change of temperature is to change the value of the force about ·077 per cent per degree C., that is to say, the electromotive force at a temperature of $t°$ C. is

$$1{\cdot}434\ \{1 - {\cdot}00077\ (t° - 15)\}\ \text{volts.}$$

The following table shows the electromotive force at various temperatures calculated from the foregoing formula:—

Temp. ° C.	S. Volts.	Temp. ° C.	S. Volts.	Temp. ° C.	S. Volts.
0	1·451	11	1·438	22	1·426
1	1·449	12	1·437	23	1·425
2	1·448	13	1·436	24	1·424
3	1·447	14	1·435	25	1·423
4	1·446	15	1·434	26	1·422
5	1·445	16	1·433	27	1·421
6	1·444	17	1·432	28	1·420
7	1·443	18	1·431	29	1·419
8	1·442	19	1·430	30	1·417
9	1·441	20	1·429	31	1·416
10	1·440	21	1·427	32	1·415

173. In order that the force in the Clark cells may preserve its value, constant care must be taken that the cells are not worked through a low resistance. It is necessary, therefore, in employing them, to take care that they are only used in circuits of a very high resistance, or for charging a condenser, or are balanced by a second battery, as in Clark's electromotive force test (see Index).

Carhart's Standard Cell.

174. In this cell zinc chloride and mercurous chloride (calomel) take the place of the zinc sulphate and mercurous sulphate used in the Clark cell. By adjusting the concentration of the solution

* Standard, or Board of Trade, volts.

Professor Carhart has been able to obtain a cell which gives almost exactly 1 volt electromotive force (1·0005 to 1·0011 volts at 20° C.), and the temperature coefficient of which is almost negligible, being about one-eighth that of the normal Clark cell.*

Electromotive Force Measurements.

175. To measure the electromotive force of a battery, we have to compare it with a standard of one or more cells, and having thus ascertained the relative values of the two, the electromotive force of the battery, in volts, is obtained by an ordinary proportion sum.

For example.

The relative electromotive forces of a battery and three standard Daniell cells were found to be as 1·25 to 1; what was the electromotive force, in volts, of the battery?

$$1{\cdot}25 : 1 :: 3 \times 1{\cdot}079 : x;$$

therefore

$$x = \frac{1 \times 3 \times 1{\cdot}079}{1{\cdot}25} = 2{\cdot}59 \text{ volts.}$$

Equal Resistance Method.

176. Let there be two batteries, whose electromotive forces E_1 and E_2 are to be compared. Join up battery E_1 with a tangent galvanometer and resistance in simple circuit, as shown by Fig. 76 (page 135). All the plugs between A and C being inserted, the infinity plug between A and D being removed, and the connections being made, depress the right-hand key, and remove a sufficient number of plugs from between D and E to obtain a convenient deflection on, say, the tangent scale of the galvanometer. Note this deflection—let it be d_1 divisions; and also note the *total* resistance (R) in circuit—that is, the resistance between D and E, plus the resistance of the galvanometer, plus the resistance of the battery (which must be determined beforehand). Now remove battery E_1 and insert battery E_2 in its place, and if this battery has a different resistance to E_1, readjust between D and E so that total resistance in circuit is the same as it was at first. Again note the deflection of the galvanometer needle—let

* 'Electrical Review,' December 18th, 1896.

it be d_2 divisions. Then if C_1 be the current producing the deflection d_1, and C_2 the current producing the deflection d_2, we must have by Ohm's law (§ 2, page 1),

$$C_1 = \frac{E_1}{R}, \quad \text{and,} \quad C_2 = \frac{E_2}{R},$$

therefore

$$E_1 : E_2 :: C_1 : C_2,$$

or since d_1 and d_2 are directly proportional to C_1 and C_2, we must have

$$E_1 : E_2 :: d_1 : d_2.$$

For example.

With a tangent galvanometer, whose resistance was 100 ohms, and battery E_1, whose resistance was 70 ohms, we obtained, with a resistance of 1830 ohms (total, 100 + 70 + 1830 = 2000), in the resistance box, a deflection of 50 divisions on the tangent scale of the galvanometer; and with battery E_2, whose resistance was 50 ohms, we obtained, with a resistance of 1850 ohms (total, 100 + 50 + 1850 = 2000, as before), in the resistance box, a deflection of 40 divisions; then

$$E_1 : E_2 :: 50 : 40,$$

or as

$$1 \cdot 25 \text{ to } 1.$$

If the deflections are read on the *degrees* scale of the tangent galvanometer, then d_1 and d_2 must be the *tangents* of the deflections.

In cases where the resistance of the batteries whose electromotive forces are to be compared are very small, we may, by using a very high resistance, practically regard the total resistance in circuit as being the same, whatever battery we use. The deflections then obtained with any number of different batteries will represent their comparative electromotive forces. The galvanometer will, in this case, of course have to be one with a high figure of merit (page 85).

177. The "Best conditions for making the test," and the "Possible degree of accuracy attainable," are almost obvious; they are

Best Conditions for making the Test.

Make the resistances in the circuits as high as possible.

Possible Degree of Accuracy attainable.

If we can be certain of the value of the two deflections to accuracies of δ'_1 and δ'_2 per cent. respectively, then we can be certain of the relative values of the two electromotive forces to accuracy of $\delta'_1 + \delta'_2$ per cent.

Equal Deflection Method.

178. Join up as in last method, and having noted the deflection and total resistance in circuit (R_1) with battery E_1, remove it and insert battery E_2 in its place. Now readjust resistance between D and E, until the deflection of the galvanometer needle becomes the same as it was at first. Note the resistance in circuit (R_2); then calling C the current,

$$C = \frac{E_1}{R_1}, \quad \text{and,} \quad C = \frac{E_2}{R_2},$$

that is

$$E_1 : E_2 :: R_1 : R_2,$$

or the electromotive forces of the batteries are directly as the total resistances that are in circuit with the respective batteries.

For example.

With a galvanometer whose resistance was 100 ohms, and a battery E_1 whose resistance was 50 ohms, we obtained, with a resistance of 2350 ohms (total, $100 + 50 + 2350 = 2500$), in the resistance box, a deflection of 40°; and with a battery E_2, whose resistance was 70 ohms, it was necessary, in order to bring the galvanometer needle again to 40°, to have a resistance of 1830 ohms (total, $100 + 70 + 1830 = 2000$), in the resistance box; then

$$E_1 : E_2 :: 2500 : 2000,$$

or as

$$5 \text{ to } 4.$$

An advantage in this test is that it can be made with a galvanometer the relative values of whose deflections are unknown.

The

Best Conditions for making the Test

and the

Possible Degree of Accuracy attainable

are the same as in the last test.

WIEDEMANN'S METHOD.

179. In Fig. 76 (page 135) join the zinc pole of battery E_1 to D, as shown, and the other pole to the zinc pole of battery E_2, whose other pole in turn is to be joined to C. Adjust the resistance so as to obtain a high deflection on the tangent scale of the galvanometer. Let the current producing this deflection be C; then

$$C = \frac{E_1 + E_2}{R},$$

where R is the total resistance in the circuit. Now reverse battery E_2 (the weaker one) so that the two batteries oppose one another—we shall then get a smaller deflection due to a current $C_{\prime}$; then

$$C_{\prime} = \frac{E_1 - E_2}{R}.$$

From these two equations we get

$$E_1\, C - E_2\, C = E_1\, C_{\prime} + E_2\, C_{\prime},$$

that is,

$$E_1 : E_2 :: C + C_{\prime} : C - C_{\prime}$$

or, substituting deflections d, $d_{\prime}$, for current strengths C, $C_{\prime}$,

$$E_1 : E_2 :: d + d_{\prime} : d - d_{\prime}.$$

For example.

Two batteries E_1 and E_2 being joined up together in simple circuit, we obtained, by adjusting the resistance in the resistance box, a deflection of 72 divisions (d) on the tangent scale of the galvanometer; and with the same resistance in circuit we obtained, on reversing battery E_2, a deflection of 8 divisions (d_1); then

$$E_1 : E_2 :: 72 + 8 : 72 - 8,$$
$$:: \quad 80 \quad : \quad 64,$$

or as

$$1 \cdot 25 \text{ to } 1.$$

If the deflections are read on the *degrees* scale of a tangent galvanometer, then d and $d_{\prime}$ must be the *tangents* of the deflections.

180. In order to make the test as accurately as possible under the last conditions, the resistance in the circuit should be so adjusted that the two deflections make approximately equal angles on opposite sides of 45° (§ 35, page 30). The more resistance it is possible to place in the circuit of the batteries the

better, since the tendency of the latter to polarise is thereby reduced to a minimum.

181. Wiedemann's method is a very satisfactory one since it is absolutely independent of the resistance of the two batteries; thus one battery might have a resistance of a fraction of an ohm only and the other a resistance of several thousand ohms, yet this would in no way affect the correctness of the results, but to avoid errors due to polarisation it is necessary with some batteries to include several thousand ohms in the circuit; if the galvanometer used be one with a high figure of merit (page 85) this can always be done.

182. The "Possible degree of accuracy attainable" in making the test is greatly dependent upon the relative values of the two electromotive forces. Let us first suppose that the deflections are read in *divisions*, and let us suppose that there is a possible error δ in both deflections. Now if we take both errors to be of similar signs, then we should have a total absolute error of $2\,\delta$ in the quantity $(d + d_{\prime})$, but if one error were plus and the other minus, then we should have a total absolute error of $2\,\delta$ in the quantity $(d - d_{\prime})$. But the latter quantity must be smaller than $(d + d_{\prime})$, therefore an absolute error $2\,\delta$ in its value must represent a greater percentage error in the relative values of E_1 and E_2 than would be the case if the same absolute error were in $(d + d_{\prime})$. As we must assume the resultant error to be the greatest *possible*, we must therefore take the error $2\,\delta$ to be in the quantity $(d - d_{\prime})$.

Let, then, λ be the error in the relative values of E_1 and E_2, that is in $\frac{E_1}{E_2}$ caused by, say, an error δ in d, and an error $-\delta$ in $d_{\prime}$, then we have

$$\frac{E_1}{E_2} - \lambda = \frac{(d + \delta) + (d_{\prime} - \delta)}{(d + \delta) - (d_{\prime} - \delta)} = \frac{d + d_{\prime}}{d - d_{\prime} + 2\,\delta},$$

therefore

$$\lambda = \frac{E_1}{E_2} - \frac{d + d_{\prime}}{d - d_{\prime} + 2\,d} = \frac{d + d_{\prime}}{d - d_{\prime}} - \frac{d + d_{\prime}}{d - d_1 + 2\,\delta}$$

$$= \frac{2\,\delta\,(d + d_{\prime})}{(d + d_{\prime})\,(d - d_{\prime} + 2\,\delta)},$$

or since $2\,\delta$ is very small we may say

$$\lambda = \frac{2\,\delta\,(d + d_{\prime})}{(d - d_{\prime})^2}.$$

If we put the percentage for the absolute value of λ, that is, if we have

$$\lambda = \frac{\lambda'}{100} \text{ of } \frac{E_1}{E_2} = \frac{\lambda'}{100} \times \frac{d + d_{\prime}}{d - d_{\prime}},$$

then we get

$$\frac{\lambda'}{100} \times \frac{d + d_{\prime}}{d - d_{\prime}} = \frac{2\,\delta\,(d + d_{\prime})}{(d - d_{\prime})^2},$$

that is to say

$$\lambda' = \frac{2\,\delta\,100}{d - d_{\prime}}. \qquad [A]$$

For example.

In the example given on page 168 the deflections could each be read to an accuracy of $\frac{1}{4}$ of a division; what was the degree of accuracy with which the value of $\frac{E_1}{E_2}$ could be determined?

$$\lambda' = \frac{2 \times \frac{1}{4} \times 100}{72 - 8} = \cdot 78 \text{ per cent.}$$

If $d_{,}$ is small compared with d, then

$$\lambda' = \frac{2\,\delta}{d}.$$

We can see from equation [A] that unless $d_{,}$ is small compared with d, the accuracy with which the test can be made will be but small; for if $d_{,}$ approaches in value to d, then $d - d_{,}$ becomes very small, that is λ' becomes large. In order that $d_{,}$ may be as much smaller than d as possible, E_1 and E_2 must be as nearly equal as possible; the test therefore will not be a satisfactory one unless such is the case.

If $d_{,}$ is small compared with d, then

$$\lambda = \frac{2\,\delta\,100}{d};$$

or if we put the percentage instead of the absolute value of δ, that is if we have

$$\delta = \frac{\delta'}{100} \text{ of } d,$$

then we get

$$\lambda' = 2\,\delta',$$

so that under the best conditions for making the test the accuracy with which the value of $\frac{E_1}{E_2}$ could be determined would be but one-half the accuracy with which the higher deflection could be observed.

183. To determine the degree of accuracy attainable in the case where the readings are made from the *degrees* scale of a tangent galvanometer, we must in the preceding investigation substitute tangents for divisions of deflections. Thus we have

$$\frac{E_1}{E_2} - \lambda = \frac{\tan(d^\circ + \delta^\circ) + \tan(d_{,}^\circ + \delta^\circ)}{\tan(d^\circ + \delta^\circ) - \tan(d_{,}^\circ + \delta^\circ)},$$

or

$$\lambda = \frac{\tan d^\circ + \tan d_{,}^\circ}{\tan d^\circ - \tan d_{,}^\circ} - \frac{\tan(d^\circ + \delta^\circ) + \tan(d_{,}^\circ - \delta^\circ)}{\tan(d^\circ + \delta^\circ) - \tan(d_{,}^\circ - \delta^\circ)}.$$

If in this equation we put

$$\tan(d^\circ + \delta^\circ) = \frac{\tan d + \tan \delta^\circ}{1 - d^\circ \tan \delta^\circ}$$

and

$$\tan(d^\circ - \delta^\circ) = \frac{\tan d^\circ - \tan \delta^\circ}{1 + \tan d^\circ \tan \delta^\circ},$$

we get

$$\lambda = \frac{2 \tan \delta^\circ \left[(\tan d^\circ + \tan d_{,}^\circ)(1 + \tan d^\circ \tan d_{,}^\circ) + X\right]}{(\tan d^\circ - \tan d_{,}^\circ)(\tan d^\circ - \tan d_{,}^\circ + Y)}$$

where X and Y are a number of factors of tan δ°. But since tan δ° is very small, we may put X and Y equal to 0, in which case we have

$$\lambda = \frac{2 \tan \delta^\circ (\tan d^\circ + \tan d_{,}^\circ)}{\tan d^\circ - \tan d_{,}^\circ} \times \frac{1 + \tan d^\circ \tan d_{,}^\circ}{\tan d^\circ - \tan d_{,}^\circ}$$

$$= \frac{2 \tan \delta^\circ (\tan d^\circ + \tan d_{,}^\circ)}{\tan d^\circ - \tan d_{,}^\circ} \times \frac{1}{\tan (d^\circ - d_{,}^\circ)}.$$

If we put the percentage for the absolute value of λ, that is, if we have

$$\lambda = \frac{\lambda'}{100} \text{ of } \frac{E_1}{E_2} = \frac{\lambda'}{100} \times \frac{\tan d^\circ + \tan d_{,}^\circ}{\tan d^\circ - \tan d_{,}^\circ},$$

then we get

$$\lambda' = \frac{2 \tan \delta^\circ \, 100}{\tan (d^\circ - d_{,}^\circ)}. \qquad [B]$$

For example.

In comparing the electromotive forces of two batteries by Wiedemann's method, the deflections obtained on the *degrees* scale of a tangent galvanometer were 71° and 18° respectively; what were the relative electromotive forces of the batteries, and what would have been the degree of accuracy with which the value of $\frac{E_1}{E_2}$ could be determined? The value of the deflections could be read to an accuracy of ¼°.

$$E_1 : E_2 :: \tan 71^\circ + \tan 18^\circ : \tan 71^\circ - \tan 18^\circ,$$

or as

$$2 \cdot 9042 + \cdot 3249 \quad \text{to} \quad 2 \cdot 9042 - \cdot 3249,$$

that is, as

$$1 \cdot 25 \quad \text{to} \quad 1;$$

also

$$\lambda' = \frac{2 \times \tan \tfrac{1}{4}^\circ \times 100}{\tan (71^\circ - 18^\circ)} = \frac{2 \times \cdot 4363}{1 \cdot 3270} = \cdot 65 \text{ per cent.}$$

Like equation [A] (page 169), equation [B] shows that unless $d_{,}^\circ$ is small compared with d°, the test cannot be made with a high degree of accuracy.

To sum up, then, we have

Best Conditions for making the Test.

184. To obtain satisfactory results, E_1 and E_2 should be as nearly as possible equal.

As much resistance should be included in the circuit as possible.

If the readings are made on the *degrees* scale of a tangent galvanometer, then the resistance in circuit should be so adjusted that the deflections, as nearly as possible, make equal angles on opposite sides of 45° (§ 35, page 30).

Possible Degrees of Accuracy attainable.

When the readings are in *divisions*, then

$$\text{Percentage of accuracy} = \frac{\frac{2}{n}\,100}{d - d_{,}}$$

where $\frac{1}{n}$ is the smallest fraction of a division to which the deflections can be read.

When the readings are in *degrees* on a tangent galvanometer, then

$$\text{Percentage of accuracy} = \frac{2 \tan \frac{1}{n}^{\circ}\,100}{\tan\,(d^{\circ} - d_{,}^{\circ})}$$

where $\frac{1}{n}^{\circ}$ is the smallest fraction of a degree to which the deflections can be read.

Kempe's Method.

185. This is merely a particular way of carrying out Wiedemann's method, and is useful when very small differences of electromotive force have to be measured.

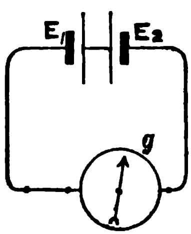


Fig. 85.

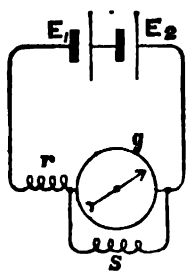


Fig. 86.

The cells to be compared are first joined up in opposition, as shown by Fig. 85, through a Thomson galvanometer, g, and the deflection, $d_{,}$, which is due to the difference in force between the two cells, is noted. One of the cells, say E_2, is now reversed (Fig. 86), and, in order to keep the deflection within the range of the scale, the galvanometer, g, is shunted with a shunt, s, a compensating resistance (§ 84, page 89) r, of the value necessary to keep the resistance of the circuit unaltered, being inserted at the same time; the deflection, d, is then noted, and we have

$$\frac{E_1}{E_2} = \frac{n\,d + d_{,}}{n\,d - d_{,}}, \qquad [A]$$

where n is the multiplying power of the shunt.

The great point in this method of testing is the high degree of accuracy with which the relative values of E_1 and E_2 (when they do not differ materially), can be determined.

For example.

Supposing the deflection obtained in the case of Fig. 85, to be 1 division and in the case of Fig. 86, to be $200 \times 1000 = 200{,}000$ (a result which would be obtained with a galvanometer of 5000 ohms resistance if the adjustment of the instrument were such that one cell through 10,000 ohms with the $\frac{1}{1000}$ shunt gave 50 divisions), then

$$\frac{E_1}{E_2} = \frac{200{,}000 + 1}{200{,}000 - 1} = 1 \cdot 00001,$$

i. e. a difference of 1 in 100,000.

It might be objected that when the cells are joined up as shown by Fig. 86, their electromotive forces would run down, this, however, is not of material consequence (even if it could take place to any marked extent with a galvanometer which has a resistance of 5000 ohms or more), for a fall in force of say one per cent. under these conditions would only mean an error of one per cent. in the value which in the above example is expressed by $\cdot 00001$, and not that percentage of error in the value expressed by $1 \cdot 00001$.

As the chief value of the method is to determine small differences of force, we may simplify equation [A] in the following manner:—

Since [A] is derived from the equation

$$\frac{E_1 - E_2}{E_1 + E_2} = \frac{d_{,}}{n\,d},$$

and since E_1 and E_2 are very nearly equal, so that in the denominator of the fraction we may put $E_1 = E_2$, therefore we get

$$\frac{E_1 - E_2}{2\,E_2} = \frac{d_{,}}{n\,d},$$

therefore

$$\frac{E_1 - E_2}{E_2} = \frac{2\,d_{,}}{n\,d},$$

therefore

$$\frac{E_1}{E_2} - 1 = \frac{2\,d_{,}}{n\,d},$$

that is,

$$\frac{E_1}{E_2} = 1 + \frac{2\,d_{,}}{n\,d}.$$

Or, if we prefer to express the difference as a percentage, then we have

$$\frac{2\,d_{,}}{n\,d} = \frac{x}{100} \text{ of } 1,$$

or

$$x = \frac{200\,d_{,}}{n\,d};$$

thus in the example taken, the percentage of difference between the two cells would be

$$x = \frac{200 \times 1}{200{,}000} = \tfrac{1}{1000}\text{th per cent.}$$

Wheatstone's Method.

186. The most elegant method of comparing the electromotive forces of batteries is that of the late Sir Charles Wheatstone.

Battery E_1 is joined up in single circuit with a galvanometer and a resistance; a deflection of a° is obtained. The resistance is now increased by ρ_1, so that a new deflection, β°, is produced.

Battery E_2 is next joined up in the place of E_1, and the resistance in circuit is adjusted until the deflection obtained is a°, as at first. The resistance is now increased by ρ_2, so that the deflection is reduced to β°, as in the first instance.

Now from the "Equal resistance method" (page 165), we see that the total resistances, R_1 and R_2, in circuit, which were required in the two cases to bring the deflections to a°, must be in direct proportion to the electromotive forces, E_1, E_2, of the two batteries. Also the total resistances, $R_1 + \rho_1$, and $R_2 + \rho_2$, in circuit which were required in the two cases to bring the deflections to β°, must be in direct proportion to the electromotive forces, E_1, E_2.

We therefore have

$$E_1 : E_2 :: R_1 : R_2,$$

or

$$E_1\,R_2 = E_2\,R_1,$$

and

$$E_1 : E_2 :: R_1 + \rho_1 : R_2 + \rho_2,$$

or

$$E_1\,R_2 + E_1\,\rho_2 = E_2\,R_1 + E_2\,\rho_1 = E_1\,R_2 + E_2\,\rho_1;$$

that is

$$E_1\,\rho_2 = E_2\,\rho_1,$$

or

$$E_1 : E_2 :: \rho_1 : \rho_2.$$

In fact, the electromotive forces of the batteries are directly proportional to the added resistances which, in both cases, were required to bring the deflections of the galvanometer needle from a° down to β°.

For example.

With a galvanometer and battery E_1 we obtained, with a resistance of 1950 ohms in the resistance box, a deflection of 54°, and by adding 2000 ohms (ρ_1), a deflection of 34°. Battery E_2 being inserted in the place of E_1, a resistance of 1650 ohms was inserted in the resistance box, which brought the galvanometer needle to 54° as at first, and by adding 1600 ohms (ρ_2), the deflection was reduced to 34° as in the first instance; then

$$E_1 : E_2 : : 2000 : 1600,$$

or as

$$1 \cdot 25 \text{ to } 1.$$

187. In this and the preceding tests we have supposed that the relative electromotive forces of any *two* batteries were being considered, but it must be evident that by noting the deflections, resistances added, &c., as the case may be, with any number of batteries, their electromotive forces may all be compared.

188. We will now proceed to determine the "Best conditions for making the foregoing test."

There are two points to be determined: first, what should be the resistances in circuit when observing the first deflections, and second, what proportion should the added resistances bear to the original resistances?

When the test is executed, there are two or more sets of observations made, viz. one for each battery. But it will be found, on examination, that the proportion between the electromotive forces, the original resistances, and the added resistances, is the same for every set; consequently, we have only to determine what relative values these quantities should have in any one set, then those in the others will be in the same proportion.

It will be convenient to consider first what proportion the added resistance should bear to the original resistance. For this purpose we will suppose ρ_1 to be the former resistance.

Now ρ_1 represents the electromotive force of the battery, and therefore in order that the test may be made as accurately as possible, it is necessary that we should be able to adjust or determine the value of ρ_1 as accurately as possible. In order to obtain the required value of ρ_1, we first adjust R_1 so as to obtain the deflection a°, and then we increase R_1 by ρ_1 so as to obtain the deflection β°; consequently, the accuracy with which we can obtain ρ_1 must be dependent upon the accuracy with which we can read both the deflections, a° and β°.

Let, then, the first deflection (a°) be due to a current, C_1, then we have

$$C_1 = \frac{E_1}{R_1}, \quad \text{or,} \quad C_1 R_1 = E_1 .$$

When the current is reduced to C_2 by the addition of ρ_1, then we get

$$C_2 = \frac{E_1}{R_1 + \rho_1}, \quad \text{or,} \quad C_2 R_1 + C_2 \rho_1 = E_1;$$

therefore

$$C_2 R_1 + C_2 \rho_1 = C_1 R_1,$$

or

$$\rho_1 = R_1 \left(\frac{C_1}{C_2} - 1 \right).$$

Now this equation is identical with equation [F] (page 131) in the "Diminished deflection shunt method" of determining the resistance of a galvanometer; consequently, we can see from the investigations there given, that ρ_1 would be most accurately obtained if

$$C_2 = \frac{C_1}{3}$$

approximately; but when this is the case

$$\rho_1 = R_1 \left(\frac{C_1}{\frac{C_1}{3}} - 1 \right) = 2\, R_1;$$

that is to say, the added resistance should be about double the original resistance.

As regards the "Possible degree of accuracy attainable," we can see from equation [H] (page 131) in the test before referred to, that the percentage of accuracy, λ', attainable must be

$$\lambda' = \frac{(C_1 c_2 + C_2 c_1)\, 100^*}{C_2 (C_1 - C_2)} \text{ per cent.}$$

As it is the relative electromotive forces of *two* batteries which have to be determined, that is to say, the value of $\frac{E_1}{E_2}$, the percentage of accuracy with which the test can be made will be double the above.

As regards the value for the original resistance there is little to be said. It does not affect the accuracy of the test, except in so far as the power of adjustment is concerned; this is evidently made as favourable as possible by making the resistance as high as convenient.

We must have therefore

Best Conditions for making the Test.

189. When making the observations with the first battery, make the original resistance as high as convenient, and make the added resistance as nearly as possible double this.

* The expression $\left[1 + G \left(\frac{1}{S_1} + \frac{1}{R} \right) \right]$ in the equation referred to [H] (page 131) becomes equal to 1 when S_1 and R are very high; this must be the case when equation [B] (page 129) becomes simplified into equation [F] (page 131).

Possible Degree of Accuracy attainable.

When the readings are in *divisions*, then

$$\text{Percentage of accuracy} = \frac{\frac{1}{m}(D - d)\,200}{d\,(D - d)}$$

where $\frac{1}{m}$ is the smallest fraction of a division to which the deflections can be read.

If the deflections are in *degrees* on a tangent galvanometer, then if we can read their values to an accuracy of $\frac{1}{m}$th of a *degree*, we have

$$\text{Percentage of accuracy} = \frac{(\tan D^\circ\,\delta_2 + \tan d^\circ\,\delta_1)\,200}{\tan d^\circ\,(\tan D^\circ - \tan d^\circ)}$$

where

$$\delta_1 = \tan D\tfrac{1}{m}^\circ - \tan D^\circ, \quad \text{and,} \quad \delta_2 = \tan d\tfrac{1}{m}^\circ - \tan d^\circ.$$

190. Wheatstone's test can be made with any form of galvanometer, as it is not necessary that the values of the deflections in terms of the currents producing them be known, except for the determination of the "Percentage of accuracy attainable." If, however, the galvanometer be "calibrated" (page 47), this percentage can be determined.

Lumsden's or Lacoine's Method.*

191. This is an excellent method of determining the comparative electromotive forces of batteries. The principle of the arrangement is shown by Fig. 87.

First Method.

The two batteries, E_1, E_2 are joined up with their opposite poles connected together, and with resistances R, ρ in their circuit, as shown by Fig. 87. A galvanometer g is connected between the points A, B. One of the resistances, say ρ, being fixed, the other R, is adjusted until no deflection is observed on the galvanometer. When this is the case we get the proportion

$$\frac{E_1}{E_2} = \frac{R}{\rho}.$$

* This method was devised by Mr. D. Lumsden (late Postal Telegraph Submarine Superintendent) in 1869, but the first description of the same appears to have been published by M. Emile Lacoine (Technical Director of the Ottoman Telegraphs) in the 'Journal Télégraphique' of Berne for January 25th, 1878, that gentleman having devised it independently of Mr. Lumsden.

The deflection, it should be remarked, will be in one direction if R is too large, and in the opposite direction if R is too small.

192. In order to understand why this is the case, let us examine the theory of the method; this may be explained by the help of Kirchoff's two laws,* viz.:—

1. *The algebraical sum of the current strengths in all the wires which meet in a point is equal to nothing.*

2. *The algebraical sum of all the products of the current strengths und resistances in all the wires forming an enclosed figure, equals the algebraical sum of all the electromotive forces in the circuit.*

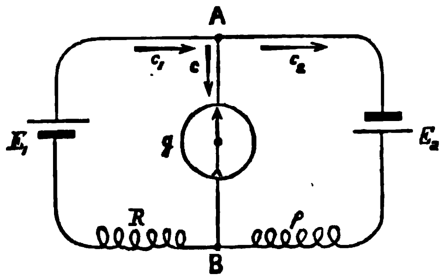


FIG. 87.

193. Supposing, at first, equilibrium not to be produced, then we have the following equations connecting the various current strengths, resistances, and electromotive forces:—

$$c_1 - c - c_2 = 0. \qquad [1]$$
$$R\, c_1 + g\, c - E_1 = 0. \qquad [2]$$
$$\rho\, c_2 - g\, c - E_2 = 0. \qquad [3]$$

From equation [1] we get, $c_1 = c + c_2$;

therefore

$$R\,(c + c_2) + g\, c - E_1 = 0.$$

From equation [3] we get

$$c_2 = \frac{E_2 + g\, c}{\rho};$$

therefore

$$R\left(c + \frac{E_2 + g\, c}{\rho}\right) + g\, c - E_1 = 0;$$

therefore

$$R\,\rho\, c + R\, E_2 + R\, g\, c + \rho\, g\, c - \rho\, E_1 = 0;$$

* For the proof of these laws see Appendix.

therefore

$$c = \frac{\rho\,E_1 - R\,E_2}{g\,(R + \rho) + R\,\rho}. \qquad [4]$$

If in this equation we put

$$c = 0,$$

then

$$\rho\,E_1 - R\,E_2 = 0,$$

or

$$\frac{E_1}{E_2} = \frac{R}{\rho},$$

that is,

$$E_1 : E_2 :: R : \rho.$$

194. Let us consider what are the "Best conditions for making the test." What we have to determine is, what are the best values to give to R and ρ? Now, since E_1 and E_2 are definite quantities, the value given to R (supposing this to be the adjustable resistance) will be determined by the value given to ρ; we must therefore determine the value to give to the latter.

The greater the accuracy with which we can adjust R, the greater will be the accuracy with which we can determine the value of $\frac{E_1}{E_2}$, that is, the relative values of E_1 and E_2. But the accuracy with which we can adjust R depends upon its range of adjustment being as great as possible, and this can only be the case when it has as high a value as possible. Thus, if R were 100 units, we could only adjust it to an accuracy of 1 unit in 100, or 1 per cent.; but if R were 10,000, then 1 unit in 10,000 represents an adjustment of $\frac{1}{100}$ per cent. But it is no use making R 10,000, unless a change of 1 unit in its value produces a perceptible deflection of the galvanometer needle.

The best value therefore to give to R *is the highest one in which a change of* 1 *unit from its correct value produces a perceptible deflection* of the galvanometer needle. Since R is dependent upon the value given to ρ, what we require to know is the highest value to give to the latter quantity.

Equation [4] shows the current, c, obtained through the galvanometer when equilibrium is not produced. If in this equation we put R — 1 in the numerator instead of R and then put

$$R\,E_2 = \rho\,E_1, \quad \text{or,} \quad R = \frac{\rho\,E_1}{E_2},$$

we shall get the current corresponding to the change of 1 unit in the correct value of R_1. Thus

$$c = \frac{\rho\,E_1 - (R-1)\,E_2}{g\,(R + \rho) + R\,\rho} = \frac{E_2}{g\left(\frac{\rho\,E_1}{E_2} + \rho\right) + \frac{\rho\,E_1}{E_2}\,\rho}$$

$$= \frac{E_2}{\rho\,\frac{E_1}{E}\left[g\left(1 + \frac{E_2}{E_1}\right) + \rho\right]}, \qquad [A]$$

or

$$\rho\left[g\left(1 + \frac{E_2}{E_1}\right) + \rho\right] + \frac{E_2^{\,2}}{E_1\,c}. \qquad [B]$$

Practically, the minimum readable deflection of a Thomson galvanometer (which is the best to employ in a test of this kind) is one division, and the current producing this deflection is the *figure of merit* of the instrument (page 85). If, therefore, in the last equation we put for c the figure of merit of the galvanometer, we can determine the highest value which can be given to ρ, E_1 and E_2 both being in *volts*.

If we wish to get the exact value of ρ, we can do so by solving the quadratic equation; but, practically, we only require to get a rough idea of what the value of ρ may be, and this we may obtain by giving different values to ρ, and trying which of them nearly satisfies the equation.

For example.

Two batteries, whose electromotive forces E_1 and E_2 were known to be of the approximate values of 2:1 (E_2 being 1 *volt*), were to be tested by the foregoing method with a Thomson galvanometer whose resistance was 5000 ohms (g) and figure of merit ·000,000,001. What was the highest value that could be given to ρ?

$$\rho\,[5000\,(1 + \tfrac{1}{2}) + \rho] = \frac{1}{\cdot 000{,}000{,}001 \times 2},$$

or

$$\rho\,[7500 + \rho] = 500{,}000{,}000.$$

From this we can see that if we make $\rho = 19{,}000$ we shall be very nearly right, for

$$19{,}000\,[7500 + 19{,}000] = 503{,}500{,}000.$$

With this value of ρ, the value which R would have when adjusted, would be

$$R = \rho\,\frac{E_1}{E_2} = 19{,}000 \times \frac{2}{1} = 38{,}000,$$

and with this value we could obtain a degree of accuracy equal to

$$\frac{1}{38{,}000} \times 100 = \cdot 0026 \text{ per cent.}$$

Having then ascertained the value to give to ρ, suppose we actually made it 19,000, and further, we found that in order to get equilibrium as nearly as possible, we had to adjust R to 36,250 ohms, then the relative values of E_1 and E_2 would be

$$E_1 : E_2 :: 36{,}250 : 19{,}000,$$

or as

$$1 \cdot 9089 \text{ to } 1,$$

and we know this is correct within ·0026 per cent.

From equation [A] (page 179) we can see that c is greatest when E_2 is larger than E_1. It is therefore best to so arrange the test that the resistance to be adjusted is the one in circuit with the strongest of the two batteries. Also we can see that the more the batteries differ in electromotive force the better, as the greater will be the value of ρ.

Second Method.

195. If the batteries consist of a large number of cells of high resistance, and also, if the galvanometer has not a high figure of merit, and consequently R and ρ have to be proportionately small,

then we cannot ignore (as in the previous test) the resistances of the batteries, and these must either be added on to R and ρ, or eliminated in the following manner.

Suppose the resistances of E_1 and E_2 to be r_1 and r_2 respectively, then, when equilibrium is produced, we have

$$E_1 : E_2 :: R + r_1 : \rho + r_2,$$

or

$$E_1 r_2 - E_2 r_1 = E_2 R - E_1 \rho. \qquad [1]$$

Now if we decrease ρ to ρ_1, and again obtain balance by decreasing R to R_1, we get a second proportion, viz.—

$$E_1 r_2 - E_2 r_1 = E_2 R_1 - E_1 \rho_1. \qquad [2]$$

By subtracting [2] from [1] we get

$$E_2 R - E_2 R_1 - E_1 \rho + E_1 \rho_1 = 0,$$

or

$$E_2 (R - R_1) = E_1 (\rho - \rho_1);$$

that is

$$\frac{E_1}{E_2} = \frac{R - R_1}{\rho - \rho_1}, \qquad [A]$$

or

$$E_1 : E_2 :: R - R_1 : \rho - \rho_1,$$

a proportion in which differences of resistance alone appear. In fact $(R - R_1)$ and $(\rho - \rho_1)$ are merely the resistances we subtracted from R and ρ, in order to get equilibrium a second time.

For example.

Two batteries whose electromotive forces E_1 and E_2 were to be compared, were joined up in circuit with a galvanometer and two resistances, as shown by Fig. 87, the resistance ρ being 500 ohms; in order to obtain equilibrium R was adjusted to 1050 ohms; ρ was then decreased to 300 ohms (ρ_1), and in order to again obtain equilibrium, R had to be reduced to 630 ohms (R_1). What were the comparative electromotive forces of the batteries?

$$E_1 : E_2 :: 1050 - 630 : 500 - 300$$
$$:: \quad 420 \quad : \quad 200$$

or as

$$2 \cdot 1 \text{ to } 1.$$

196. What are the best values to give to R_1 and ρ_1, or rather to ρ_1, for the value given to the latter will determine the value given to R_1?

In order to work out the problem let us suppose, in the equation

$$\frac{E_1}{E_2} = \frac{R - R_1}{\rho - \rho_1},$$

there is a small error λ in $\frac{E_1}{E_2}$ caused by a definite error $-\phi$ in R_1, that is, let

$$\frac{E_1}{E_2} + \lambda = \frac{R - (R_1 - \phi)}{\rho - \rho_1} = \frac{R - R_1}{\rho - \rho_1} + \frac{\phi}{\rho - \rho_1}. \qquad [B]$$

By subtracting [A] (page 181) from [B] we get

$$\lambda = \frac{\phi}{\rho - \rho_1}.$$

This shows that with the definite error ϕ, λ is as small as possible when ρ_1 is as small as possible. λ would be very great if ρ_1 approached in value to ρ; but it would be small when ρ_1 is about equal to $\frac{\rho}{2}$, and but little less if ρ_1 is made very much smaller still. Although, therefore, we should make ρ_1 small, there is but little advantage in making it very much smaller than $\frac{\rho}{2}$: in fact, there is an actual disadvantage, for when ρ_1 is very small, R_1 is proportionately small, and its range of adjustment is correspondingly limited.

From equation [A] (page 179) we can see that in the present case the currents flowing through the galvanometer when equilibrium is not established, in consequence of R and R_1 being each 1 unit out of adjustment, are

$$c_1 = \frac{E_2}{(\rho + r_2)\frac{E_1}{E_2}\left[g\left(1 + \frac{E_2}{E_1}\right) + \rho + r_2\right]}$$

and

$$c_2 = \frac{E_2}{(\rho_1 + r_2)\frac{E_1}{E_2}\left[g\left(1 + \frac{E_2}{E_1}\right) + \rho_1 + r_2\right]},$$

respectively; and from these equations it is evident that if c_1 is a perceptible deflection when R is 1 unit out, c_2 will be a still more perceptible deflection when R_1 is 1 unit out, since R_1 must be smaller than R; consequently the value we give to R_1 will not be limited by any considerations with regard to a perceptible deflection being obtained.

As in the first test, c_1 and c_2 are both greatest when E_1 is larger than E_2; the batteries should therefore be so arranged that this is the case.

With regard to the *Possible degree of accuracy attainable* with this test, we can see first of all that R cannot be adjusted quite so accurately as in the case where the resistance of the batteries was negligible; we can, however, ascertain the exact degree attainable by putting $\rho + r_2$ instead of ρ in equation [B] (page 179). Thus, to take the example given on page 179, suppose the battery E_2 had a resistance of 5000 ohms (r_2) approximately, then we should have

$$(\rho + 5000)[5000(1 + \tfrac{1}{2}) + \rho + 5000] = \frac{1}{\cdot 000{,}000{,}001 \times 2},$$

or

$$(\rho + 5000)[12{,}500 + \rho] = 500{,}000{,}000.$$

If in this equation we make $\rho = 14,000$, we get

$$(14,000 + 5000)\,[12,500 + 14,000] = 503,500,000,$$

which is close to the correct value. In other words, if ρ does not exceed 14,000 ohms, we can be sure of the value of R within 1 unit.

The degree of accuracy with which we can determine the value of $\frac{E_1}{E_2}$ from the equation

$$\frac{E_1}{E_2} = \frac{R - R_1}{\rho - \rho_1}$$

depends upon the degree of accuracy with which we can adjust both R and R_1, and as the errors in either of them may be either + or −, the greatest possible total error is that which will be produced by a + error in R, and a − error in R_1, or *vice versâ*. Let these errors be both 1 unit, and let the corresponding error in $\frac{E_1}{E_2}$ be λ, then we have

$$\frac{E_1}{E_2} + \lambda = \frac{R + 1 - (R_1 - 1)}{\rho - \rho_1} = \frac{R - R_1}{\rho - \rho_1} + \frac{2}{\rho - \rho_1},$$

and

$$\frac{E_1}{E_2} = \frac{R - R_1}{\rho - \rho_1},$$

therefore

$$\lambda = \frac{2}{\rho - \rho_1}.$$

Since we require to know what *percentage* (λ') of error this represents, we have

$$\lambda = \frac{\lambda}{100} \text{ of } \frac{E_1}{E_2},$$

or

$$\lambda' = 100\,\lambda\,\frac{E_1}{E_2} = \frac{200}{\rho - \rho_1} \cdot \frac{E_1}{E_2}. \qquad [C]$$

To take the example we have just considered, we see that the possible percentage of accuracy attainable, supposing ρ_1 to equal $\frac{\rho}{2}$, is

$$\lambda' = \frac{200}{14,000 - 7000} \times \tfrac{1}{2} = \cdot 014 \text{ per cent.}$$

197. With a Thomson galvanometer of ordinary sensitiveness it is evident from the foregoing investigation, that if we have two batteries, one E_2 having an electromotive force of 1 volt or more, and E_1 an electromotive force of twice that value or more, we can without difficulty determine their relative electromotive forces to an accuracy of, at least, ·015 per cent.; and if the resistance of the batteries be very low, we can be certain of the accuracy within, say, ·003 per cent.

198. It is possible to get a still greater accuracy by employing a set of resistance coils adjustable to $\frac{1}{10}$th or $\frac{1}{100}$th of a unit, for in this case we can make both R and R_1 low without losing the range of adjustment, whilst by making these quantities low we increase the value of the galvanometer deflection when exact adjustment is not obtained; this is only the case, however, when the resistances of the batteries and of the galvanometer are low.

We can easily determine to what extent the degree of accuracy is increased by using submultiples of the units; first by ascertaining from equation [B] (page 179) what value ρ can have, $\frac{E_2^2}{E_1 c}$ being divided by 10 if R is adjustable to $\frac{1}{10}$ths, and by 100 if R is adjustable to $\frac{1}{100}$ths; and second by working out the value of λ' from equation [C] (page 183) which gives the required percentage of accuracy.

Of course when great accuracy is required, the test must be made by the method in which the resistances of the batteries are eliminated; it is no use making the test by the first method, since the accuracy attainable by having R adjustable to $\frac{1}{10}$th or $\frac{1}{100}$th of an ohm is more than counterbalanced by the error produced by not taking into account the resistance of the batteries.

To summarise the results we have obtained, we have

Best Conditions for making the Test.

First Method.

199. First make a rough test to ascertain the approximate values of E_1 and E_2, then make ρ of such a value that

$$\rho\left[g\left(1+\frac{E_2}{E_1}\right)+\rho\right]=\frac{E_2^2}{E_1 c}$$

approximately, c being the figure of merit of the galvanometer, and E_1 the stronger of the two batteries, E_1, and E_2 being in *volts.*

Second Method.

Make ρ of such a value that

$$(\rho+r_2)\left[g\left(1+\frac{E_2}{E_1}\right)+\rho+r_2\right]=\frac{E_2^2}{E_1 c}$$

approximately.

If R is adjustable to $\frac{1}{10}$th or $\frac{1}{100}$th of an ohm, the right-hand side of the equation should be $\frac{E_2^2}{E_1\, 10\, c}$ or $\frac{E_2^2}{E_1\, 100\, c}$ respectively.

ρ_1 should be about equal to $\frac{\rho}{2}$.

In both methods E_1 should be the larger of the two batteries.

Possible Degree of Accuracy attainable.

First Method.

Where resistance of battery is very small,

$$\text{Percentage of accuracy} = \frac{100}{\rho}\times\frac{E_2}{E_1}.$$

Second Method.

$$\text{Percentage of accuracy} = \frac{200}{\rho - \rho_1} \times \frac{E_2}{E_1}.$$

Or, if ρ_1 is nearly equal to $\frac{\rho}{2}$,

$$\text{Percentage of accuracy} = \frac{400}{\rho} \times \frac{E_2}{E_1}.$$

200. A great point in these methods of determining the comparative electromotive forces of batteries, lies in the fact that both batteries are working under exactly the same conditions; moreover, if the resistances R and ρ are high there is but little tendency for the batteries to polarise. If one of the batteries be a constant one, such as a Daniell, then by varying the values of R and ρ we can test how the other battery behaves when worked through different resistances.

Poggendorff's Method.

201. In this method one battery is balanced against the other. The method is shown by Fig. 88. In this figure E_1 and E_2 are the electromotive forces to be compared. R and ρ are adjustable resistances, r_1 and r_2 being the resistances of the batteries. G is the resistance of the galvanometer.

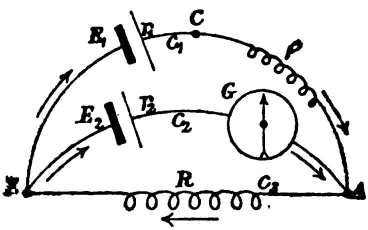


Fig. 88.

Before equilibrium is obtained we have

$$c_1 + c_2 - c_3 = 0 \qquad [1]$$

$$(r_1 + \rho)\, c_1 + R\, c_3 - E_1 = 0 \qquad [2]$$

$$(r_2 + G)\, c_2 + R\, c_3 - E_2 = 0 \qquad [3]$$

By substituting the value of c_1 obtained from equation [1], in equation [2], and then again the value of c_3 obtained from equation [2], in equation [3], we shall find that

$$c_2 = \frac{(r_1 + \rho)\, E_2 - R\,(E_1 - E_2)}{R\,(r_2 + G + r_1 + \rho) + (r_1 + \rho)\,(r_2 + G)}. \qquad [4]$$

If we put $c_2 = 0$, we get

$$(r_1 + \rho) E_2 - R(E_1 - E_2) = 0,$$

or

$$E_2 (R + r_1 + \rho) = E_1 R;$$

that is

$$E_1 : E_2 :: R + r_1 + \rho : R, \qquad [5]$$

or

$$\frac{E_1}{E_2} = 1 + \frac{r_1 + \rho}{R}. \qquad [6]$$

It will be observed that in order to get the ratio of E_1 to E_2 from this proportion, we must know the resistance r_1 of the battery E_1. If, however, we decrease ρ to ρ_1, and again get equilibrium by readjusting R to R_1, we get a second proportion, viz.

$$E_1 : E_2 :: R_1 + r_1 + \rho_1 : R_1, \qquad [7]$$

and by combining the two proportions, r_1 is eliminated in the manner shown in the last test (page 181) and we get,

$$\frac{E_1}{E_2} = \frac{(R - R_1) + (\rho - \rho_1)}{R - R_1},$$

or

$$E_1 : E_2 :: (R - R_1) + (\rho - \rho_1) : (R - R_1), \qquad [A]$$

a proportion in which differences of resistance alone enter.

For example.

Two batteries whose electromotive forces E_1 and E_2 were to be compared, were joined up in circuit with a galvanometer and two resistances as shown in Fig. 88. The resistance ρ being 200 ohms, it was necessary in order to obtain equilibrium to adjust R to 500 ohms. ρ was then reduced to 100 ohms (ρ_1), and in order again to get equilibrium R had to be readjusted to 400 ohms (R_1), then

$$E_1 : E_2 :: (500 - 400) + (200 - 100) : (500 - 400);$$

or as

$$2 : 1.$$

202. In making this test practically, the connections with the set of resistance coils shown by Fig. 6, page 14 would be as shown by Fig. 89. Having depressed the left-hand key, then, according to the example, we should take out the two 100 plugs between A and C, and proceed to adjust between D and E. This being

done, we should insert one of the 100 plugs between A and C and readjust the resistance between D and E.

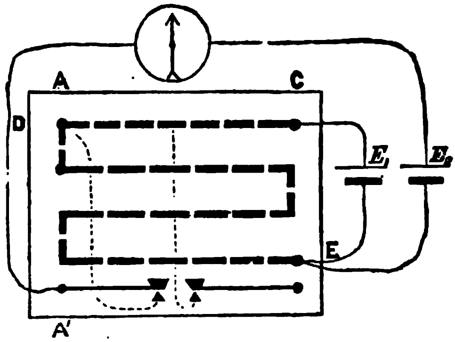


FIG. 89.

203. As only one of the batteries (the smaller) in this test has its electromotive force balanced, the other one should be a constant battery, whose electromotive force does not fall off on being worked continuously, such as a Daniell.

204. It is evident that the test can be made either by making ρ a fixed resistance and R an adjustable one, or by making R fixed and ρ adjustable. In order therefore to determine the *Best conditions for making the test*, one point for consideration will be—should R or ρ be the adjustable quantity?

Now by a similar reasoning to that given in § 194, page 179, we can see that in either case the value of the adjustable resistance should be *the highest one in which a change of* 1 *unit from its correct resistance produces a perceptible deflection of the galvanometer needle.*

If we refer to equation [6] (page 186) we can see that if $E_1 = 2\,E_2$ then $r_1 + \rho$ must be equal R, and that according as E_1 is greater or less than E_2, so will $r_1 + \rho$ be greater or less than R. It is evident that the larger we make the adjustable resistance the greater will be the range of adjustment of which it is capable, therefore for this reason it follows that if E_1 is greater than 2 E_2 then $r_1 + \rho$ should be the resistance in which the adjustment is effected, whereas if E_1 is less than 2 E_2 then R should be the adjustable resistance.

Now if R be the adjustable resistance, then inasmuch as the value which it will have will depend upon the value given to ρ, therefore we must determine the highest value we can give to ρ.

Equation [4] (page 185) shows the current c_2 obtained through the galvanometer, when equilibrium is not produced. If in this equation we put R − 1 in the numerator instead of R, and then put

$$\frac{E_1}{E_2} = \frac{R + r_1 + \rho}{R}, \quad \text{or,} \quad R = (r_1 + \rho)\frac{E_2}{E_1 - E_2},$$

we shall get the current, c_2, corresponding to the change of 1 unit in the correct value of R. Thus

$$c_2 = \frac{(E - E_2)^2}{(r_1 + \rho)[(r_1 + G) E_1 + (r_1 + \rho) E_2]}, \qquad [1]$$

or

$$(r_1 + \rho)[(r_2 + G) E_1 + (r_1 + \rho) E_2] = \frac{(E_1 - E_2)^2}{c_2}. \qquad [A]$$

And if in this equation we make c_2 the figure of merit (page 85) of the galvanometer, then the value of ρ which satisfies the equation will be the highest value which it should have; as explained in the last test, ρ can be obtained by trial.

If ρ be the adjustable resistance, then what we have to determine is the value which R should have. To do this we must put $\rho + 1$* instead of ρ in the numerator of equation [4] (page 185) and then put

$$\frac{E_1}{E_2} = \frac{R + r_1 + \rho}{R}, \quad \text{or,} \quad r_1 + \rho = \frac{R(E_1 - E_2)}{E_2},$$

we shall then get the current c'_2, corresponding to the change of 1 unit in the current value of ρ. Thus

$$c'_2 = \frac{E_2^2}{R[(r_2 + G) E_1 + R(E_1 - E_2)]},$$

or

$$R[(r_2 + G) E_1 + R(E_1 - E_2)] = \frac{E_2^2}{c'_2}, \qquad [B]$$

from which, as in the previous case, R can be obtained by trial.

We have next to determine the value which should be given to R, or to ρ_1. Let us in the first instance take R_1 to be the adjustable resistance, then what we have to do is to find the proper value to give to ρ_1. If, then, we suppose in the equation

$$\frac{E_1}{E_2} = \frac{(R - R_1) + (\rho - \rho_1)}{R - R_1}, \qquad [2]$$

or

$$\frac{E_1}{E_2} = 1 + \frac{\rho - \rho_1}{R - R_1}, \qquad [3]$$

that there is a small error λ in $\frac{E_1}{E_2}$ caused by a corresponding error $-\phi$ in R_1; then we have

$$\frac{E_1}{E_2} + \lambda = 1 + \frac{\rho - (\rho_1 - \phi)}{R - R_1}. \qquad [4]$$

By subtracting [3] from [4] we get

$$\lambda = \frac{\rho - (\rho_1 - \phi)}{R - R_1} - \frac{\rho - \rho_1}{R - R_1} = \frac{\phi}{R - R_1};$$

but from [3]

$$R - R_1 = \left(\frac{E_2}{E_1 - E_2}\right)(\rho - \rho_1);$$

* We put $\rho + 1$ in this case in preference to $\rho - 1$, simply in order to avoid giving c_2 a minus value. The general result obtained, however, would be similar whether the 1 be plus or minus.

therefore

$$\lambda = \frac{\phi}{\left(\frac{E_1}{E_1 - E_2}\right)(\rho - \rho_1)}.$$

This shows that with the definite error ϕ, λ is as small as possible when ρ_1 is as small as possible. λ would be very great if ρ_1 approaches in value to ρ, but it would be small when ρ_1 is about equal to $\frac{\rho}{2}$, and but little less if ρ_1 is made very small indeed. As our range of adjustment of R_1 is limited by making ρ_1 very small, it is advisable not to make it smaller than $\frac{\rho}{2}$.

A similar investigation would have proved that if ρ_1 were the adjustable resistance, then R_1 should be made small, though not smaller than $\frac{R}{2}$.

205. From equation [1] (page 188) we can see that the test is impossible if E_1 and E_2 are equal, since $c_2 = 0$ with any value we can give to the resistances.* We can further see that the more the batteries differ in electromotive force the better; and also that it does not matter materially which is the stronger of the two.

206. As regards the *Possible degree of accuracy attainable*, this depends upon the degree of accuracy with which we can adjust both R and R_1 (or ρ and ρ_1, if R and R_1 are the fixed resistances), and as the errors in either of them may be + or −, the greatest possible error is that which will be produced by a + error in R and a − error in R_1 or *vice versâ*. Let these errors be both 1 unit, and let the corresponding error in $\frac{E_1}{E_2}$ be λ, then we have from equation [3] (page 188)

$$\frac{E_1}{E_2} + \lambda = 1 + \frac{\rho - \rho_1}{R - 1 - (R_1 + 1)} = 1 + \frac{\rho - \rho_1}{R - R_1 - 2},$$

and

$$\frac{E_1}{E_2} = 1 + \frac{\rho - \rho_1}{R - R_1}, \quad \text{or,} \quad R - R_1 = \frac{E_2}{E_1 - E_2}(\rho - \rho_1);$$

therefore

$$\lambda = \frac{\rho - \rho_1}{R - R_1 - 2} - \frac{\rho - \rho_1}{R - R_1} = \frac{2(\rho - \rho_1)}{(R - R_1 - 2)(R - R_1)};$$

or, since $R - R_1$ is very large,

$$\lambda = \frac{2(\rho - \rho_1)}{(R - R_1)^2} = \frac{2}{\rho - \rho_1} \times \left(\frac{E_1 - E_2}{E_2}\right)^2.$$

Since we require to know what *percentage* (λ') of error this represents, we have

$$\lambda = \frac{\lambda'}{100} \text{ of } \frac{E_1}{E_2},$$

or

$$\lambda' = 100\,\lambda\frac{E_2}{E_1} = \frac{200}{\rho - \rho_1} \times \frac{(E_1 - E_2)^2}{E_1\,E_2}.$$

---

* This is not the case with Lumsden's test.

In the case where ρ and ρ_1 are the adjustable resistances, we should get

$$\lambda = \frac{\rho - \rho_1 + 2}{R - R_1} - \frac{\rho - \rho_1}{R - R_1} = \frac{2}{R - R_1},$$

and calling, as before, λ' the *percentage* of error, we get

$$\lambda = \frac{200}{R - R_1} \times \frac{E_2}{E_1}.$$

To sum up, then, we have

Best Conditions for making the Test.

207. First make a rough test to ascertain the approximate values of E_1, E_2, r_1, and r_2; then if E_1 is less than 2 E_2, make ρ a fixed resistance, and of such a value that

$$(r_1 + \rho)\,[(r_2 + G)\,E_1 + (r_1 + \rho)\,E_2] = \frac{(E_1 - E_2)^2}{c} \quad [A]$$

approximately.

If R is adjustable to $\frac{1}{n}$th of an ohm, then the right-hand side of the last equation should be

$$\frac{(E_1 - E_2)^2}{c} \times \frac{1}{n},*$$

c being the figure of merit of the galvanometer, and E_1 and E_2 both being in *volts*.

ρ_1 should be about equal to $\frac{\rho}{2}$.

If E_1 is greater than 2 E_2 then make R a fixed resistance, and of such a value that

$$R\,[(r_2 + G)\,E_1 + R\,(E_1 - E_2)] = \frac{E_2^2}{c} \quad [B]$$

approximately.

If ρ is adjustable to $\frac{1}{n}$th of an ohm, then the right-hand side of the last equation should be

$$\frac{E_2^2}{c} \times \frac{1}{n},$$

c being the figure of merit of the galvanometer, and E_1 and E_2 being both in *volts*.

R_1 should be about equal to $\frac{R}{2}$.

* § 198, page 183.

Possible Degree of Accuracy attainable.

When R and R_1 are the adjustable resistances, then

$$\text{Percentage of accuracy} = \frac{200}{\rho - \rho_1} \times \frac{(E_1 - E_2)^2}{E_1\,E_2};$$

or if ρ_1 nearly equals $\frac{\rho}{2}$

$$\text{Percentage of accuracy} = \frac{400}{\rho} \times \frac{(E_1 - E_2)^2}{E_1\,E_2}.$$

When ρ and ρ_1 are the adjustable resistances, then

$$\text{Percentage of accuracy} = \frac{200}{R - R_1} \times \frac{E_2}{E_1};$$

or if R_1 nearly equals $\frac{R}{2}$

$$\text{Percentage of accuracy} = \frac{400}{R} \times \frac{E_2}{E_1}.$$

208. If the test is made by obtaining the result from formula [6] (page 186), the resistance r_1 of the battery being very small, then it is not difficult to see, from the investigation given in "Lumsden's test" (page 177) that when R is the adjustable resistance,

$$\text{Percentage of accuracy} = \frac{100}{\rho} \times \frac{(E_1 - E_2)^2}{E_1\,E_2}.$$

Also we should make ρ of such a value that

$$\rho\,(G\,E_1 + \rho\,E_2) = \frac{(E_1 - E_2)^2}{c}$$

approximately.

When ρ is the adjustable resistance, then

$$\text{Percentage of accuracy} = \frac{100}{R} \times \frac{E_2}{E_1}.$$

Also we should make R of such a value that

$$R\,[G\,E_1 + R\,(E_1 - E_2)] = \frac{E_2}{c}$$

approximately.

Fahie's Method of Measuring Battery Resistance.

209. It may be pointed out* that the foregoing test also affords a means of ascertaining the *resistance* r_1, of the battery

* See Sabine's 'The Electric Telegraph,' p. 323.

E_1; thus from equations [5] and [7] (page 186) we can see that

$$R + r_1 + \rho : R :: R_1 + r_1 + \rho_1 : R_1;$$

therefore

$$R_1 R + R_1 r_1 + R_1 \rho = R_1 R + R r_1 + R \rho_1,$$

therefore

$$r_1 (R - R_1) = R_1 \rho - R \rho_1,$$

or

$$r_1 = \frac{R_1 \rho - R \rho_1}{R - R_1};$$

thus if we take the example given on page 186, in which we have

$$\begin{aligned} R_1 &= 400 \\ R &= 500 \\ \rho &= 200 \\ \rho_1 &= 100. \end{aligned}$$

we get

$$r_1 = \frac{(400 \times 200) - (500 \times 100)}{500 - 100} = 75 \text{ ohms.}$$

210. A resistance test made in this way, however, would not be an accurate one if the resistance r_1 of the battery were small in comparison with the resistance ρ_1 (which is in the same circuit with r_1), for in this case the high value of the latter would swamp, as it were, the low value of r_1. If, however, as suggested by Mr. Fahie,* we commence the test by having no resistance at first in circuit with the battery E_1, that is to say, if we have ρ_1, equal to 0, then we can obtain more satisfactory results; in this case we get

$$r_1 = \frac{R_1 \rho}{R - R_1}. \qquad [A]$$

211. With regard to the *Best conditions for making the test* according to formula [A], the resistance R_1 is the resistance required to produce balance in the first instance, and it can have but one value; R, however, is dependent upon ρ, so that what is required is the value which should be given to the latter quantity. Now from formula [A] we can see that the larger we make ρ the larger will be the value of R, and the larger we make the latter the greater will be its range of adjustment; consequently, as in the electromotive force test, we should give it *the highest value in which a change of* 1 *unit from its correct resistance produces a perceptible deflection of the galvanometer needle;* this resistance we shall obtain by giving ρ such a value that

$$(r_1 + \rho)[(r_2 + G) E_1 + (r_1 + \rho) E_2] = \frac{(E_1 - E_2)^2}{c}$$

approximately,† c being the figure of merit of the galvanometer.

* See 'Electrical Review,' vol. xii. p. 203.

† Equation [A], p. 188.

As regards the *Possible degree of accuracy attainable*, this we shall obtain, as in previous cases, by supposing that there is an error of $+1$ in R and an error of -1 in R_1, these errors causing a corresponding total error λ in r_1: thus

$$r_1 + \lambda = \frac{(R_1 + 1)\rho}{(R - 1) - (R_1 + 1)} = \frac{(R_1 + 1)\rho}{R - R_1 - 2},$$

and since

$$r_1 = \frac{R_1 \rho}{R - R_1}$$

we get

$$\lambda = \frac{(R_1 + 1)\rho}{R - R_1 - 2} - \frac{R_1 \rho}{R - R_1} = \frac{\rho(R + R_1)}{(R - R_1 - 2)(R - R_1)},$$

or since $R - R_1$ is very large, we may say

$$\lambda = \frac{\rho(R + R_1)}{(R - R_1)^2}; \qquad [B]$$

but

$$\frac{E_1}{E_2} = \frac{R_1 + r_1}{R_1} = \frac{R + \rho + r_1}{R},$$

or

$$R_1 = \frac{E_2 r_1}{E_1 - E_2}, \quad \text{and} \quad R = \frac{E_2(r_1 + \rho)}{E_1 - E_2},$$

therefore

$$R + R_1 = \frac{E_2}{E_1 - E_2}(2 r_1 + \rho), \quad \text{and} \quad R - R_1 = \frac{E_2 \rho}{E_1 - E_2};$$

and by substituting these values of $R + R_1$ and $R - R_1$ in equation [B] we get

$$\lambda = \frac{E_1 - E_2}{E_2}\left(\frac{2 r_1}{\rho} + 1\right).$$

Or if we call λ' the *percentage* of error, then

$$\lambda = \frac{\lambda'}{100} \text{ of } r_1,$$

or

$$\lambda' = \frac{100\,\lambda}{r_1} = \frac{E_1 - E_2}{E_2}\left(\frac{2}{\rho} + \frac{1}{r_1}\right) 100.$$

212. The relative electromotive forces of the batteries, it may be pointed out, are given by the proportion

$$E_1 : E_2 :: (R - R_1) + \rho : (R - R_1),$$

which is the same as proportion [A], page 186, except that ρ_1 is put equal to 0.

To sum up, then, we have

Best Conditions for making the Test.

213. Make ρ of such value that

$$(r_1 + \rho)\left[(r_2 + G)\,E_1 + (r_1 + \rho)\,E_2\right] = \frac{(E_1 + E_2)^2}{c}$$

approximately, c being the figure of merit of the galvanometer.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{E_1 - E_2}{E_2}\left(\frac{2}{\rho} + \frac{1}{r_1}\right) 100.$$

Fahie's Combined Method of Comparing Electromotive Forces and Measuring Battery Resistance.

214. This is an extremely ingenious and elegant method, and although its application is rather limited it is well worth being noticed. The arrangement is a combination of Poggendorff's method of comparing electromotive forces (page 185) and Mance's method of measuring battery resistances (page 143).

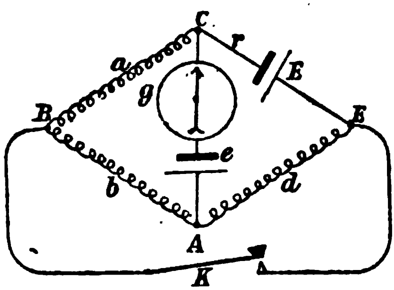


Fig. 90.

Referring to Fig. 90, the following is the mode of making the test:—E is the stronger battery whose electromotive force is to be compared with the battery e, and whose internal resistance is to be measured; d is a variable and $a + b$ a slide, resistance, B being the slider by the movement of which the ratio of a to b can be varied. The key K being open, the resistance d is adjusted until the needle of the galvanometer shows that no current is passing through the latter; when this is the case, then, as in Poggendorff's method (page 185), we have

$$E : e :: r + d + a + b : a + b. \qquad [1]$$

Balance being thus obtained, the key K is alternately depressed and raised and the slider B moved until the latter is brought to such a position that the movement of the key K ceases to affect the galvanometer needle, as in Mance's test (page 143). Now, inasmuch as the battery e merely acts as a counteracting force to the current which in Mance's test would cause a permanent

deflection of the galvanometer needle, it must be evident that when the movement of the key K ceases to affect g, then we must have

$$r = \frac{a\,d}{b}, \qquad [2]$$

or

$$r + d = \frac{a\,d}{b} + d = \frac{d}{b}(a + b).$$

Substituting this value of $r + d$ in equation [1], we get

$$\mathrm{E} : e :: \frac{d}{b}(a + b) + a + b : a + b,$$

or

$$\mathrm{E} : e :: d + b : b. \qquad [3]$$

Equation [2], therefore, gives the resistance of the battery E, and equation [3] gives the relative electromotive forces of the two batteries.

For example.

The key K being raised, balance was obtained on the galvanometer g by adjusting d to 200 ohms. When the key K was alternately raised and depressed, the balance on g was disturbed until the slider B was moved to the position at which b was equal to 100 ohms; the total resistance of the slide resistance $a + b$ was 400 ohms, that is to say, a was equal to 300 ohms; then

$$r = \frac{300 \times 200}{100} = 600 \text{ ohms},$$

and

$$\mathrm{E} : e :: 200 + 100 : 100,$$

or as

$$3 : 1.$$

215. The conditions for making this test so as to obtain accurate results must evidently be similar to those specified in the cases of Poggendorff's test and Mance's test made with a slide resistance. The nature of the method, however, is such that we cannot obtain the conditions which are best for the Poggendorff test without impairing the conditions necessary for making the Mance test accurately, so that practically we must arrange the resistances so as to suit the conditions necessary for making the latter satisfactorily; at the same time it may be pointed out that these conditions are such as to enable the Poggendorff test to be made with a considerable, though n t with a *very* high, degree of accuracy. As in the case of Mance's test with a slide wire (page 146), the conditions required are that d shall be as large as possible, but not so large that the range of adjustment of the slider becomes excessively reduced. Now, practically, a slide resist-

ance would not consist of more than about 100 coils; consequently if d were of such a value that the slider had to be set so that b was about 10 times as large as a (as would be the case when a slide wire is used), then the accuracy with which the latter could be adjusted would be extremely small, being only about 1 in 10, or 10 per cent. To make the test satisfactorily, therefore, it would be necessary to arrange so that the slider would have to come near the centre of its traverse, even though the sensitiveness of the whole arrangement became reduced in consequence. As long, however, as *sufficient* sensitiveness is obtained, that is to say, a sensitiveness such that a movement of the slider from its correct position to either of the contiguous coils produces a perceptible disturbance of the balance, then the nearer we can get the slider to the centre the better. It would not do, however, in any case to pass beyond the centre point; for in this case, although the error made in a by the slider being one coil out of adjustment is small, yet the error made in b becomes comparatively large. Now, in order that we may be able to get the slider near the centre of its traverse, it would be necessary that d should be approximately equal to r, but since, in order to obtain balance in the first instance, we must have

$$\mathrm{E} : e :: a + b + d + r : a + b,$$

or

$$\frac{\mathrm{E}}{e} = 1 + \frac{d + r}{a + b},$$

d could not be made equal to r unless

$$\frac{\mathrm{E}}{e} = 1 + \frac{2\,r}{a + b}, \quad \text{or,} \quad e = \frac{\mathrm{E}\,(a + b)}{a + b + 2\,r}.$$

Now if E and e were both fixed quantities and were not of such relative values that the above equation held good, then it would be impossible to obtain the conditions necessary for making the test favourably; the method of testing we are considering, however, would usually be employed for the purpose of measuring the electromotive force of a battery in terms of the electromotive force of one or more standard cells whose number could be varied to suit any particular requirement; in such a case it would usually be possible to give to e the value which would enable the above equation to be satisfied. Thus, for example, suppose the resistance, r, of the battery E were estimated to be about 100 ohms, and suppose the slide resistance $a + b$ consisted of 100 coils of 10 ohms each, that is, 1000 ohms in all, then we must have

$$e = \frac{\mathrm{E}\,1000}{1000 + (2 \times 100)} = \frac{\mathrm{E}\,10}{12},$$

that is to say, the electromotive forces of the batteries E and e should be in the proportion of 10 to 12. Now, it is evident that if E were a battery of one or two cells only, then it would practically be impossible to give to e the required value; but if E consisted of a considerable number of elements, 20 or 30 for example, then there would be no difficulty in adjusting e. From these considerations it must be evident that Fahie's method, although extremely ingenious and elegant, and in some special cases very convenient, is very limited in its application.

216. With respect to the *Possible degree of accuracy attainable*, this as regards the *resistance* test is directly dependent upon the accuracy with which we can adjust the ratio of a to b; thus if $a + b$ consisted of 100 coils, then if the ratio of E to e were such that the slider when adjusted stood near the centre position of its traverse, the error caused by the slider being 1 coil out of position would

be 1 in 50 in a, and 1 in 50 in b, consequently the total error would be 1 in 25, or 4 per cent. With n coils, in fact, the *Possible degree of accuracy attainable* would be 1 in $\frac{n}{4}$, or, $\frac{100 \times 4}{n}$ per cent.

To determine the degree of accuracy attainable in the electromotive force test, we must suppose that d is 1 unit, and b 1 coil, out of adjustment. If we call λ the error caused in $\frac{E}{e}$, then we must have

$$\frac{E}{e} + \lambda = 1 + \frac{d+1}{b - \frac{a+b}{n}}, \quad \text{or,} \quad \lambda = 1 + \frac{d+1}{b - \frac{a+b}{n}} - \frac{E}{e};$$

and since

$$\frac{E}{e} = 1 + \frac{d}{b},$$

we get

$$\lambda = 1 + \frac{d+1}{b - \frac{a+b}{n}} - 1 - \frac{d}{b} = \frac{b(n+d) + a\,d}{b\,[b\,(n-1) - a]}.$$

If λ' be the *percentage* of error, then we have

$$\lambda = \frac{\lambda'}{100} \text{ of } \frac{E}{e}, \quad \text{or,} \quad \lambda' = 100\,\lambda \frac{b}{b+r};$$

therefore

$$\lambda' = \frac{100\,[b\,(n+d) + a\,d]}{(b+r)\,[b\,(n-1) - a]}.$$

If the test is made under the best conditions, that is, if we have $a = b$, and $d = r$, approximately, then we get

$$\lambda = \frac{100\,[b\,(n+r) + b\,r]}{(b+r)\,[b\,(n-1) - b]} = \frac{100\,(n + 2\,r)}{(b+r)\,(n-2)};$$

or since n is large, we may say

$$\lambda' = \frac{100\,(n + 2\,r)}{n\,(b+r)}.$$

For example.

In determining the relative electromotive forces, E and e, of two batteries by Fahie's method, the resistance, r, of E being approximately 100 ohms, a slide resistance having 100 coils (n) of 10 ohms each was employed. What was the *greatest possible degree* of accuracy attainable?

$$\lambda' = \frac{100\,[100 + (2 \times 100)]}{100\,(500 + 100)} = \tfrac{1}{2} \text{ per cent.}$$

To sum up, then, we have

Best Conditions for making the Test.

217. Make

$$e = \frac{E\,(a+b)}{a + b + 2\,r},$$

approximately, r being the approximate resistance of the battery E.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{100\,[b\,(n + d) + a\,d]}{(b + r)\,[b\,(n - 1) - a]}.$$

If $a = b$, and $d = r$, and n is large, then

$$\text{Percentage of accuracy} = \frac{100\,(n + 2\,r)}{n\,(b + r)},$$

n in both cases being the number of coils of which the slide resistance is composed.

218. It may be as well to point out that Fahie's test cannot be made (except under very exceptional circumstances, rarely met with in practice) with a slide *wire;* for, as a rule, the latter has such an extremely low resistance that it would be impossible to obtain equilibrium in the first instance; the proportion which is necessary

$$E : e :: r + d + a + b : a + b,$$

for equilibrium, could not, in fact, be satisfied unless the resistance of the battery E and the resistance d were both extremely small; in which case, moreover, the latter would have to be adjustable to a very small fraction of an ohm.

Negreau's Method.

219. This method, devised by M. D. Negreau, is shown by Fig. 91. The arrangement consists in adjusting the resistances R and ρ (these resistances including the resistances of the batteries and of the galvanometer) until the galvanometer deflection remains unaltered when the key is raised or depressed.

Assuming first that the key is raised, then the current through the galvanometer is

$$\frac{E_1 + E_2}{R + \rho};$$

if, now, the key is depressed, then the current through the galvanometer is

$$\frac{E_2}{\rho};$$

and if the currents in the two cases are the same, then

$$\frac{E_1 + E_2}{R + \rho} = \frac{E_2}{\rho},$$

therefore

$$E_1 \rho + E_2 \rho = E_2 R + E_2 \rho,$$

or

$$E_1 \rho = E_2 R,$$

that is

$$E_1 : E_2 :: R : \rho. \qquad [A]$$

220. As regards the "Best conditions for making the test," this would be considered in the same way as the investigation in the case of Thomson's method of measuring galvanometer resistance (page 115).

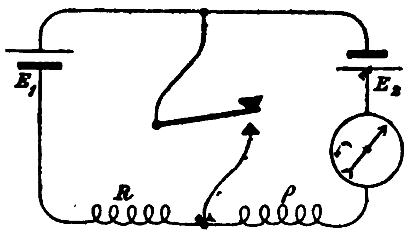


FIG. 91.

Let, then, C_1 and C_2 be the currents which are obtained, respectively, when key is raised and depressed and when balance is not obtained, and let C be the current when the balance is correct. Then if R_1 be the incorrect value of R, we have

$$E_1 + \lambda = \frac{E_2 R_1}{\rho}$$

or

$$E_2 R_1 = E_1 \rho + \rho \lambda,$$

and

$$\frac{C_2 - C_1}{C} = \frac{\frac{E_2}{\rho} - \frac{E_1 + E_2}{R_1 + \rho}}{\frac{E_2}{\rho}} = \frac{\frac{E_2 R_1 - E_1 \rho}{\rho (R_1 + \rho)}}{\frac{E_2}{\rho}}$$

$$= \frac{E_2 R_1 - E_1 \rho}{E_2 (R_1 + \rho)} = \frac{E_1 \rho + \rho \lambda - E_1 \rho}{E_2 (R_1 + \rho)} = \frac{\rho \lambda}{E_2 (R + \rho)}$$

(since R_1 and R are very nearly the same)

$$= \frac{\lambda}{E_2 \left(\frac{R}{\rho} + 1\right)} = \frac{\lambda}{E_2 \left(\frac{E_1}{E_2} + 1\right)} = \frac{\lambda}{E_1 + E_2}.$$

We see from this that E_1, say, being the electromotive force to be measured, then the smaller E_2 is made the better, though there is no advantage in making E_2 extremely small. The relative values of R and ρ would of course be governed by the relative values of E_1 and E_2, but their actual values are immaterial so long as the larger of the two is sufficiently great to be adjustable to the required degree of accuracy. The galvanometer deflection, of course, should be brought as near as possible to the angle of maximum sensitiveness (page 25).

If the resistance of the cells is unknown and cannot be neglected, then a second measurement should be made with, say, ρ increased by r and R increased by r', so that we get a second equation,

$$E_1 : E_2 :: R + r' : \rho + r,$$

so that, as in the case of Wheatstone's method (page 174), we have

$$E_1 : E_2 :: r' : r.$$

Clark's Method.

221. This is a valuable modification of Poggendorff's method, and is shown in theory by Fig. 92. $a\,b$, which takes the place of R in Poggendorff's method (page 185), is a slide resistance; E_3 is a third battery which is connected to a slider through a galvanometer G_3.

Now if we suppose equilibrium to be obtained in both galvanometers, we must have from [5], page 186,

$$E_1 : E_2 :: r_1 + \rho + a + b : a + b,$$

and also

$$E_1 : E_3 :: r_1 + \rho + a + b : a;$$

from which we get

$$E_2 : E_3 :: a + b : a.$$

If then we take $a + b$ to represent the electromotive force of the standard battery E_2, a will represent the electromotive force of the battery E_3.

In making this test practically, the battery E_3, which would be the trial battery, being disconnected from the slide resistance, balance would be obtained with the standard battery E_2 by adjusting ρ until no deflection is observed on the galvanometer G_1.

E_3 would then be connected up and the slider moved until no deflection is observed on the second galvanometer G_3.

The great advantage of Clark's method is that both the standard and the trial battery are compared under the same conditions, that is, when no current is flowing in either of them; this is a great point, as errors due to polarisation are avoided.

222. It must be evident that if equilibrium is not produced with the trial cell, then the balance in the standard cell circuit

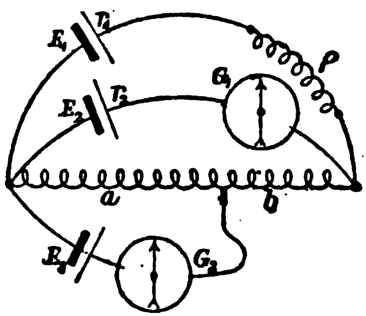


FIG. 92.

will also be disturbed; it would therefore seem to be possible to dispense with the galvanometer G_3, but inasmuch as the current which would flow through the galvanometer G_1 would only be a fraction of that flowing out of the battery E_3, we should not be able to make a measurement with nearly such a degree of accuracy as we could if we employed the galvanometer G_3, which would be acted upon by the full force of the current.

223. To determine the best arrangement of resistances, &c., for making the test, let us suppose that there is a small error, λ, in E_3, caused by a corresponding small error in a, and let us find what effect this error has upon the current which would flow through the galvanometer G_3. Supposing then that a_1 is the new value of a which causes this error, then, keeping in mind that $a + b$ being a slide resistance is not altered by changing a, we have

$$E_3 + \lambda = \frac{E_2 a_1}{a + b}, \qquad [1]$$

or

$$a_1 = \frac{(E_3 + \lambda)(a + b)}{E_2}. \qquad [2]$$

We next have to determine what the current flowing through the galvanometer, when equilibrium is disturbed, is equal to.

Referring to Fig. 98, in which m, n, a, b, and g represent the resistances, and c_1, c_2, c_3, κ_1, and κ_2, the current strengths in the various circuits, we have by Kirchoff's laws (page 178)

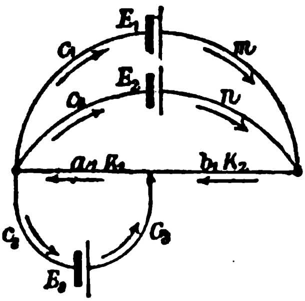


Fig. 98.

$$c_1 + c_2 + c_3 - \kappa_1 = 0$$
$$\kappa_1 - c_3 - \kappa_2 = 0$$
$$\kappa_2 - c_1 - c_2 = 0$$
$$c_1 m + \kappa_1 a_1 + \kappa_2 b_1 - E_1 = 0$$
$$c_2 n + \kappa_1 a_1 + \kappa_2 b_1 - E_2 = 0$$
$$c_3 G_3 + \kappa_1 a_1 - E_3 = 0.$$

We know also that

$$E_1 : E_2 :: m + a_1 + b_1 : a_1 + b_1,$$

and

$$a_1 + b_1 = a + b$$

By finding then the value of c_1 from the first equation and substituting its value throughout the others, and then again the value of c_2 from any other equation, and again substituting throughout, and so on, and also substituting the value of E_1 obtained from the proportion, and also the value of $a_1 + b_1$, we shall find that

$$c_3 = \frac{E_3 - E_2 \frac{a_1}{a+b}}{G_3 + \frac{a_1\left(b_1 + \frac{m\,n}{m+n}\right)}{a + b + \frac{m\,n}{m+n}}} = \frac{E_3 - E_2 \frac{a_1}{a+b}}{K}. \qquad [3]$$

If in this equation we substitute the value of a_1 given by equation [2] (page 201), we shall get

$$c_3 = \frac{\lambda}{G_3 + \frac{a_1\left(b_1 + \frac{m\,n}{m+n}\right)}{a + b + \frac{m\,n}{m+n}}};$$

or as a_1 and b_1 are very nearly equal to a and b, we may say

$$c_3 = \frac{\lambda}{G_3 + \frac{a\left(b + \frac{m\,n}{m+n}\right)}{a + b + \frac{m\,n}{m+n}}}. \qquad [4]$$

On examining this equation we see that to make c_3 as *large* as possible we must make $\frac{a\left(b + \frac{m\,n}{m+n}\right)}{a + b + \frac{m\,n}{m+n}}$ as *small* as possible, but we also see that it is no use making it much smaller than G_3, as c_3 is but very little increased by so doing.

Now the quantity $\dfrac{a\left(b+\dfrac{mn}{m+n}\right)}{a+b+\dfrac{mn}{m+n}}$ is the resistance a combined in multiple arc with the resistance b plus m and n combined in multiple arc, consequently this quantity can never be greater than a. As long therefore as a is smaller than G_3, the highest values that can be given to the other resistances cannot make c_3 less than $\dfrac{\lambda}{G_3+a}$, whilst, on the other hand, however low we make these resistances, we can never make c_3 greater than $\dfrac{\lambda}{G_3}$. The value therefore we give to a practically determines the sensitiveness of the system. But as a is only a portion of the slide resistance $a+b$, and as it may include the whole of the latter, as for instance when the slider is moved quite to the end of $a+b$, the sensitiveness is practically dependent upon the value given to $a+b$. This must then be made as much lower than G_3 as may be desirable.

It would not, however, do to have the resistance excessively low, for the following reason:—

In order to get equilibrium on the galvanometer G_2, it is necessary that the relation

$$E_1 : E_2 :: r_1+\rho+a+b : a+b,$$

or

$$a+b=\frac{r_1+\rho}{\dfrac{E_1}{E_2}-1},$$

should hold good. This cannot be the case, however, if $\dfrac{r_1+\rho}{\dfrac{E_1}{E_2}-1}$ is greater than $a+b$; that is to say, if $a+b$ is very small $\dfrac{r_1+\rho}{\dfrac{E_1}{E_2}-1}$ must be very small also; but to make the latter small we must make E_1 large and $r_1+\rho$ small, but since r_1, the resistance of E_1, will increase by increasing E_1, it may be impossible to do this. Practically we may say the resistance of $a+b$ should be a fractional value of G_3.

224. Let us now determine the *Possible degree of accuracy attainable* by the method. In equation [1] (page 201) we have supposed that an error λ has been caused in E_3 by a being out of adjustment; that is to say, from the slider being moved a little too far, so that a becomes a_1. If we call ϕ the distance the slider has been moved beyond its correct position, then we have

$$E_3+\lambda=\frac{E_2(a+\phi)}{(a+b)}=\frac{E_2\,a}{a+b}+\frac{E_2\,\phi}{a+b};$$

but

$$E_3=\frac{E_2\,a}{a+b},$$

therefore

$$\lambda=\phi\frac{E_2}{a+b},$$

that is to say, the distance the slider is out of position represents directly the error λ in E_3. The degree of accuracy therefore with which we adjust the position of the slider will be the degree of accuracy with which we can measure E_3.

We have pointed out that if $a + b$ is small, then

$$\frac{a\left(b + \frac{m\,n}{m+n}\right)}{a + b + \frac{m\,n}{m+n}}$$

will be smaller still; if, therefore, G_3 is large compared with $a + b$, equation [4] (page 202) becomes

$$c_3 = \frac{\lambda}{G_3}.$$

If in this equation we put the value of λ, given above, we have

$$c_3 = \frac{\phi\,E_2}{G_3\,(a+b)},$$

or

$$\frac{\phi}{a+b} = \frac{c_3\,G_3}{E_2}.$$

This equation enables us to determine what movement of the slider produces a perceptible deflection on the galvanometer. With a Thomson galvanometer of 5000 ohms resistance and figure of merit = ·000,000,001 (page 85) we should have, supposing E_2 to be 1 volt,

$$\frac{\phi}{a+b} = 5000 \times \cdot000{,}000{,}001 = \cdot000{,}005 = \frac{1}{200{,}000},$$

or a movement of the slider equal to $\frac{1}{200,000}$th of the length of $a + b$ would produce a perceptible deflection; that is to say, we could determine the accuracy of an electromotive force E_3 of about 1 volt to an accuracy of $\frac{1}{200,000}$th.

To obtain this accuracy, however, it would be necessary to have the wire $a + b$ graduated into 200,000 parts, each of which would be very small, unless indeed the wire were very long. If a lesser number of graduations were employed, we could practically subdivide each of them by noting what the galvanometer deflections were when the slider stood, first at one division mark, and then at the contiguous mark, as follows:—

Suppose the slider stood at a distance a from the end of the slide wire, and a deflection due to a current c_1 was produced to one side of zero; and suppose that when the slider was moved 1 division forward, that is to $a + 1$, the deflection was on the other side of zero, or was produced by a current $- c_2$. Then we have from equation [3] (page 202), since a and $a + 1$ are very nearly equal,

$$c_1 = \frac{E_3 - E_2\dfrac{a}{a+b}}{K}$$

and

$$-c_2 = \frac{E_3 - E_2\dfrac{a+1}{a+b}}{K} = \frac{E_3 - E_2\dfrac{a}{a+b} - \dfrac{E_2}{a+b}}{K};$$

therefore

$$c_1 E_3 - c_1 E_2 \frac{a}{a+b} - c_1 \frac{E_2}{a+b} = - c_2 E_3 + c_2 E_2 \frac{a}{a+b},$$

or

$$E_3 (c_1 + c_2) = E_2 \frac{a}{a+b}(c_1 + c_2) + \frac{E_2}{a+b} c_1;$$

therefore

$$E_2 : E_3 :: a + b : a + \frac{c_1}{c_1 + c_2}.$$

The subdivision of the division beyond a is therefore given by the fraction $\frac{c_1}{c_1 + c_2}$. We have seen that we could get a deflection of 1 division on the galvanometer if the slider were moved a distance of $\frac{1}{200000}$th beyond the distance required to give equilibrium. If the wire $a + b$ were divided into 20,000 parts, then a movement of the slider through 1 part or division would give 10 divisions of deflection on the galvanometer, each division representing a tenth of one of the wire graduations. If in making a measurement we got a deflection of 7 divisions (c_1) to the left when the slider stood at a distance a from the end of the wire, and a deflection of 3 divisions (c_2) to the right when the slider was moved 1 wire graduation beyond a, then the position of the slider for exact equilibrium would be

$$a + \frac{7}{7+3} = a + \cdot 7.$$

The galvanometer can thus be made to act as a *vernier;* and the greater the deflection produced by a movement of the slider through one division of the graduated wire, the greater will be the accuracy with which a test can be made.

The general results that we arrive at from the foregoing investigations are as follows:—

Best Conditions for making the Test.

225. Let the slide wire $a + b$ be a fractional value of the resistance of the galvanometer G_3, but not so low that it is less than $\dfrac{r_1 + \rho}{\dfrac{E_1}{E_2} - 1}$.

The values given to the other resistances and electromotive forces do not affect the sensitiveness of the arrangement.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{c_3 \, G_3 \, 100}{E_3}.$$

226. The late Mr. Latimer Clark employed a platinum-iridium wire of 40 ohms resistance, wound spirally on an ebonite cylinder, for the slide resistance. The edge of the cylinder being divided into 1000 equal parts, and there being twenty turns to the cylinder, the whole wire is divided into 20,000 equal parts. By employing with this instrument (which combined with the batteries and resistances is called a "Potentiometer") a galvanometer with a high figure of merit (page 85), and a standard battery E_2 of one Daniell cell, a 1 division movement of the slider, after equilibrium has been produced, will produce a deflection of 50 divisions. It is possible, therefore, with the apparatus to measure an electromotive force of one Daniell cell to an accuracy of

$$\frac{1}{20{,}000 \times 50} = \frac{1}{1{,}000{,}000}\text{th.}$$

POTENTIOMETER DIRECT METHOD.

227. This method, which is an exceedingly simple and satisfactory one, is shown by Fig. 94. $a\ b$, is a slide resistance (page 17), E is a battery of constant electromotive force connected to the end of $a\ b$.

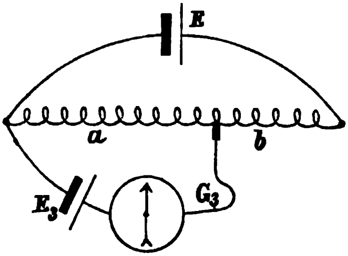


FIG. 94.

A standard cell E_3, is first connected up through a galvanometer G_3, as shown, and the slider is moved until the galvanometer needle comes to zero. E_3 is now removed, and the battery E_2, whose force is required, is connected up in its place; the slider is then moved until equilibrium is again produced on G_3. If then a_1 be the new value of a corresponding to the changed position of the slider, we have

$$E_2 : E_3 :: a_1 : a,$$

the truth of which is obvious.

DIRECT READING POTENTIOMETER. See Appendix.

CHAPTER VIII

THE WHEATSTONE BRIDGE.

228. The theoretical arrangement of the Wheatstone Bridge, or Balance, is shown by Fig. 95. It consists of four resistances a, b, d, and x, arranged in the form of a parallelogram, a battery occupying the place of one, and a galvanometer the place of the other, diameter. When the four resistances are so adjusted

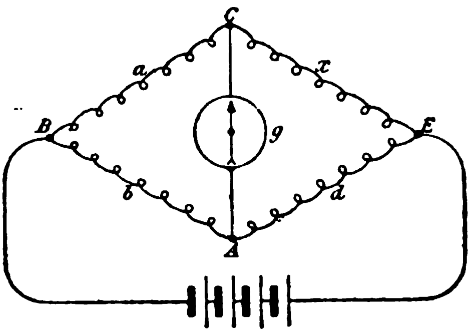


Fig. 95.

that equilibrium is produced, that is to say, when no current passes through the galvanometer, then these resistances bear a certain relation to one another. This relation may be thus determined:—

When equilibrium is produced, then since there is no tendency for a current to flow between the points A and C, the galvanometer may be removed without altering the strength of current in the other parts of the bridge; and, further, we may join the points A and C without affecting the strengths. Let us first suppose the points A and C to be separated; then the joint resistance given by the four resistances between the points B and E will be

$$\frac{(a+x)(b+d)}{a+x+b+d}.$$

If, now, we join A and C, the resistance may be written

$$\frac{a\,b}{a+b} + \frac{d\,x}{d+x},$$

which must be equal to the former expression, that is,

$$\frac{(a+x)(b+d)}{a+x+b+d} = \frac{a\,b}{a+b} + \frac{d\,x}{d+x}.$$

By multiplying up and simplifying we get

$$a^2\,d^2 + b^2\,x^2 - 2\,a\,b\,d\,x = 0;$$

therefore

$$(a\,d - b\,x)^2 = 0,$$

or

$$a\,d - b\,x = 0,$$

that is

$$a\,d = b\,x,$$

from which

$$\frac{a}{b} = \frac{x}{d}.^*$$

If, now, three of the quantities in this equation are known, the fourth can be determined; thus:—

$$x = \frac{a\,d}{b}.$$

229. In the most general form of bridge, two of the resistances are fixed, and a third is adjustable, the fourth being the resistance whose value is to be determined.

As a rule, a and b are the fixed resistances, x the resistance whose value it is required to find, and d the adjustable resistance.

In the simplest method of measuring we should make a and b of equal value, in which case

$$x = d;$$

that is to say, the resistance between A and E when equilibrium is produced, gives the value of the unknown resistance.

It is absolutely necessary that there be some resistance in a and b, for otherwise the galvanometer is short-circuited, and equilibrium will apparently be always produced, no matter what resistances we have in the other two branches.

* See also Appendix for a geometrical proof of the principle.

230. Besides using equal resistances in a and b, we can make one of the two to be 10 or 100 times as great as the other; or, in fact, any multiple of it we like, but multiples of 10 are those most commonly used. If, when we are measuring a resistance x, we make b 10 times as large as a, then every unit of resistance in d represents $\frac{1}{10}$th of a unit in x, for in this case

$$x = \frac{d}{10}.$$

We can, therefore, by this device determine the value of a resistance to an accuracy of $\frac{1}{10}$th of a unit, although d is adjustable only to units. In like manner, if we make b 100 times as large as a, then every unit of resistance in d represents $\frac{1}{100}$th of a unit in x; for in this case

$$x = \frac{d}{100},$$

and we can thus determine the value of a resistance to an accuracy of $\frac{1}{100}$th of a unit. In the first instance, however, the value of d when adjusted would be 10 times that of x; we could not, therefore, in that case measure a resistance whose value was greater than $\frac{1}{10}$th of the total resistance we could insert in d; and in the second instance d would be 100 times as great as x; we could not, therefore, in that case measure a resistance greater than $\frac{1}{100}$th of the total resistance in d. In fact, the larger we make d the closer will be the degree of accuracy with which a measurement can be made, but, at the same time, the smaller will be the resistance which can be measured, unless extra resistance coils are added in between A and E.

There is, however, a limit to the degree of accuracy with which a resistance can be thus measured, which is dependent upon the figure of merit (page 85) of the galvanometer; of this we shall speak hereafter.

If, now, we wish to measure a resistance which is greater than the total resistance we can insert in d, we must make a larger than b. If a be made 10 times as great as b, we can then measure any resistance which is not greater than 10 times the resistance we can insert in d, but as in this case 1 unit in d represents 10 units in x, we can only be certain of the value of x within 10 units. Similarly, if we make a 100 times as great as b, we can measure any resistance not greater than 100 times d, but we can only determine its value within 100 units.

231. The practical method of joining up one form of the bridge (Figs. 6 and 7, pages 14 and 15) is shown by Fig. 96. When the connections are made, and the proper plugs removed from A B (b) and B C (a), the *right*-hand key must be pressed down to put on the battery current. Plugs are now removed from E A (d) until we have inserted a resistance, as near as we can guess, equal to

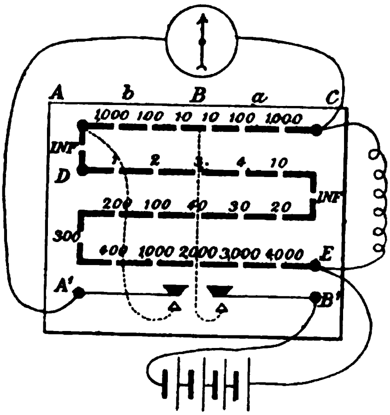


Fig. 96.

the resistance we are going to measure. The left-hand (galvanometer) key is next pressed down, and plugs removed or shifted from E A (d) until no movement of the galvanometer needle is produced upon raising and depressing the key. The connections in the case of the set of coils shown by Figs. 4 and 5 (page 13) would be similar to the foregoing, but separate keys, in circuit with the battery and galvanometer respectively, would have to be employed.

232. If the galvanometer used has a high figure of merit, and has a fine fibre suspension, the key must not, at first, be pressed firmly down, but only snapped down sharply; for otherwise, if equilibrium is not very nearly produced when the key is depressed, there is a danger of breaking the fibre of the galvanometer needle by the violent deflection. When, however, after repeated trials, we have very nearly obtained equilibrium, then the key may be firmly held down, and the final adjustment of plugs made.

233. Fig. 97 shows a plan of the internal connections of the set of resistance coils which were shown in general view by Fig. 8, page 15. The method of joining up these coils to form a bridge

would be as follows: The resistance to be measured is connected between C and E, the "Infinity" plug between the two being removed; the galvanometer is joined between A and C; the battery is connected between B and E. The "Infinity" plug between A and D is inserted firmly in its place. Besides the connections referred to, it is necessary to have a key in circuit with the galvanometer, and another in circuit with the battery.

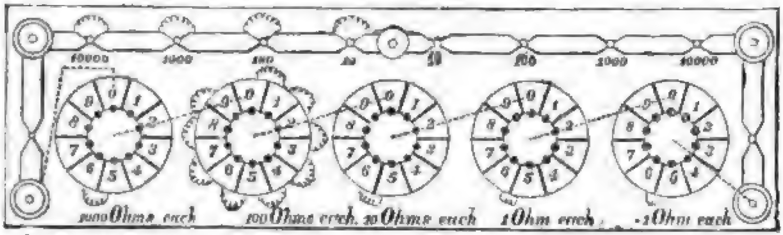


FIG. 97.

In this form of bridge, when balance is obtained, we have

$$\text{Resistance to be measured} = R\,\frac{a}{b}.$$

We may, if we please, insert the resistance to be measured between terminals A and D instead of between C and E, a plug being inserted between the latter, and the plug between A and D being removed; in this case, when balance is obtained, we should have

$$\text{Resistance to be measured} = R\,\frac{b}{a}.$$

An advantage of the foregoing set of coils lies in the fact that there are only five plugs to be shifted, for the insertion of these plugs brings the resistances into circuit, instead of short-circuiting them as in the ordinary coils. The reading, also, of the total value of the resistance in circuit is a very easy matter, as must be obvious.

In addition to the connections shown in Fig. 97 the "9" block of each dial is connected to the centre block by a resistance equal to one of those forming that particular dial. By this arrangement the withdrawal of a plug instead of causing a disconnection merely

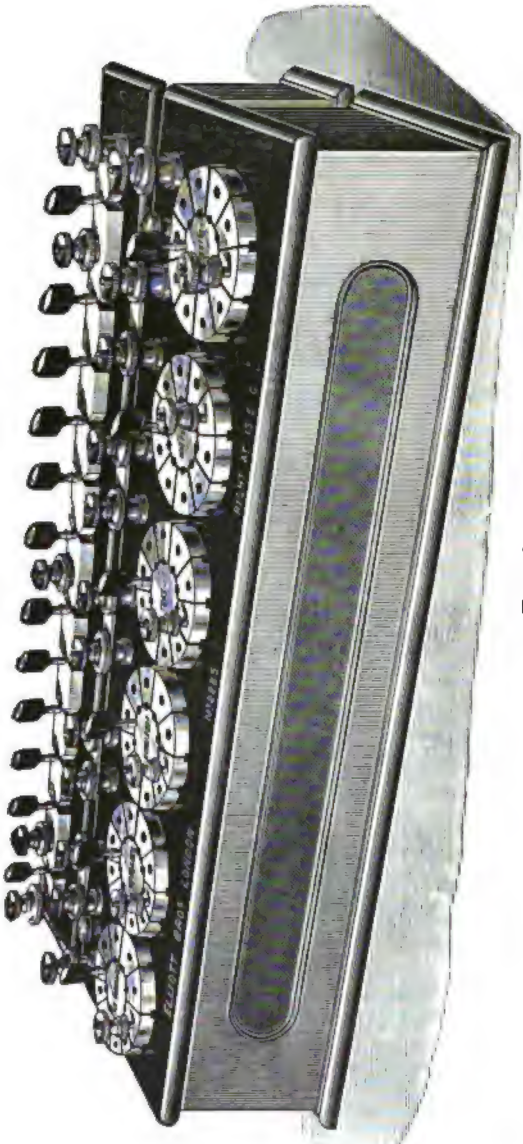


Fig. 98.

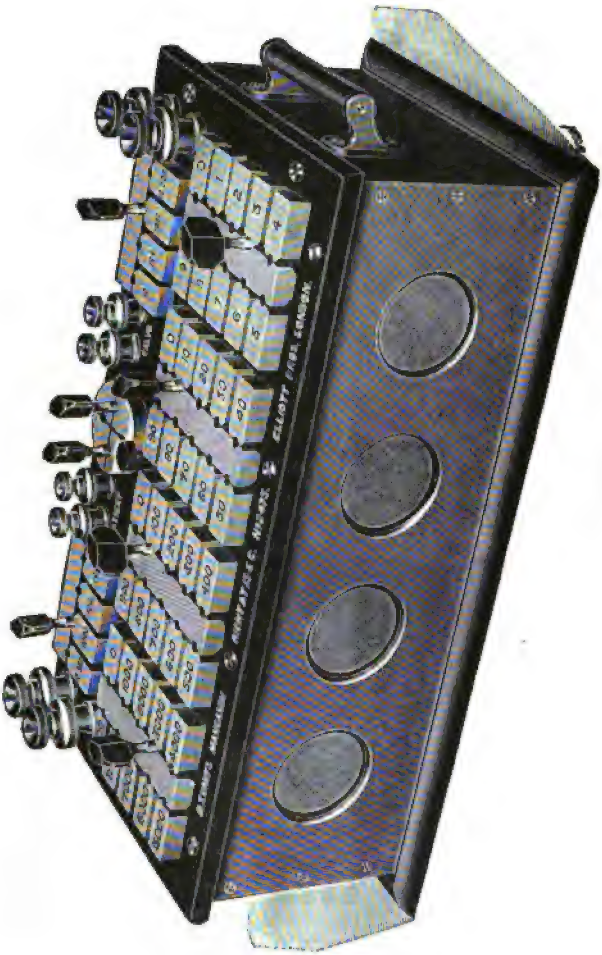


Fig. 99.

introduces a resistance equal to *one* of the coils of the next higher dial, and forms a good test of their accuracy.

The "Dial" pattern shown by Fig. 10, § 18, page 16, is in many cases a very satisfactory instrument for use as a Wheatstone Bridge.

234. Modified forms of the bridge are shown by Figs. 98 and 99.

Conditions for Accurate Measurements.

235. Besides the method of joining up, as shown by Fig. 96, we may also join up by placing the battery between A′ and C, and the galvanometer between B′ and E; this, under certain conditions, renders the action of the galvanometer more sensitive than by the common arrangement. What these conditions are, and what should be the general arrangement of the resistances in the bridge in order that a test may be made under the best possible conditions for ensuring accuracy, we will now proceed to consider.

To investigate these questions it is first of all necessary to find what relation the current which flows through the galvanometer when equilibrium is not produced, bears to the different resistances which make up the bridge.

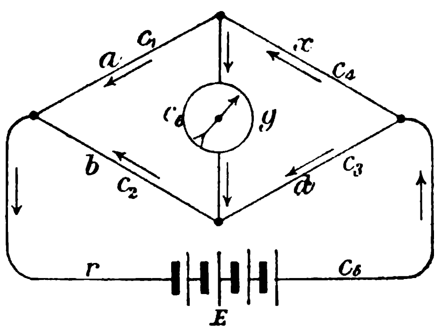


Fig. 100.

In Fig. 100 let a, b, d_1, x, r, and g be the resistances of the different parts of the bridge, also let c_1, c_2, c_3, c_4, c_5, and c_6 be the current strengths in the same, and let E be the electromotive force of the battery.

Applying Kirchoff's laws (page 178), we get the following six

equations as representing the connection between the resistances, current strengths, and the electromotive force:—

$$c_5 - c_1 - c_2 = 0. \qquad [1]$$
$$c_4 - c_6 - c_1 = 0. \qquad [2]$$
$$c_3 + c_6 - c_2 = 0. \qquad [3]$$
$$c_5 r + c_3 d_1 + c_2 b - E = 0. \qquad [4]$$
$$c_1 a - c_2 b - c_6 g = 0. \qquad [5]$$
$$c_3 d_1 - c_4 x - c_6 g = 0. \qquad [6]$$

From these equations we have to find the value of c_6, the current flowing through the galvanometer.

By finding the value of c_1 from equation [1] and substituting its value in equations [2] and [5] we get rid of c_1; and in like manner, by finding the value of c_2 from equation [3] and substituting throughout, we get rid of c_2. By adopting the same process with respect to c_3 and c_4 we shall finally get equations [5] and [6] to become

$$c_5 a - c_6 a - c_6 b - c_6 g - (a + b)\frac{E - c_6 b - c_5 r}{b + d_1} = 0. \qquad [5]$$

$$c_5 x + c_6 g - (d_1 + x)\frac{E - c_6 b - c_5 r}{b + d_1} = 0. \qquad [6]$$

From these two equations we get

$$\frac{c_5\{r(d_1+x)+x(b+d_1)\}}{c_5\{r(a+b)+a(b+d_1)\}} = \frac{-c_6(bg+d_1g+bx+bd_1)+E(d_1+x)}{c_6(ad_1+bd_1+bg+d_1g)+E(a+b)}. \qquad [7]$$

from which

$$c_6 = \frac{E\,(ad_1 - bx)}{g\,\{\,(a+x)(b+d_1)+r(a+b+d_1+x)\,\}\,+r(d_1+x)(a+b)+bd_1(a+x)+ax(b+d_1)} = \frac{A}{B_1}. \qquad [8]$$

This equation gives the strength of the current which would flow through the galvanometer if the resistances were arranged as shown by Fig. 100.

236. Suppose now the battery occupied the place taken by the galvanometer and *vice versâ*, or, in other words, suppose the galvanometer connected the junctions of a with b, and d_1 with x, and the battery connected the junctions of a with x, and b with d_1, then the current (c_7) flowing through the galvanometer would be

$$c_7 = \frac{E\,(ad_1 - bx)}{g\,\{\,(d_1+x)(a+b)+r(a+b+d_1+x)\,\}\,+r(b+d_1)(a+x)+ab(d_1+x)+d_1x(a+b)} = \frac{A}{B_2}. \qquad [9]$$

If we subtract equation [9] from equation [8] we get

$$c_6 - c_7 = \frac{A}{B_1} - \frac{A}{B_2} = \frac{A}{B_1 B_2}(B_2 - B_1),$$

and if in $(B_2 - B_1)$ we substitute the values of B_1 and B_2 given in equations [8] and [9], respectively, and then multiply up, cancel, &c., we finally get

$$c_6 - c_7 = \frac{A}{B_1 B_2}(g - r)(a - d)(x - b).$$

In this equation, if g is larger than r, and both a and x are respectively larger or smaller than d_1 and b; or if r is greater than g and at the same time both a and b are greater than d_1 and x, then $c_6 - c_7$ will be a positive quantity, that is, c_6 will be greater than c_7.

But c_6 is the current obtained by the arrangement of the bridge indicated by Fig. 100; and on examination it will be found that when the resistances have the relative magnitudes indicated, the greater of the two resistances g and r connects the junction of the two greater with the junction of the two lesser resistances: consequently, as this arrangement gives the greatest current through the galvanometer when equilibrium is not produced, it must be the best one to employ.

In practice it is almost always the case that the galvanometer has a higher resistance than the testing battery.

237. We have next to consider what should be the relative values of a, b, d, and x, in order that the bridge test may be made under the best possible conditions.

There are several different considerations involved in these questions, but we will investigate the problem from a general point of view first.

Equation [8] (page 215) shows the relation between the current and the resistances. In this equation, as equilibrium is very nearly produced, we may, except where differences are concerned, put

$$a\,d_1 = a\,d = b\,x, \quad \text{or,} \quad b = \frac{a\,d}{x},$$

d being the adjusted resistance when equilibrium is exactly produced.

We then get

$$c_6 = \frac{E\,x\,(a\,d_1 - b\,x)}{\{g\,(a + x) + a\,(d + x)\}\ \{r\,(d + x) + d\,(a + x)\}}. \quad [1]$$

Let us now suppose that the error in d_1, which causes the current c_6, produces an error λ in x, or that

$$x + \lambda = \frac{a\,d_1}{b}, \quad \text{or,} \quad a\,d_1 = b\,x + b\,\lambda,$$

and as

$$b = \frac{a\,d}{x},$$

therefore

$$c_6 = \frac{\mathrm{E}\,a\,d\,\lambda}{\{g(a+x) + a(d+x)\}\ \{r(d+x) + d(a+x)\}}$$

$$= \frac{\mathrm{E}\,\lambda}{\left\{d + g + x + \frac{g\,x}{a}\right\}\ \left\{r + a + x + \frac{r\,x}{d}\right\}} \qquad [2]$$

or

$$\lambda = \frac{c_6}{\mathrm{E}}\left\{d + g + x + \frac{g\,x}{a}\right\}\ \left\{r + a + x + \frac{r\,x}{d}\right\}. \qquad [3]$$

Now our object is to make λ as small as possible, and this we shall do by making the error in d as small as possible. But the accuracy with which we can adjust d is limited by the degree of closeness with which the movement of the galvanometer needle from zero can be observed. In other words, if c_6 is the smallest current which will produce a perceptible deflection on the galvanometer, that is to say, if c_6 is its "figure of merit" (page 85), then the value of λ which corresponds to c_6 will be the amount of the error which we are likely to make in x.

If we write equation [3] in the form

$$\lambda = \frac{c_6\,r\,x\left\{d + \left(g + x + \frac{g\,x}{a}\right)\right\}\left\{\frac{1}{d} + \frac{1}{\left(\frac{r\,x}{r + x + a}\right)}\right\}}{\mathrm{E}} \qquad [4]$$

we can see that λ is smallest when the numerator of the fraction is smallest, and we must determine the values of d and a, which make this numerator as small as possible.

In order to do this let us simplify the above equation by putting

$$\left(g + x + \frac{g\,x}{a}\right) = \mathrm{X}, \text{ and, } \left(\frac{r\,x}{r + x + a}\right) = \mathrm{Y};$$

we then get

$$\lambda = \frac{c_6\, r\, x \left\{d + \mathrm{X}\right\} \left\{\frac{1}{d} + \frac{1}{\mathrm{Y}}\right\}}{\mathrm{E}},$$

or

$$\lambda = \frac{c_6\, r\, x \left\{\frac{\mathrm{Y} + \mathrm{X} + \left(d + \frac{\mathrm{X\,Y}}{d}\right)}{\mathrm{Y}}\right\}}{\mathrm{E}}. \qquad [\mathrm{A}]$$

From this equation we can see that to make λ as small as possible we must make

$$d + \frac{\mathrm{X\,Y}}{d}$$

as small as possible.

Now

$$d + \frac{\mathrm{X\,Y}}{d} = 2\sqrt{\mathrm{X\,Y}} + \left(\sqrt{d} - \frac{\sqrt{\mathrm{X\,Y}}}{\sqrt{d}}\right)^2,$$

and in order to make the right-hand side of the equation as small as possible we must make

$$\sqrt{d} - \frac{\sqrt{\mathrm{X\,Y}}}{\sqrt{d}}$$

as small as possible; that is to say, we must have

$$\sqrt{d} - \frac{\sqrt{\mathrm{X\,Y}}}{\sqrt{d}} = 0, \quad \text{or,} \quad \sqrt{d} = \frac{\sqrt{\mathrm{X\,Y}}}{\sqrt{d}},$$

from which we get

$$d = \sqrt{\mathrm{X\,Y}};$$

that is to say, we must make d equal to the *geometric mean* of the quantities $\left(g + x + \frac{g\,x}{a}\right)$ and $\left(\frac{r\,x}{r + x + a}\right)$.

Now, although the value "$d = \sqrt{\mathrm{X\,Y}}$" is one which gives a minimum value to λ, yet it is not the value which makes λ an *absolute* minimum, for X and Y both contain the variable quantity a. In order, therefore, to make λ an absolute minimum, we must determine what value a should have.

If in equation [A] we put $d = \sqrt{XY}$, we get

$$\lambda = \frac{c_6\, r\, x \left\{ \dfrac{Y + X + \left(\sqrt{XY} + \dfrac{XY}{\sqrt{XY}}\right)}{Y} \right\}}{E}$$

$$= \frac{c_6\, r\, x \left\{ \dfrac{Y + X + 2\sqrt{XY}}{Y} \right\}}{E} = \frac{c_6\, r\, x \left\{ 1 + \sqrt{\dfrac{X}{Y}} \right\}^2}{E}. \quad [B]$$

In order to make λ an absolute minimum, we can see that we must make $\dfrac{X}{Y}$ a minimum. Now

$$\frac{X}{Y} = \frac{\left(g + x + \dfrac{g\,x}{a}\right)}{\left(\dfrac{r\,x}{r + x + a}\right)} = \left(g + x + \frac{g\,x}{a}\right)\left(\frac{r + x + a}{r\,x}\right)$$

$$= \frac{g}{r}\left\{a + (r + x)\right\}\left\{\frac{1}{a} + \frac{1}{\left(\dfrac{g\,x}{g + x}\right)}\right\},$$

consequently we can see from the reasoning in the previous investigation, that to make $\dfrac{X}{Y}$ a minimum we must make

$$a = \sqrt{(r + x)\frac{g\,x}{g + x}}, \quad [C]$$

Having now obtained the required value of a in terms of the known quantities, r, g, and x, we can also determine the value of d in terms of r, g, and x; for we have

$$d = \sqrt{XY} = \sqrt{\left(g + x + \frac{g\,x}{a}\right)\left(\frac{r\,x}{r + x + a}\right)}$$

$$= \sqrt{\left\{g + x + \frac{g\,x}{\sqrt{\dfrac{(r + x)\,g\,x}{g + x}}}\right\}\left\{\frac{r\,x}{r + x + \sqrt{\dfrac{(r + x)\,g\,x}{g + x}}}\right\}}$$

$$= \sqrt{\left\{\frac{(g+x)\sqrt{r+x} + \sqrt{(g+x)gx}}{\sqrt{r + x}}\right\}\left\{\frac{r\,x\sqrt{g + x}}{(r+x)\sqrt{g+x} + \sqrt{(r+x)gx}}\right\}}$$

$$= \sqrt{\frac{g+x}{r+x}\left\{\sqrt{g+x}\sqrt{r+x} + \sqrt{g\,x}\right\}\left\{\frac{r\,x}{\sqrt{r+x}\sqrt{g+x} + \sqrt{g\,x}}\right\}}$$

or

$$d = \sqrt{(g + x)\frac{r\,x}{r + x}}. \qquad [D]$$

If we multiply equations [C] and [D] together, $(r + x)$ and $(g + x)$ cancel out, and we get

$$a\,d = \sqrt{g\,r\,x^2}, \quad \text{or,} \quad \frac{a\,d}{x} = \sqrt{g\,r}$$

that is

$$b = \sqrt{g\,r}.$$

238. Although equations [C] and [D] show the values of a and d which are necessary for making the error λ an *absolute* minimum, yet practically we may make both a and d to vary considerably from these exact values without increasing λ to any great extent.

As regards d it is preferable to make it as high as possible, so that its range of adjustment may be as high as possible. Referring to equation [4] (page 217), we have proved that for an absolute minimum we must make d equal to the geometric mean of the quantities $\left(g + x + \frac{g\,x}{a}\right)$ and $\left(\frac{r\,x}{r + x + a}\right)$, consequently we can see that in this case d must be less than $\left(g + x + \frac{g\,x}{a}\right)$. If we suppose the value of d for a minimum to be very small compared with $\left(g + x + \frac{g\,x}{a}\right)$, then we can see that even if we increase d up to an equality with $\left(g + x + \frac{g\,x}{a}\right)$, we cannot increase λ beyond twice its minimum value, especially if we consider that by increasing d we diminish the value of $\left\{\frac{1}{d} + \frac{1}{\left(\frac{r\,x}{r + x + a}\right)}\right\}$. If we only increase d up to $g + x$, then λ will of course be increased still less. Should the value of d for a minimum happen to be only a little less than $\left(g + x + \frac{g\,x}{a}\right)$, then of course the increase of d referred to will have but little effect on λ. In any case, however, by keeping d below $g + x$, the increase in λ must be less, and may be considerably less, than $2\,\lambda$. The importance of this fact may be seen if we suppose g, x, and r to have the following values:—

$$g = 4899,\ x = 1,\ r = 100;$$

then for the minimum we must take

$$d = \sqrt{(4899 + 1)\frac{100 \times 1}{100 + 1}} = 70;$$

but we have proved that if we may make "$d = (4899 + 1) = 5000$," then by so doing we cannot possibly increase λ to more than 2λ, and actually the increase must be to less than 2λ.

We next have to consider to what extent we may vary a. To do this let us write equation [3] (page 217) in the form:—

$$\lambda = \frac{c_s\, g\, x \left\{a + \left(r + x + \frac{r\,x}{d}\right)\right\}\left\{\frac{1}{a} + \frac{1}{\left(\frac{g\,x}{g + x + d}\right)}\right\}}{\text{E}}.$$

From this equation we can see that if d has the value necessary to make λ a minimum, then as long as we do not make a less than $\frac{g\,x}{g + x}$ we cannot possibly increase λ to more than 2λ. But then the question arises—Suppose we have already increased λ by making d as great as $g + x$, under these conditions what will be the effect of also decreasing a to $\frac{g\,x}{g + x}$?

If we refer to the last equation, we can see from the investigation made in the case of equation [4] (page 217), that the value of a which makes λ a minimum must be

$$a = \sqrt{\left(r + x \frac{r\,x}{d}\right)\left(\frac{g\,x}{g + x + d}\right)},$$

and this value is one which makes λ a minimum whatever be the value of d, though to make λ an *absolute* minimum we must also have

$$d = \sqrt{(g + x)\frac{r\,x}{r + x}}.$$

Now, if we increase d, we can see that to make λ a minimum we shall have to decrease the value of a, for by increasing d we decrease both $\left(r + x + \frac{r\,x}{d}\right)$ and $\left(\frac{g\,x}{g + x + d}\right)$; consequently a decrease in a after d has been increased will tend to decrease again the increased value of λ. We cannot, however, bring back λ to its original absolute minimum, although we may bring it near to it; for after a certain point the decrease in the value of a causes

λ to increase again; as long, however, as we avoid making a less than $\frac{g\,x}{g+x}$ this increase cannot be great.

As the value which d has must depend upon the value given to b, therefore after we have determined what values to give to a and d, we must ascertain the value of b from the equation "$b = \frac{a\,d}{x}$."

For example.

It being required to measure exactly a resistance x, whose value was found by a rough test to be about 500 ohms, a ten-cell Daniell battery ($E = 10 \cdot 7$), whose resistance was 200 ohms (r), was used for the purpose, and also a galvanometer whose resistance was 5000 ohms (g) and figure of merit $\cdot 000{,}000{,}001$ (c_6). What resistances should be given to the arms a and b of the bridge in order that the test may be made under the most favourable conditions, also what percentage of accuracy would be obtainable under these conditions?

$$x = 500$$
$$g = 5000$$
$$r = 200;$$

therefore

$$a = \sqrt{(200 + 500)\frac{5000 \times 500}{5000 + 500}} = 560 \text{ ohms},$$

$$d = \sqrt{(5000 + 500)\,\frac{200 \times 500}{200 + 500}} = 890 \text{ ohms};$$

also we must have

$$b = \frac{560 \times 890}{500} = 1000 \text{ ohms}.$$

In practice we could make d as high as 5500 ohms $(g + x)$, and a as low as 450 ohms $\left(\frac{g\,x}{g+x}\right)$, without seriously increasing λ.

Supposing, however, we actually gave a and d their best values, then by equation [3] (page 217) we should have

$$\lambda = \frac{\cdot 000{,}000{,}001}{10 \cdot 7}\left\{890 + 5000 + 500 + \frac{5000 \times 500}{560}\right\}$$

$$\left\{200 + 560 + 500 + \frac{500 \times 200}{890}\right\} = \cdot 0014$$

that is to say, we may be $\cdot 0014$ units out when we measure x exactly; this is equivalent to an error of $\frac{\cdot 0014 \times 100}{500} = \cdot 0003$ per cent. approximately.

In order to make the test as accurately as this, it would be necessary that d be adjustable to a small fraction of a unit; if we call ϕ the value of the latter, then we should have

$$x + \lambda = \frac{a(d + \phi)}{b} = \frac{a\,d}{b} + \frac{a\,\phi}{b}$$

and

$$x = \frac{a\,d}{b};$$

therefore

$$\lambda = \frac{a\,\phi}{b}, \text{ or, } \phi = \frac{b\,\lambda}{a}.$$

We therefore have

$$\phi = \frac{1000 \times \cdot 0014}{500} = \cdot 003,$$

showing that d ought to be adjustable to $\cdot 003$ of an ohm or less. If we make it adjustable to $\cdot 001$ or $\frac{1}{1000}$th of an ohm therefore, we shall be able to make the test properly.

239. The facts we have arrived at by the foregoing investigation are these, that with $a = 560$ ohms and $b = 1000$ ohms, then when equilibrium is exactly produced, an alteration in the value of d equal to $\cdot 003$ of an ohm (which quantity would mean an error λ, of $\cdot 0014$ units, or $\cdot 0003$ per cent. approximately, in x) would produce a perceptible deflection (1 division) on the galvanometer.

We have, then,

Best Conditions for making the Test.

240. First make a rough test to ascertain approximately the value of x.

Make d not greater than $g + x$, or less than $\sqrt{(g + x)\frac{r\,x}{r + x}}$, and preferably make it as near to the latter quantity as possible, provided the range of adjustment of d is not reduced to too great an extent by so doing.

Make a not less than $\frac{g\,x}{g+x}$ and not greater than $\sqrt{(r+x)\frac{g\,x}{g+x}}$ and preferably make it as near to the latter quantity as possible in the case where d is made nearly equal to $\sqrt{(g+x)\frac{r\,x}{r+x}}$; but if d is made more nearly equal to $g+x$, then a should preferably be made more nearly equal to $\frac{g\,x}{g+x}$.

It is clearly advantageous that E should be as large and r as small as possible.

Possible Degree of Accuracy attainable.

Percentage of accuracy $= \frac{\lambda\,100}{x}$, where

$$\lambda = \frac{c_6}{\mathrm{E}}\left\{d+g+x+\frac{g\,x}{a}\right\}\left\{r+a+x+\frac{r\,x}{d}\right\},$$

c_6 being the figure of merit of the galvanometer.

In order to obtain this percentage of accuracy, d must be adjustable to not less than $\frac{d\,\lambda}{x}$ units, or $\frac{1}{\left(\frac{x}{d\,\lambda}\right)}$th of a unit.

241. In the foregoing investigation we have considered the exact conditions required for a maximum degree of accuracy, and we have seen that in order to attain this it is necessary that d be adjustable to a fraction of a unit. At the commencement of the chapter (§ 231, page 210), however, we saw that if d is only adjustable to units, then in order to obtain the greatest possible accuracy we should make d as much larger than x as possible, as by so doing we get a great range of adjustment. But, as we also stated, there is a limit to thus increasing d, for unless we are able to adjust d accurately, we can gain nothing by having the range of adjustment so large. Now to adjust d we note the deflection of the galvanometer needle, and when this becomes 0 we know that d is adjusted exactly right; but if an alteration of several units produces no perceptible effect on the deflection we may just as well have d of a smaller value. Thus, supposing we have b 10 times as great as a, that is d 10 times x; then if an alteration of 10 units in d only just affects the galvanometer needle, it is evident that we

cannot adjust d to a closer accuracy than 10 units, and consequently we cannot obtain the value of x to a closer accuracy than 1 unit. If we have b equal to a, that is, d equal to x, then if we can adjust d within 1 unit, we shall in this case obtain the value of x to an accuracy of 1 unit, that is, with just as much accuracy as we could in the first case, when d was 10 times x. It is even possible that we could obtain the value of x more accurately in the latter case, for it may be that an alteration of 1 unit in d when b equals a may produce a much greater movement of the galvanometer needle than does the alteration of 10 ohms when b is 10 times a. Whether this is so or not is a point we have to determine.

We have also to find what should be the absolute values of a and b.

We have seen that in order to obtain accuracy it is necessary to make d as high as possible, but the highest useful value we could give to d would be *that which produces the smallest perceptible deflection when it is one unit out of adjustment.*

Now if λ be the error in x caused by d being 1 unit of adjustment, we must have

$$x + \lambda = \frac{a\,(d+1)}{b} = \frac{a\,d}{b} + \frac{a}{b}, \qquad [A]$$

and since

$$x = \frac{a\,d}{b}, \text{ or, } \frac{a}{b} = \frac{x}{d},$$

therefore

$$\frac{a\,d}{b} + \lambda = \frac{a\,d}{b} + \frac{x}{d},$$

or

$$\lambda = \frac{x}{d}.$$

We have, then, from equation [2] (page 217)

$$c_6 = \frac{\mathrm{E}\,x}{d\left\{d+g+x+\frac{g\,x}{d}\right\}\left\{r+a+x+\frac{r\,x}{d}\right\}}. \qquad [B]$$

From this equation we have to determine the highest value we can give to d; this will be limited by the "figure of merit" (page 85) of the galvanometer, and also by the value of a. Let us write the above equation in the form

$$d\left\{a+r+x+\frac{r\,x}{d}\right\}\left\{\frac{1}{a}+\frac{1}{\frac{g\,x}{g+x+d}}\right\} = \frac{\mathrm{E}}{g\,c_6}.$$

Now since $\frac{E}{g\,c_8}$ is a fixed quantity, therefore in order that d may have as large a value as possible we must give a such a value that

$$\left\{a + r + x + \frac{r\,x}{d}\right\}\left\{\frac{1}{a} + \frac{1}{\frac{g\,x}{g + x - d}}\right\}$$

is as small as possible. From the investigation given in § 237 (page 216) we can see that if we make a as low as possible, but not lower than, say, $\frac{g\,x}{g + x}$, then

$$\left\{a + r + x + \frac{r\,x}{d}\right\}\left\{\frac{1}{a} + \frac{1}{\frac{g\,x}{g + x + d}}\right\}$$

will be very close to its minimum value, no matter how high d may be.

For the purpose of determining the actual numerical value which d can have, let us write equation [B] in the form

$$\left\{d + g + x + \frac{g\,x}{a}\right\}\left\{d + \frac{r\,x}{r + x + a}\right\} = \frac{E\,x}{c_8\,(r + x + a)};$$

this equation, being an ordinary quadratic,* would enable the value of d to be obtained in terms of the other quantities in the usual manner, but inasmuch as we only require to determine the value of d within, say, 10 per cent., it is a much simpler and

* The solution of the quadratic equation is as follows:—

Let

$$g + x + \frac{g\,x}{a} = A,$$

$$\frac{r\,x}{r + x + a} = B,$$

$$\frac{E\,x}{c_8\,(r + x + a)} = K,$$

then we get

$$\{d + A\}\ \{d + B\} = K,$$

therefore

$$d^2 + d\,(A + B) = K - A\,B,$$

or

$$d^2 + d\,(A + B) + \left(\frac{A + B}{2}\right)^2 = K - AB + \frac{A^2 + 2AB + B^2}{4} = \frac{4K + (A - B^2)}{4},$$

therefore

$$d = \frac{\sqrt{4\,K + (A - B)^2}}{2} - \frac{A + B}{2} = \frac{\sqrt{4\,K + (A - B)^2} - (A + B)}{2}.$$

shorter operation to adopt the "trial" method; that is to say, to give d different values until we arrive at one which approximately satisfies the equation.

For example.

Suppose, as in the last example,

$$\begin{aligned} E &= 10, \\ x &= 500 \text{ (from a rough test)}, \\ g &= 5000, \\ r &= 200, \\ c_6 &= \cdot 000{,}000{,}01; \end{aligned}$$

then make $a = 500$;
we then get

$$\left\{d + 5000 + 500 + \frac{5000 \times 500}{500}\right\} \left\{d + \frac{200 \times 500}{200 + 500 + 500}\right\} = \frac{10 \times 500}{\cdot 000{,}000{,}01\,(200 + 500 + 500)},$$

or,

$$\{d + 10{,}500\} \quad \{d + 83 \cdot 3\} = 4{,}170{,}000{,}000.$$

If we make $d = 60{,}000$ we shall very nearly satisfy the equation, for

$$\{60{,}000 + 10{,}500\} \quad \{60{,}000 + 83 \cdot 3\} = 4{,}236{,}000{,}000.$$

As the value which d will have will depend upon the value given to b, the latter must be made equal to

$$b = \frac{500 \times 60{,}000}{500} = 60{,}000.$$

As regards the *Possible degree of accuracy* with which the test can be made, we have seen on page 225 that

$$\lambda = \frac{x}{d};$$

we therefore have

$$\lambda = \frac{500}{60{,}000} = \cdot 0083,$$

which equals

$$\frac{\cdot 0083 \times 100}{500} = \cdot 0017 \text{ per cent.};$$

this compares unfavourably with the result obtained when the

test was made with d of a low value and adjustable to $\frac{1}{1000}$th of a unit, the percentage of accuracy in the latter case being ·0003 per cent. To summarise the results of the investigation, we have

Best Conditions for making the Test.

242. Make a as low as possible, but not lower than $\frac{g\,x}{g+x}$.

Make d as high as possible, but not so high that

$$\left\{d+g+x+\frac{g\,x}{a}\right\}\left\{d+\frac{r\,x}{r+x+a}\right\}$$

is greater than

$$\frac{\mathrm{E}\,x}{c_6\,(r+x+a)},$$

c_6 being the figure of merit of the galvanometer.

Possible Degree of Accuracy Attainable.

$$\text{Percentage of accuracy} = \frac{\frac{x}{d}\times 100}{x} = \frac{100}{d}.$$

If we make d adjustable to any particular fraction of a unit, we can tell the degree of accuracy with which x could be measured, for if in equation [A] (page 225) we put $\frac{1}{n}$ instead of 1, we get

$$\lambda = \frac{x}{n\,d};$$

and equation [B] (page 225) becomes

$$\left\{d+g+x+\frac{g\,x}{a}\right\}\left\{d+\frac{r\,x}{r+x+a}\right\}' = \frac{1}{n}\cdot\frac{\mathrm{E}\,x}{c_6\,(r+x+a)}. \quad \text{[B]}$$

If in this last equation we give to $\frac{1}{n}$ the fractional value to which d is adjustable, we determine the degree of accuracy with which we can make the test.

For example.

Suppose d was adjustable to $\frac{1}{10}$th of a unit $\left(\frac{1}{n}\right)$, then we have (giving to x, a, g, and r the values used in the previous examples)

$$\{d+10{,}500\}\;\{d+83\cdot 3\} = 417{,}000{,}000.$$

If we make $d = 16{,}000$, we shall very nearly satisfy the equation, and the percentage of accuracy, λ', with which x would be measured would be

$$\lambda' = \frac{\frac{x}{n\,d} \times 100}{x} = \frac{100}{n\,d} = \frac{100}{10 \times 16{,}000} = \cdot 00062 \text{ per cent.}$$

243. At the commencement of the chapter (§ 230, page 209) we saw that by making b 10 or 100 times as great as a, and consequently d 10 or 100 times as great as x, we were enabled to measure x to an accuracy of $\frac{1}{10}$th or $\frac{1}{100}$th of a unit, although d was adjustable to units only. Every unit in d, in fact, represented $\frac{1}{10}$th or $\frac{1}{100}$th of a unit in x. But to measure to an accuracy of $\frac{1}{100}$th of a unit, with the forms of bridge shown in Chapter II., pages 13, 14, 15, and 16, the resistances in a and b have to be 10 and 1000 respectively, we have no other choice. In the investigation we have made, we have seen that a should be not less than $\frac{g\,x}{g + x}$, but in the bridge as usually arranged, if we wished to have a and b in the proportion of 1 to 100, so that we could measure to the accuracy of $\frac{1}{100}$th of a unit, we might find that we should have to very considerably transgress the rule of not making a smaller than $\frac{g\,x}{g + x}$, unless, indeed, x were a low resistance; for inasmuch as we could adjust the resistances in the bridge so as to theoretically measure a resistance of 100 ohms to an accuracy of $\frac{1}{100}$th of a unit, if the resistance were as high, or nearly as high, as 100, it might be 10 times, or nearly 10 times, as high as we could make a. Under these conditions, then, the bridge is not in a favourable condition for ensuring an accurate test.

We say it is not in a favourable condition for ensuring accuracy, but it does not follow therefore that we *cannot* measure a resistance of 100 ohms accurately to an accuracy of $\frac{1}{100}$th of a unit with such an arrangement. A galvanometer if it has a high figure of merit may, although the conditions are unfavourable, still give a sufficient deflection to enable us to exactly adjust.

What, then, it may be asked, is the practical value of the results we have theoretically arrived at? The value is this: if we find we have *not* got sufficient sensitiveness to obtain a good test, then we can see what may be the cause of it, and therefore how we can remedy it. The results further show that the values given to a

and b in the bridges as ordinarily arranged are such that only certain resistances can be measured under the best conditions for ensuring accuracy.

244. It should not be overlooked that the conditions for obtaining a good test are, to a very great extent, dependent upon the resistance of the galvanometer used, since the value which a must have is dependent upon both g and x. But it must not therefore be imagined that we can make these conditions anything we please by employing a galvanometer of a low resistance, for such galvanometers have a low figure of merit, and consequently what is gained in one direction by having g low, is more than counterbalanced by having the figure of merit low. It must be evident, then, that the whole question of the accuracy with which a bridge test can be made is dependent, in the first instance, upon both the resistance and figure of merit of the galvanometer, and, as we shall see, in certain cases it is absolutely necessary that the resistance be very low, although the figure of merit has consequently to be low also.

Measurement of a Resistance when exact Equilibrium cannot be obtained.

245. It very often happens, especially when measuring small resistances, that exact equilibrium cannot be obtained in the bridge; thus one unit too much in d may give a deflection to one side of zero, and one unit too little, a deflection to the other side of zero, and as no nearer adjustment can be made, the exact value of x is not directly determinable. If, however, the values of the deflections be noted, the true value of x can be obtained very closely.

On page 215 we have an equation [8] which gives the value of the current (c_6) passing though the galvanometer when equilibrium is not produced.

Let, then, c' be the current which produces, say, a left-hand deflection of the galvanometer needle, and let this current be caused by d being too small; also let c'' be the current which produces a right-hand deflection, and let this current be caused by d being too large. Then if d' and d'' be the smaller and larger resistances respectively, we have two equations, viz.

$$c' = \frac{a\,d' - b\,x}{B'}, \text{ and } - c'' = \frac{a\,d'' - b\,x}{B''},$$

where B′ and B″ are quantities corresponding to B_1 in equation [8].

Now, since d' and d'' are very nearly equal, B′ and B″ may be taken as being equal without sensibly altering the relative values of c' and c''; therefore we may say

$$-\frac{c'}{c''} = \frac{a\,d' - b\,x}{a\,d'' - b\,x},$$

that is,

$$c'\,a\,d'' - c'\,b\,x = c'\,b\,x - c''\,a\,d',$$

or

$$x = \frac{a\,(c'\,d'' + c''\,d')}{b\,(c' + c'')}.$$

But as d'' would be only 1 unit larger than d', that is, as

$$d'' = d' + 1,$$

therefore

$$x = \frac{a\,(c'\,d' + c' + c''\,d')}{b\,(c' + c'')} = \frac{a\,(d'\,(c' + c'') + c')}{b\,(c' + c'')} = \frac{a}{b}\left(d' + \frac{c'}{c' + c''}\right).$$

For example.

a and b being 10 and 1000 ohms respectively, when d' was adjusted to 156 ohms a deflection of 15 divisions (c') was obtained to one side of zero, and when d' was increased to 157 ohms, a deflection of 20 divisions (c'') to the other side of zero, was observed. What was the exact value of x?

$$x = \frac{10}{1000}\left(156 + \frac{15}{15 + 20}\right) = 1\cdot5643 \text{ ohms.}$$

Slide Resistance Coils Bridge.

246. Instead of fixing a and b and varying d, we may make a a fixed resistance, and $b + d$ a slide resistance, and vary the ratio of b to d. Either a slide wire or a set of slide resistance coils, such as that indicated by Fig. 11 (page 17), may be used. The former would be employed if $b + d$ is required to be a low resistance, the latter if a high resistance is necessary.

A set of coils allows of but few different ratios being given to b and d, unless indeed the number of coils is very large, which would be both a cumbersome and an expensive arrangement. The late Mr. Varley, by means of a movable derived circuit, reaching across two of the coils, devised a means of subdividing each of the latter. This arrangement is shown by means of Figs. 101 and 102 (pages 232 and 234). Referring to Fig. 101, let us suppose that

equilibrium is produced so that no current circulates through the galvanometer. This being the case, the points C and A may be joined without altering the current strengths in the various circuits.

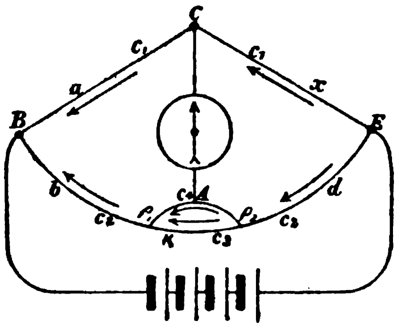


FIG. 101.

Let us suppose this junction to be effected; then by applying Kirchoff's laws (page 178), we have the following relations existing between the current strengths and the resistances in the system:—

$$\begin{aligned}
c_2 \quad - c_3 \quad - c_4 \quad &= 0. \\
c_1 a_1 - c_2 b - c_4 \rho_1 &= 0. \\
c_1 x \quad - c_2 d - c_4 \rho_2 &= 0. \\
c_4 \rho_1 + c_4 \rho_2 - c_3 \kappa &= 0.
\end{aligned}$$

By substitution we get

$$c_1 a = c_4 \left(\frac{\rho_1 + \rho_2}{\kappa} b + b + \rho_1 \right),$$

$$c_1 x = c_4 \left(\frac{\rho_1 + \rho_2}{\kappa} d + d + \rho_2 \right).$$

If we divide one equation by the other, then we have

$$\frac{a}{x} = \frac{b(\rho_1 + \rho_2 + \kappa) + \kappa \rho_1}{d(\rho_1 + \rho_2 + \kappa) + \kappa \rho_2}. \qquad [A]$$

Now if in this equation we make $\kappa = \rho_1 + \rho_2$, we get

$$\frac{a}{x} = \frac{2b(\rho_1 + \rho_2) + (\rho_1 + \rho_2)\rho_1}{2d(\rho_1 + \rho_2) + (\rho_1 + \rho_2)\rho_2} = \frac{2b + \rho_1}{2d + \rho_2} = \frac{b + \frac{\rho_1}{2}}{d + \frac{\rho_2}{2}}.$$

This equation shows that if the slide resistance $\rho_1 + \rho_2$ be made equal to the portion κ of the slide resistance $b + \kappa + d$ which it encloses, then the values of the resistances between the points B A

and E A will be to one another, as the resistance b plus half the resistance ρ_1, is to the resistance d plus half the resistance ρ_2.*

If, therefore, we have $b + \kappa + d$ formed of 101 coils of, say, 1000 ohms each, and $\rho_1 + \rho_2$ of 100 coils of 20 ohms each, that is, 2000 ohms $(\rho_1 + \rho_2)$ in all, and further, if the slider s_2 (Fig. 102) bridges across two of the 1000-ohm coils so as to enclose a resistance of 2000 ohms (κ), then a movement of slider s_1 from one contact to the next represents an alteration of 10 ohms in the ratio of B A to E A, whilst a similar movement of the slider s_2 represents an alteration of 1000 ohms. We can thus, by means of the 201 coils, 101 of 1000 ohms each, and 100 of 20 ohms each, obtain 10,000 ratios of B A and E A, each differing from the next by 10 ohms.

247. A consideration of the following note (*) will make it evident that the Varley slide principle can be applied to the Ayrton-Mather Universal Shunt (page 96); this has been done by Mr. J. Rymer Jones, and also by Mr. H. W. Sullivan (see Appendix).

* This follows almost as a matter of course from the following consideration :— Since κ is equal to $\rho_1 + \rho_2$, the current flowing through κ must be equal to the current flowing through $\rho_1 + \rho_2$, and, further, the point on κ which the slider A touches must have an exactly corresponding point on $\rho_1 + \rho_2$, which two points if connected together cannot affect the flow of the currents in the resistances, that is, cannot affect the balance on the galvanometer, and, moreover, it must be evident that as κ is equal to $\rho_1 + \rho_2$, the portions into which κ is divided on either side of the connected points must be equal to ρ_1 and ρ_2, i.e. the resistance on either side of the connected points must be $\frac{\rho_1}{2}$ and $\frac{\rho_2}{2}$ respectively.

It may be added that if κ is not equal to $\rho_1 + \rho_2$, then the point on κ corresponding to the position of the slider on $\rho_1 + \rho_2$ must be such that κ is divided into the two parts $\kappa \frac{\rho_1}{\rho_1 + \rho_2}$ and $\kappa \frac{\rho_1}{\rho_1 + \rho_2}$ respectively, and the resistances on either side of the connected points must be

$$\frac{\kappa \frac{\rho_1}{\rho_1 + \rho_2} \times \rho_1}{\kappa \frac{\rho_1}{\rho_1 + \rho_2} + \rho_1} = \frac{\kappa \rho_1}{\kappa + (\rho_1 + \rho_2)},$$

and

$$\frac{\kappa \rho_2}{\kappa + (\rho_1 + \rho_2)},$$

respectively. So that if, for example, $\rho_1 + \rho_2$ were equal to 2κ, then $\frac{\kappa \rho_1}{\kappa + (\rho_1 + \rho_2)}$ and $\frac{\kappa \rho_2}{\kappa + (\rho_1 + \rho_2)}$ would have the respective values $\frac{\rho_1}{3}$ and $\frac{\rho_2}{3}$; so that the equation for equilibrium would be

$$\frac{a}{x} = \frac{b + \frac{\rho_1}{3}}{d + \frac{\rho_2}{3}},$$

and so on.

We could, if required, have a second slider like s_2 to move along $\rho_1 + \rho_2$ (Fig. 101), and connected to a third set of coils along which the slider s_1 would move; by this means the differences of 10 ohms

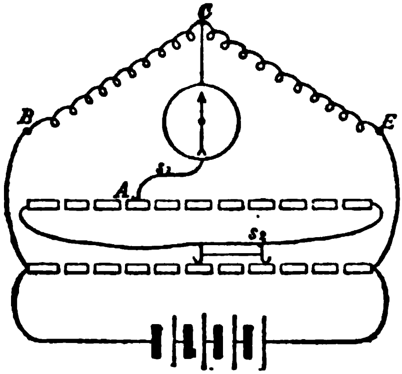


FIG. 102.

could be subdivided into differences of $\frac{1}{10}$th of an ohm. In fact, we could have any number of sets of coils with sliders, each carrying out the subdivision to any required degree.

When we come to make very small subdivisions, such, for instance, as subdividing $\frac{1}{10}$th of an ohm into 100 parts of $\frac{1}{1000}$th of an ohm each, it would be inconvenient to employ a set of small resistances, as they are difficult to adjust exactly; slide wires (§ 20, page 17) may therefore be employed with advantage for the purpose.

248. Fig. 103 shows a convenient arrangement of the Slide Resistance Coils Bridge; the coils in this case are arranged in a circle instead of in a straight line as represented by the theoretical diagram, Fig. 102. The left-hand dial contains the contacts and double slider for the 1000-ohm coils, and the right-hand dial the contacts and single slider for the 20-ohm coils.

Fig. 104 shows a theoretical arrangement of the foregoing Slide Resistance Coils Bridge; the connections in this diagram differ from those shown in Fig. 102 in so far that the relative positions of the battery and galvanometer are reversed, but this reversal is not essential to the principle, as either arrangement can be employed.

A less elaborate set of slide coils (arranged by Dr. Muirhead) which can be used for the same purpose as the foregoing, is shown by Fig. 105. In this set there are 43 resistances only, and consequently the whole can be made at a considerably less cost than the arrangement shown in Figs. 103 and 104.

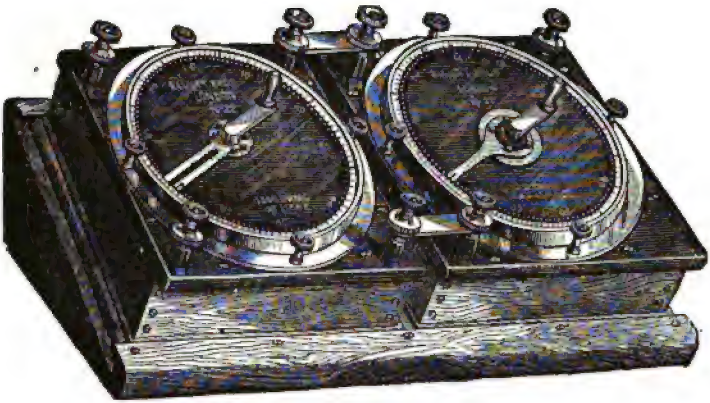


FIG. 103.

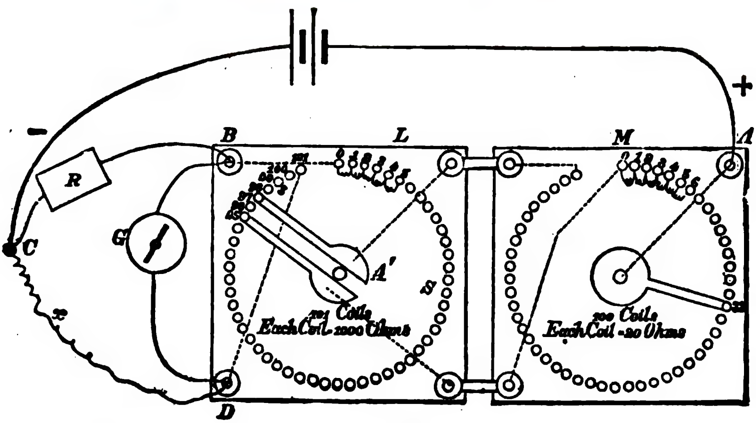


FIG. 104.

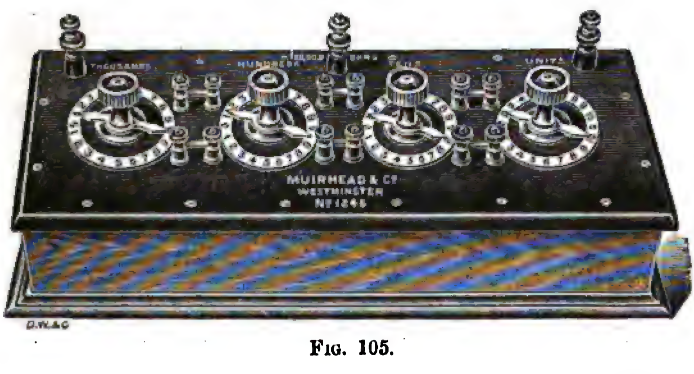


FIG. 105.

Measurement of Low Resistances.

SLIDE WIRE OR METRE BRIDGE.

249. The simple slide wire bridge is a very useful arrangement, as a very close adjustment can be made by means of it, and great accuracy of measurement thereby be obtained. It is especially useful for measuring small resistances accurately.

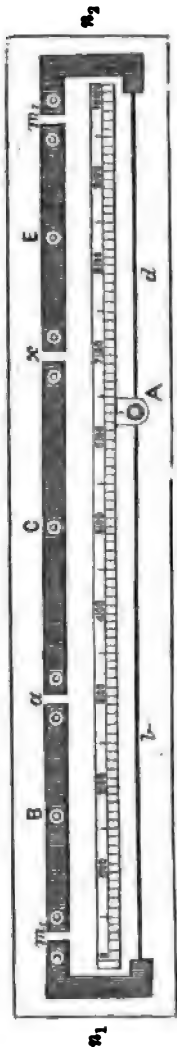


Fig. 106.

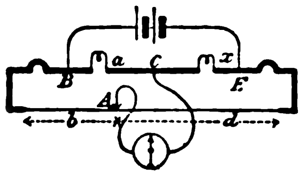


Fig. 107.

A form in which this description of bridge is very generally constructed is shown by Fig. 106.

The slide wire, which is 1 metre long and about 1·5 mm. in diameter, is stretched upon an oblong board (forming the base of the instrument) parallel to a metre scale divided throughout its whole length into millimetres, and so placed that its two ends are as nearly as possible opposite to the divisions 0 and 1000 respectively of the scale.

The ends of the wire are soldered to a broad, thick copper band, which passes round each end of the graduated scale, and runs parallel to it on the side opposite to the wire.

This band is interrupted by four gaps at m_1, a, x, and m_2. On each side of these gaps, and also at B, C, and E, are terminals.

In the ordinary use of the apparatus (Fig. 107), the wires from the battery are attached to the terminals B and E, and the galvanometer is connected between C and the slider A; by pressing down a knob this latter is put in contact with the wire.

The conductor whose resistance has to be measured, and a standard resistance, are placed in the gaps at x and a respectively.

The two gaps at m_1 and m_2 (Fig. 106) can either be bridged across by thick copper straps, or resistances of known values can be inserted in them; it is easy to see that these resistances are simply ungraduated prolongations of the slide wire.

250. If we have no resistance in these gaps, then when we have equilibrium,

$$\frac{x}{a} = \frac{d}{b}, \quad \text{or,} \quad x = a\frac{d}{b}.$$

As $\frac{d}{b}$ is merely a ratio, we do not require to know the absolute values of d and b, but only their relative values, that is to say, we only require to know the *lengths* of the portions on either side of the slider A, and not the *resistances* of those portions.

The length k of the slide wire is constant, that is,

$$b + d = k, \quad \text{or,} \quad d = k - b,$$

therefore

$$x = a\frac{k-b}{b} = a\left(\frac{k}{b} - 1\right);$$

but $k = 1000$ millimetres, and b is usually called the *scale reading*, therefore we have

$$x = a\left(\frac{1000}{\text{scale reading}} - 1\right). \qquad [A]$$

For example.

The standard resistance a being 1 ohm, equilibrium was obtained when the scale reading was 510; what was the value of the unknown resistance x?

$$x = 1\left(\frac{1000}{510} - 1\right) = \cdot 961 \text{ ohms.}$$

251. It has been pointed out by Mr. Martin F. Roberts that equation [A] is the same as

$$x = a\left(\begin{Bmatrix}\text{reciprocal of}\\ \text{scale reading}\end{Bmatrix} \times 1000 - 1\right),$$

and that consequently, by the use of a table of reciprocals, calculations can be considerably simplified in working out the value of x.

252. Equation [A] is only true if the resistances between the ends of the slide wire and the terminals B and E are zero. But, although it may not appear so, it is by no means easy to make these resistances inappreciable; even the careful soldering of the ends of the wire to the copper straps introduces a resistance which is sufficient to affect very accurate tests. Referring to Fig. 106, in which n_1 and n_2 are these resistances, we know that, strictly speaking,

$$\frac{x}{a} = \frac{d + n_2}{b + n_1};$$

or that

$$x = a\left(\frac{1000 + n_1 + n_2}{\text{scale reading} + n_1} - 1\right).$$

To make a strictly accurate test, then, we must know the values of n_1 and n_2 in terms of the equivalent length of the slide wire. These may be obtained in the following manner:—

Having bridged across the gaps at m_1 and m_2 with thick copper straps, taking care that the surfaces in contact are scraped bright, insert known resistances at a and x, a being rather larger than x; then having obtained equilibrium, we have

$$a(d + n_2) = x(b + n_1)$$

now reverse a and x, and again obtain equilibrium. Let the new scale readings be b_1 and d_1; we then have

$$x(d_1 + n_2) = a(b_1 + n_1).$$

By multiplying up and arranging the quantities, we have

$$a\,n_2 = x\,b + x\,n_1 - a\,d$$

and

$$x\,n_2 = a\,b_1 + a\,n_1 - x\,d_1;$$

therefore

$$\frac{a}{x} = \frac{x\,b + x\,n_1 - a\,d}{a\,b_1 + a\,n_1 - x\,d_1};$$

that is

$$a^2 n_1 - x^2 n_1 = x^2 b - a\,x\,d - a^2 b_1 + a\,x\,d_1,$$

therefore

$$n_1 = \frac{a\,x(d_1 - d) + x^2 b - a^2 b_1}{a^2 - x^2}.$$

In a similar manner we should find

$$n_2 = \frac{a\,x(b - b_1) + x^2 d_1 - a^2 d}{a^2 - x^2};$$

or since

$$b + d = b_1 + d_1 = 1000,$$

that is,

$$d = 1000 - b, \quad \text{and} \quad d_1 = 1000 - b_1,$$

we have

$$n_1 = \frac{a\,x\,(b - b_1) + x^2 b - a^2 b_1}{a^2 - x^2} = \frac{b\,x - b_1\,a}{a - x},$$

and

$$n_2 = \frac{(1000 - b_1)\,x - (1000 - b)\,a}{a - x}.$$

For example.

In order to determine n_1 and n_2, resistances were inserted at a and x equal to 3 and 2 ohms respectively. Balance was obtained when the scale reading b was 603. On reversing a and x, balance was obtained when the scale reading b_1 was 399. What were the values of n_1 and n_2?

$$n_1 = \frac{(603 \times 2) - (399 \times 3)}{3 - 2} = 9 \text{ mm.}$$

$$n_2 = \frac{(1000 - 399)\,2 - (1000 - 603)\,3}{3 - 2} = 11 \text{ mm.}$$

The value of x, then, would be given by the equation

$$x = a\left(\frac{1000 + 9 + 11}{\text{scale reading} + 9} - 1\right) = a\left(\frac{1020}{\text{scale reading} + 9} - 1\right).$$

253. Although perfectly satisfactory results may be obtained with the metre bridge when the latter is properly made, and when the measurements are carefully carried out, yet considerable trouble is often occasioned to inexperienced persons by results being obtained which are obviously erroneous. One most frequent cause of error is that occasioned by imperfect contacts; great care should therefore be taken that the important connections, viz. those at the gaps, should be well made; this should be ensured by having the various surfaces in contact made clean and bright by scraping. Good contacts are best assured by having mercury cups at the gaps instead of screw term'nals; care should be taken that the mercury in these cups is in good metallic contact with them, that is to say, it should *wet* the metallic surfaces. The mercury should also, of course, be in similar good contact with the ends of the wires or rods (the latter are usually attached to the standard resistances), which may be dipped into the cups.

The amalgamation of the metallic surfaces is best effected by scouring the latter with emery paper, and then moistening them with a solution of nitrate of mercury.

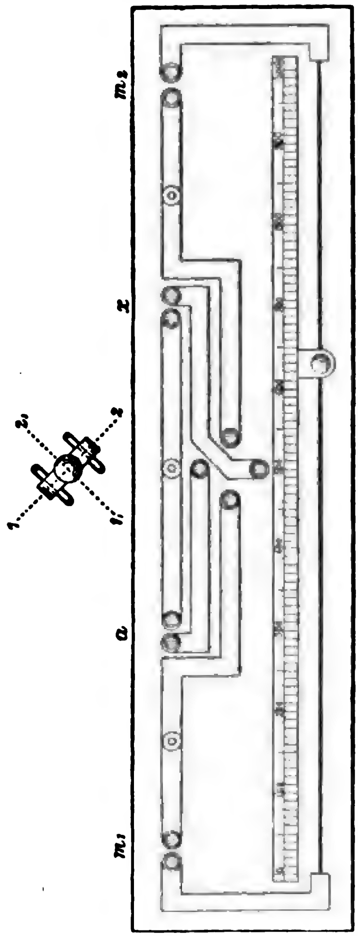


Fig. 108.

254. A form of bridge in which mercury cups are used in the place of terminals for the more important connections is shown by Fig. 108. This apparatus is also provided with a commutator for reversing the resistances placed at a and x. This commutator is formed of four mercury cups (seen in the centre of the figure) forming the corners of a square. These cups can be connected by means of the connector shown in the upper part of the figure. This connector is simply a short bar of ebonite with short copper rods at its extremities and at right angles to the latter; the ends of these rods are bent down so that they can dip into the cups when the arrangement is placed over the latter. If the connector is placed over the cups so that the ebonite bar is in the position shown by the dotted line, 1–2, then it will be seen that the left-hand cup at a is connected to the right-hand cup at m_1, and the right-hand cup at x to the left-hand cup at m_2; if, however, the ebonite bar is in the position shown by the dotted line, $1_{,}$–$2_{,}$ then the left-hand cup at a is connected to the left-hand cup at m_2, and the right-hand cup at x to the right-hand cup at m_1.

Even if good contacts be assured, correct results cannot be obtained if the standard resistances are incorrect, or if the slide wire is not uniform in its resistance throughout its length.

Standard Resistances.

255. Where great accuracy is not required, standard resistances similar to those shown by Fig. 109 may be used, but for precise measurements a more elaborate pattern is necessary.

256. The form of standard resistance generally used for exact measurements with the metre bridge is similar to that shown by Fig. 110.

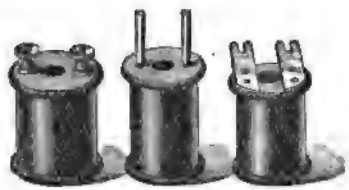


FIG. 109.

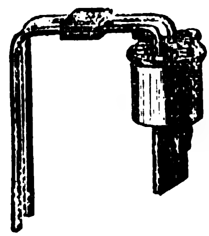


FIG. 110.

The ends of the brass rods to the left of the figure dip into the mercury cups; the resistance itself is enclosed in a brass box and bedded in paraffin wax.

The particular pattern shown is an arrangement devised by Professor Chrystal to show whether the temperature of the interior of the brass box is the same as that of the surrounding air. It contains a thermo-electric couple with one junction outside and one junction inside the box; by connecting this couple (whose terminals are seen on the upper part of the box) to a galvanometer of low resistance, no deflection would be produced if the two junctions, that is, the paraffin inside and the air outside the box, are at the same temperature.

257. There are several objections to the form of standard shown by Fig. 110; in the first place the shell cannot be wholly immersed in water (in order to determine the temperature of the resistance) without partially short-circuiting the electrodes; and moreover, when the arrangement is used as intended, then whilst the narrow or bottom portion of the shell is in the water, the upper and more massive portion is in the air, and therefore may be at a different temperature to the bottom portion; hence arises the doubt as to the actual temperature of the enclosed resistance. It has to be borne in mind that the limitation of accuracy in

comparisons of standards of resistance is determined by the difficulty of ascertaining temperature and not in the mere measurement of resistance; uncertainty as to the actual temperature of the wire to the extent of one or two-tenths of a degree Centigrade renders nugatory elaborate arrangements for very accurate measurement of resistance. Again, if the standard is immersed in melting ice or snow, and, therefore, cooled to 0° C., deposition of dew will take place upon the upper surface through which the rod electrodes protrude, and will thereby partially short-circuit the resistance.

To avoid these defects Dr. J. A. Fleming has devised the very excellent form of standard shown by Fig. 111. The case or shell which contains the coil is in the form of a ring. This ring consists of a pair of square-sectioned circular troughs provided with flanges which are screwed together with rubber packing, so as to form a square-sectioned, hollow, water-tight, circular ring. From this ring proceed upwards two brass tubes about 5 or 6 inches in length. Down these brass tubes pass the copper electrodes or rods, and these rods are insulated from the tubes at the top and bottom by ebonite insulators. The insulator at the bottom of the tube, where it enters the ring, is a simple collar, that at the top has the form of a funnel corrugated on its outer surface. The actual resistance coil is a length of platinum silver wire, threefold silk covered, saturated in ozokerit, and enclosed in the groove in the ring. The resistance-coil is thus contained in a thin ring of metal, and can be placed wholly below the surface of water or ice. In order to prevent surface leakage from one electrode to the other, an insulating oil can be placed in the ebonite funnels.

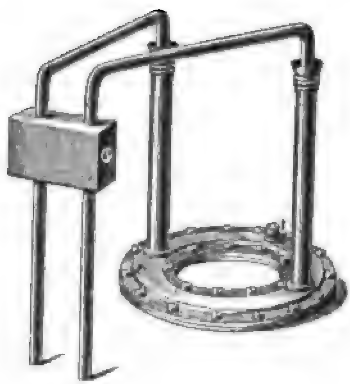


Fig. 111.

The ring-coils when in use are placed on rather shallow zinc troughs, which can be filled with water, and are closed with a wooden lid. The large metallic mass of the ring assists in quickly bringing the whole to the temperature of the surrounding water in which of course an accurate thermometer is placed.

258. A very good pattern of standard resistance is that shown

by Fig. 112, designed by Messrs. Nalder. This form reduces to a minimum the various elements of error to which the ordinary pattern is liable, such, for instance, as the conduction of heat to the enclosed resistance down the copper legs from the outside.

Fig. 112.

259. The accuracy with which a test may be made depends upon the values of the various resistances, and amongst these upon the value given to k. In order to be able to vary the value of this quantity, the gaps at m_1 and m_2 (Fig. 106, page 236) are provided.

As the resistances placed in these gaps are simply prolongations of the slide wire, it is necessary that their values should be known in terms of equivalent lengths of the slide wire; that is, we must know how many millimetres of the wire they are equal to. This is best done in the following manner :—

Close the gaps at m_1 and m_2 with the thick copper straps, and place resistances of known values at a and x. Adjust the slides so that equilibrium is produced, then

$$x = a\left(\frac{1000 + n_1 + n_2}{b + n_1} - 1\right),$$

or

$$x\,(b + n_1) + a\,(1000 + n_2 - b);$$

Now insert one of the resistances, whose equivalent length m_1 in millimetres is required, at the left-hand gap, and again obtain equilibrium; calling the new scale reading b_1 we then have

$$x\,(b_1 + n_1 + m_1) = a\,(1000 + n_2 - b_1);$$

by subtracting the one equation from the other we get

$$x\,(b - b_1) - x\,m_1 = a\,(b_1 - b),$$

that s,

$$(b - b_1)\,(a + x) = m_1\,x,$$

or

$$m_1 = (b - b_1)\frac{a + x}{x}.$$

For example.

It being required to know how many millimetres of the slide wire a resistance m_1 was equal to, the scale reading b, with the two gaps closed, was 500 mm., and the scale reading b_1, with m_1 inserted, was 480 mm., the resistances at a and x being 6 and 4 ohms respectively. What was the value of m_1?

$$m_1 = (500 - 480)\frac{6 + 4}{4} = 50 \text{ mm.}$$

If we have a and x equal, we get the simplification

$$m_1 = (b - b_1)\,2.$$

There are other methods of determining the value of m_1, but the one given, besides being extremely simple, is very accurate, as it is independent of the quantities n_1 and n_2.

The millimetre values of the resistances to be placed at m_1 and m_2 being thus determined, the value of x is given by the equation

$$x = a\left(\frac{1000 + n_1 + n_2 + m_1 + m_2}{\text{scale reading} + n_1 + m_1} - 1\right).$$

260. Let us now consider the *Best arrangement of resistances*, &c., for making a test with the metre bridge under favourable conditions.

Now a mistake of a millimetre in the position of the slider will make a much greater error in the result of x worked out from the formula, when the slider is near the ends of the wire than when it is near the middle. Thus, for example, suppose x was 1 ohm and a was also 1 ohm, then we should have the slider standing exactly at 500 if it were properly adjusted. Suppose, however, it was 1 millimetre out, then the apparent value of x would be

$$x = 1\left(\frac{1000}{501} - 1\right) = \cdot 996,$$

that is, we make x, $1 - \cdot 996$, or $\cdot 004$ ohms, too small.

Next suppose $a = 9$ ohms, then for equilibrium the scale reading would be 900, and if we make a mistake of 1 millimetre we should have

$$x = 9\left(\frac{1000}{901} - 1\right) = \cdot 990;$$

that is, we make x, $1 - \cdot 990$, or $\cdot 010$ ohms, too small.

Lastly, let us suppose $a = \frac{1}{9}$ ohm, then the scale reading for exact equilibrium would be 100, and supposing there to be an error of 1 millimetre, we have

$$x = \frac{1}{9}\left(\frac{1000}{101} - 1\right) = \cdot 989\,;$$

that is, we make x, $1 - \cdot 989$, or $\cdot 011$ ohms, too small.

To summarise the results, then, we see that with

a larger than	x,	error	was	·010, or 1	per cent.
„ equal to	„	„	„	·004, or $\frac{2}{5}$	„
„ smaller than	„	„	„	·011, or 1	„

The error, in fact, was smallest when the slider was at the middle of the wire. We must, however, determine whether the middle is really the point at which the error is least.

Calling k' the resistance of the slide wire and its prolongations m_1, m_2 and b' the scale reading plus the prolongation m_1, let there be an error λ in x caused by an error $-\delta$ in b', then

$$x + \lambda = a\left(\frac{k'}{b' - \delta} - 1\right) \quad \text{or} \quad \lambda = a\left(\frac{k'}{b' - \delta} - 1\right) - x.$$

But

$$x = a\left(\frac{k'}{b'} - 1\right), \quad \text{or,} \quad a = \frac{x}{\dfrac{k'}{b'} - 1}\,;$$

therefore

$$\lambda = x\left[\frac{\dfrac{k}{b' - \delta} - 1}{\dfrac{k'}{b'} - 1} - 1\right] = x\,\frac{c'\,\delta}{(b' - \delta)\,(k' - b')},$$

or since δ is a very small quantity, we may say,

$$\lambda = x\,\frac{k'\,\delta}{b'\,(k' - b')}. \qquad [A]$$

Now we have to make λ as *small* as possible; this we shall do, since x and k' are constant quantities, by making $b'\ (k' - b')$ as *large* as possible.

But

$$b'\,(k' - b') = \frac{k'^2}{4} - \left(\frac{k'}{2} - b'\right)^2,$$

and to make this expression as large as possible, we must make $\frac{k'}{2} - b'$ as small as possible; that is, since b' must be positive, we must make it equal to 0, or

$$\frac{k'}{2} - b' = 0; \text{ that is, } b' = \frac{k'}{2};$$

which proves the truth of the supposition.

To obtain the slider as near to the middle of the wire as possible when equilibrium is produced, we must make a as nearly as possible equal to x.

If in equation [A] we put

$$\lambda = \frac{\lambda'}{100} \text{ of } x, \quad \text{and,} \quad b' = \frac{k'}{2},$$

we get

$$\lambda' = \frac{400\,\delta}{k'};$$

so that if when the slider is near to the centre of k' we can adjust the slider to an accuracy of 1 division (δ), then if k' consisted of 1000 parts (as would be the case if there were no prolongations m_1, m_2), we could measure the value of x to an accuracy of

$$\frac{400 \times 1}{1000} = \cdot 4 \text{ per cent.}$$

261. In order to make a measurement in this manner, as we have seen, it is necessary for a to be approximately equal to x. Now in many cases there would be no difficulty in arranging that such should be the case. Thus, for example, suppose it were required to measure the conductivity of a sample of wire, then in this case we should take a sufficient length of the wire to give a resistance *approximately* equal to a, and then having measured the exact length taken, we should ascertain its *exact* resistance by adjusting the slider until equilibrium was obtained.

262. If we wish the measurement to be made to a higher percentage of accuracy than can be made with the slide wire k alone, then we must add equal resistances, m_1 and m_2, at each end of the wire so as to increase the value of k.

Since

$$\lambda' = \frac{400\,\delta}{k'},$$

therefore

$$k' = \frac{400\,\delta}{\lambda'};$$

so that if we wish to measure x to an accuracy, say, of $\cdot 1$ per cent., then we must make k' equal to

$$\frac{400 \times 1}{\cdot 1} = 4000;$$

that is to say, we must add resistances m_1 and m_2 at each end of k, each equivalent to 1500 millimetres of the wire k. It must be recollected, however, that there will be no advantage in thus increasing the length of k, unless the figure of merit of the galvanometer employed is sufficiently high to enable a movement of the slider to a distance of 1 division from its correct position, to produce a perceptible movement of the needle.

If the resistance to be measured is not one which admits of adjustment, then in order to obtain a satisfactory measurement we must add a resistance on to one or other of the ends of k, according as x is larger or smaller than a; or we may add resistances to both ends, their values being unequal.

If in equation [A] (page 245) we put

$$x = a\left(\frac{k'}{b'} - 1\right), \quad \text{or,} \quad k' = b'\frac{a + x}{a}, \qquad [1]$$

then we get

$$\lambda = \frac{(a + x)\,\delta}{b'};$$

or if we put

$$\lambda = \frac{\lambda'}{100} \text{ of } x,$$

we have

$$\lambda' = \frac{100\,(a + x)\,\delta}{b'\,x}, \quad \text{or,} \quad b' = \frac{100\left(\frac{a}{x} + 1\right)\delta}{\lambda'}.$$

From this equation we can see that no matter what are the relative values of a and x, still b' can always have a value which will enable x to be obtained to any percentage of accuracy λ'; that is, of course, provided the figure of merit of the galvanometer be sufficiently high for the purpose.

For example.

It is required to measure the exact value of a resistance x, whose approximate value is five times that of the resistance a; what must be the value of b' in order that the measurement may be made to an accuracy of $\cdot 5$ per cent.? The adjustment of the slider can be determined to an accuracy of 1 division.

$$b' = \frac{100\left(\frac{1}{5} + 1\right)1}{\cdot 5} = 240.$$

From equation [1] (page 247) we get

$$k' = 240\left(1 + \frac{5}{1}\right) = 1440,$$

consequently since k consists of 1000 divisions we must add a prolongation m_2 equal to not less than 440 divisions, on to k.

We may of course make the prolongation larger than 440; in fact, in practice we should have to do so unless we had a resistance available of the exact required value; but it must not be too large, otherwise the position of balance for the slider would be at some point on m_2 instead of on the wire k. In fact, m_2 must not be greater than $\frac{k\,x}{a}$.

If it should happen that in order to obtain a particular percentage of accuracy it is necessary that b' should exceed k, then in this case it would be necessary to have a prolongation m_1 in addition to the prolongation m_2; the latter quantity in this case must not exceed $(k + m_1)\frac{x}{a}$.

In the last example we have supposed x to be *less* than a. If, however, x is *greater* than a, then b' will probably have to be greater than k, in which case of course we should have to add the prolongation m_1 in the place of the prolongation m_2, the value of m_1 being such that it does not exceed $k\,\frac{a}{x}$, unless we also add a prolongation m_2 in addition to m_1, in which case m_1 must not exceed $(k + m_2)\frac{a}{x}$.

We have seen that by means of m_1 and m_2—the values of which can be determined in the manner shown in § 259 (page 243)—we can theoretically arrange that the value of x can be assured to

any required degree of accuracy, no matter what the relative values of x and a may be. This, however, can only be the case provided the figure of merit of the galvanometer is such as to enable the slider to be adjusted to an accuracy of 1 division. The figure of merit of the galvanometer, therefore, as in other tests, is the limit to the "Possible degree of accuracy attainable." This limit can be determined from equation [2] (page 217) in the following manner:—

Let λ be the error in x, caused by b' being $\frac{1}{n}$th of a unit out of adjustment, then we have

$$x + \lambda = a\,\frac{d' + \frac{1}{n}}{b' - \frac{1}{n}}, \quad \text{or,} \quad \lambda = a\,\frac{d' + \frac{1}{n}}{b' - \frac{1}{n}} - x = \frac{a\,d' + \frac{a}{n} - b'\,x + \frac{x}{n}}{b' - \frac{1}{n}};$$

and since $a\,d' = b'\,x$, and $\frac{1}{n}$ is a very small quantity, we get

$$\lambda = \frac{1}{n} \cdot \frac{a + x}{b'};$$

we have then from equation [2] (page 217) by putting $d' = \dfrac{b\,x}{a}$,

$$c_6 = \frac{\mathrm{E}\,\frac{1}{n} \cdot \frac{a + x}{b'}}{\left\{\frac{x}{a}(b' + g) + x + g\right\}\left\{r + a + x + \frac{r\,a}{b'}\right\}}.$$

In order, therefore, that b' may be able to have the value necessary to ensure x being measured to the required degree of accuracy, the value of c_6 must not be less than that given by the above equation.

As the values of g, d, x, and r are mostly easily obtained in ohms, the value of b' corresponding to the number of divisions of which it would consist must be in ohms also; $\frac{1}{n}$, likewise, will have to be the resistance, in the fraction of an ohm, corresponding to 1 division (or fraction of a division, if the slider can be adjusted to a closer accuracy than 1 division) of the wire k.

For example.

In the last example it was required to be known whether a galvanometer whose resistance was 1 ohm (g), and whose figure of merit was ·0002 (c_6) would be suitable for the purpose of making the measurement in question. The resistance of the slide

wire, which was divided into 1000 divisions (k), was 5 ohms; the resistance a was 1 ohm, and the resistance x, 5 ohms approximately. The actual value of the prolongation added to k was such as to make k' equal to 1560. The resistance of the battery was 5 ohms (r), and its electromotive force 2 volts (E) approximately.

Since $k = 1000$, therefore $\frac{1}{n} = \frac{\cdot 5}{1000} = \cdot 0005$.

Also (from equation [1], page 247) we have

$$b' = \frac{a\,k'}{a+x} = \frac{1 \times 1560}{1+5} = 260 \text{ divisions} = \frac{\cdot 5 \times 260}{1000} = \cdot 13 \text{ ohms};$$

therefore

$$c_6 = \frac{2 \times \cdot 0005 \times \dfrac{1+5}{\cdot 13}}{\left\{\frac{5}{1}(\cdot 13 + 1) + 5 + 1\right\}\left\{5 + 1 + 5 + \dfrac{5 \times 1}{\cdot 13}\right\}}$$

$$= \frac{\cdot 046}{(11\cdot 65)\,(49\cdot 46)} = \cdot 0008,$$

which is greater than $\cdot 0002$, the figure of merit of the galvanometer in question, consequently the latter instrument is well suited for the purpose for which it is required.

263. The resistance of the galvanometer employed in making a bridge test is an important point, especially as regards the measurement of small resistances.

In the case of the ordinary bridge test, we can adjust within 1 unit, and in the case of the slide wire bridge, we can adjust within 1 millimetre of the wire; if then the galvanometers employed in these cases are such that when we are 1 unit or 1 millimetre from exact equilibrium we obtain perceptible deflections of the needles, then we have what we require, whatever the resistances of the galvanometers may be.

In the ordinary form of bridge, where the adjustable resistances are not capable of being adjusted to a greater accuracy than 1 unit, a Thomson's galvanometer, like those shown on pages 51 to 53, and which has a resistance of about 5000 ohms, gives, under all circumstances, a very large deflection when the adjustment is only 1 unit from equilibrium. In the case of the slide wire bridge, however, where to be 1 millimetre from exact equilibrium means to be only $\frac{1}{5000}$th of an ohm, or even less, out,

a galvanometer of such a high resistance as 5000 ohms would not be found to give a perceptible deflection.

The reason of this is, that such a galvanometer is practically short-circuited by the very low resistance it has between its terminals.

The question of galvanometer resistance is considered at length in Chapter XXV., and it is there shown that it is best that the instrument should have a resistance not more than about 10 times, or less than about $\frac{1}{10}$th, $\frac{a(d+x)}{a+x}$. Of course in practice we cannot adjust the resistance to meet every particular case, but the limits given are sufficiently wide to enable an instrument to be made which would prove satisfactory for most purposes for which the metre bridge is adapted; moreover, if a particular galvanometer does not prove to be suitable for a particular purpose, we can ascertain, by the help of the above rule, whether the cause is due to its resistance being too high or too low.

It should be clearly understood that when we speak of the resistance of the galvanometer we mean the resistance of the instrument itself, and not the resistance in its circuit; thus, if according to calculation it were proved that the galvanometer resistance should be 1 ohm, then it would not be carrying out the rule if we took an instrument having a resistance of, say, $\frac{1}{4}$ of an ohm, and added a resistance of $\frac{3}{4}$ of an ohm in its circuit, for this $\frac{3}{4}$ of an ohm would be an addition to the *external* circuit and not a part of the resistance of the galvanometer itself.

Under no conditions should the battery be joined between A and C, and the galvanometer between B and E, for in such a case the battery current in passing from the slider to the wire would be liable to injure the surface of the latter.

To sum up, then, we have

Conditions necessary for making the Test to any required Degree of Accuracy.

264. The number of divisions of which b' must consist in order that x may be measured to an accuracy of λ' per cent. must be not *less* than

$$\frac{100\left(\frac{a}{x}+1\right)\delta}{\lambda'},$$

δ being the number of divisions, or the fraction of a division, to which it is possible to adjust the slider.

If prolongations are necessary, then m_1 must not exceed $\left(k + m_2\right)\frac{a}{x}$, and m_2 must not exceed $\left(k + m_1\right)\frac{x}{a}$.

The figure of merit of the galvanometer must be not less than

$$\frac{\mathrm{E}\,\delta . \frac{a+x}{b'}}{\left\{\frac{x}{a}\left(b'+g\right)+x+g\right\}\left\{r+a+x+\frac{r\,a}{b'}\right\}}$$

where E is in volts and *all* the other quantities (including b' and d') are in ohms.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{100}{b'}\left(\frac{a}{x}+1\right)\delta.$$

Measurements by Carey Foster's Method.

265. This method, devised by Prof. Carey Foster,* consists in determining the value of the unknown resistance in terms of an equivalent length of the slide wire; this is effected in the following way:—

The resistance, x, whose value is to be determined, is placed in the left-hand gap (Fig. 113), and resistances r_1, r_2, the ratio of

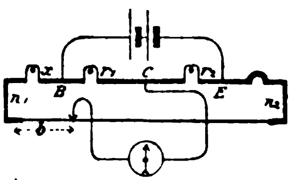


Fig. 113.

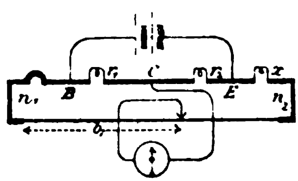


Fig. 114.

whose values does not differ from unity more than does that of the resistance to be measured and the resistance of the whole slide wire, are placed in the two centre gaps; the right-hand gap is closed by a conductor without sensible resistance.

The slider is now adjusted until equilibrium is obtained, and the reading b is noted. x is then transferred to the right-hand gap, and the left-hand gap is closed by a conductor without

* 'Journal of the Society of Telegraph-Engineers,' vol. i. p. 196.

sensible resistance (Fig. 114); the slider is again adjusted and the reading b_1 noted.

Calling n_1 the resistance of the portion of the copper strap between B and the left-hand end of the slide wire, and n_2 the resistance of the portion of the strap between E and the right-hand end of the slide wire; also calling r_1 the total resistance between the points B and C, and r_2 the total resistance between the points C and E; finally calling b and b_1 the respective *resistances* of the portions of the slide wire in the two tests, and calling l the total *resistance* of the slide wire, we have

$$\frac{r_1}{r_2} = \frac{x + n_1 + b}{l - b + n_2}$$

and also

$$\frac{r_1}{r_2} = \frac{n_1 + b_1}{l - b_1 + n_2 + x};$$

therefore

$$\frac{x + n_1 + b}{l - b + n_2} = \frac{n_1 + b_1}{l - b_1 + n_2 + x};$$

therefore

$$\frac{x + n_1 + b}{l - b + n_2} + 1 = \frac{n_1 + b_1}{l - b_1 + n_2 + x} + 1;$$

therefore

$$\frac{x + n_1 + b + l - b + n_2}{l - b + n_2} = \frac{n_1 + b_1 + l - b_1 + n_2 + x}{l - b_1 + n_2 + x};$$

therefore

$$l - b + n_2 = l - b_1 + n_2 + x,$$

or

$$x = b_1 - b.$$

In order to make this formula useful we must know the resistance per millimetre of the slide wire, since b_1 and b on the scale represent not resistances but lengths. The simplest method of doing this is to take a test in the foregoing manner, giving the resistance x a known value, ·1 ohm for example; in the latter case, since

$$\cdot 1 = b_1 - b$$

the difference between the two scale readings multiplied by 10 gives the number, ν, of millimetres corresponding to 1 ohm resistance, and therefore when we make a test to determine an unknown resistance, x, we get

$$x = \frac{b - b_1}{\nu}.$$

The accuracy of the test depends upon the conductor with which the unknown resistance, x, is interchanged, having practically no resistance; it should, therefore, be made of as massive and short a piece of copper as possible, and the connections should be made by means of mercury cups.*

The great merit of Professor Foster's method lies in the fact, that the measurements are independent of the resistances of the various parts of the copper band.

266. Professor Foster points out that inasmuch as by his method the value of a resistance, x, can be determined in terms of a certain length of the slide wire, therefore if x be made a known resistance and the slide wire itself be formed of a portion of wire whose resistance per unit length is required, this latter resistance can easily be determined. Such a method would give very accurate results, and is as good as "Thomson's Bridge" method which was devised by Sir William Thomson for the same purpose, and is as follows:—

Thomson's Bridge.

267. The arrangement of this bridge is shown by Fig. 115; its object is the accurate measurement of the resistance of a portion of a conductor of low resistance, lying between two points, errors

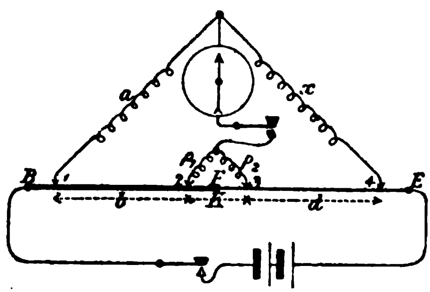


Fig. 115.

due to imperfect connections being avoided. In the figure, B F is the conductor, the resistance b of the corresponding length, 1–2, of which requires to be determined. F E is a standard slide wire

* As a rule the cups at each side of a gap are too small and are not put close enough together, the consequence being that a conductor used for bridging over a gap is comparatively long and has a sensible resistance. The cups ought to be of large dimensions, and so close together as almost to touch, the bridge piece could then be made so massive and short as to be practically of a negligible resistance. The ends of this piece should be quite flat, so as to lie closely in contact with the bottom of the cups.

whose resistance per unit length is accurately known. Now when we have equilibrium we see from equation [A] (page 232) that we have

$$\frac{a}{x} = \frac{b(\rho_1 + \rho_2 + K) + K\rho_1}{d(\rho_1 + \rho_2 + K) + K\rho_2};$$

by multiplying up and arranging we get

$$a d - b x = \frac{K(x\rho_1 - a\rho_2)}{\rho_1 + \rho_2 + K}.$$

Now if we have

$$\frac{a}{\rho_1} = \frac{x}{\rho_2}, \quad \text{that is,} \quad x\rho_1 = a\rho_2$$

we get

$$a d - b x = 0,$$

or

$$b = d\frac{a}{x};$$

from which we see that the value of b is independent of the resistance of any of the connections provided the contacts at the points 1, 2, 3, and 4 are small compared with the resistances

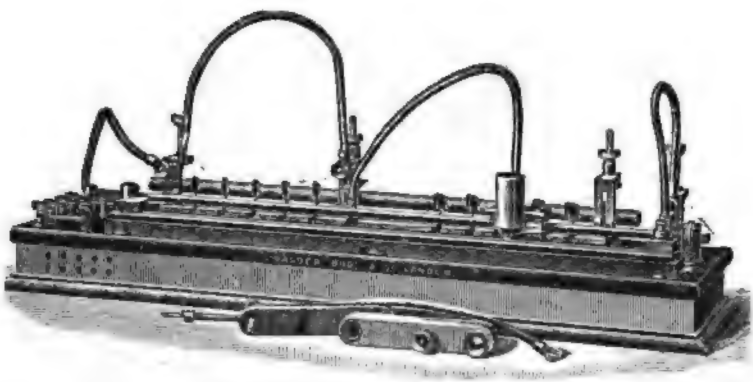


FIG. 116.

a, ρ_1, ρ_2, and x, which by making these resistances high enough, will practically be the case. The points 1, 2, 3, and 4 should be knife edges, so that the exact distance between 1 and 2, and between 3 and 4 can be properly determined.

Fig. 116 shows a practical form of the apparatus.

Measurement of Low Resistances by the Potentiometer.

268. One of the most satisfactory methods of measuring a low resistance is by means of a potentiometer (§ 226, page 206). This is effected by placing the resistance to be measured in series with a resistance of standard value, and then passing a constant current through the two resistances, wires being led from the terminals of the latter to the potentiometer. By measuring accurately the values of the potential differences, that is the electromotive forces, between the ends of the two resistances the relative values of the latter are at once obtained, since the resistance values are exactly proportional to the potential differences. To ensure accuracy it is, of course, necessary that the current be maintained absolutely constant, and that the standard resistance be perfectly correct. A "direct reading potentiometer," which is very convenient and suitable for the purpose, is described in the Appendix.

Measurement of the Conductivity Resistance of a Telegraph Line.

Direct Method.

269. When, by means of the bridge (Fig. 96, page 210), we are measuring the *conductivity* resistance of a wire whose further end is not at hand, we should join one end to terminal C, put the further end to earth, put terminal E to earth, and then measure in the usual way.

Loop Method.

270. It is always as well, however, when possible, to measure without using an earth, by looping two wires together at their further ends, the nearer ends being joined to terminals E and C respectively; this gives the joint conductivity resistance of the two. Errors consequent from earth currents, or a defective earth, &c., are thereby avoided. We cannot, however, by this means, obtain the conductivity resistance of each wire separately.

Measurement of the Individual Resistance of Three Wires.

If, however, we have three wires at hand, we can by three measurements obtain the conductivity resistance of each wire, without using an earth. This is effected as follows:—

Let the three wires be numbered respectively 1, 2, and 3. First loop wires 1 and 2, at their further ends, and let their resistance be R_1. Next loop wires 1 and 3, and let their resistance be R_2. Lastly, loop 2 and 3, and let their resistance be R_3. Supposing the respective resistances of 1, 2, and 3 to be r_1, r_2, and r_3, we get

$$\begin{aligned} r_1 + r_2 &= R_1 \\ r_1 + r_3 &= R_2 \\ r_2 + r_3 &= R_3 \end{aligned}$$

Now, since each of the wires is looped first with one and then with the other of the other two, it is evident that the sum of the three measurements will be the sum of the individual resistances of the three wires taken twice over, and consequently $\frac{R_1 + R_2 + R_3}{2}$ must be the sum of the resistances of the three wires. If, then, we subtract R_1 from this result, the remainder must be the resistance of r_3. Similarly, if we subtract R_2 from the same, the remainder will give us r_2; and lastly, by subtracting R_3, we get the value of r_1.

For example.

The conductivity resistance of each of three wires, Nos. 1, , and 3, was required. Nos. 1 and 2 being looped, the resistance (R_1) was found to be 300 ohms. Nos. 1 and 3 looped gave a resistance (R_2) of 400 ohms. Lastly, Nos. 2 and 3 looped gave a resistance (R_3) of 500 ohms. Then :—

Added resistance of the three wires will be

$$\frac{300 + 400 + 500}{2} = 600 \text{ ohms};$$

therefore,

Resistance (r_1) of No. 1 wire $= 600 - 500 = 100$ ohms.
„ (r_2) „ 2 „ $= 600 - 400 = 200$ „
„ (r_3) „ 3 „ $= 600 - 300 = 300$ „

By this device, then, we are enabled to eliminate all sources of error without making a greater number of measurements than would be required if we measured each wire separately, by using an earth. (See also § 272.)

MEASUREMENT OF THE RESISTANCE OF AN EARTH.

271. By means of a method very similar to the foregoing we can, if we have two wires at our disposal, measure the resistance of the *earths* at the ends of the lines. The following is the way in which this can be done:—

Let the two wires be numbered respectively 1 and 2. First loop the two wires at their further ends, and let the measured resistance of the loop be R_1. Next have No. 1 wire put to the earth at its further end, and measure the resistance, which will be that of the wire and earths combined; let this total resistance be R_4. Lastly, have wire No. 2 put to the earth at the distant station, and measure the total resistance, which we will call R_5; then by adding R_1, R_4, and R_5 together, and dividing the result by 2, we get the sum of the resistances of the two wires and the earth; by subtracting from this result the resistances of the two looped wires the remainder will be the resistance of the earths.

272. By means of a test made in this manner we can determine not only the resistance of an earth, but also the individual resistance of two wires; for if we subtract R_4 from $\frac{R_1 + R_4 + R_5}{2}$, the result will be the resistance of wire No. 2, and if we subtract R_5 instead of R_4, then the result will be the resistance of wire No. 1. Such a test, however, although it eliminates errors due to defective earths, does not eliminate errors due to earth currents. But inasmuch as it is a test which is applicable when only *two* wires can be had, it is useful, since the earth current errors can be eliminated by a method which we shall investigate (§ 277, page 261).

*Pomeroy's Method.**

273. This method is shown by Figs. 117 and 118. Any line wire (Line 1) is connected up with the Wheatstone Bridge and the "Earth" (A) which is to be tested (Fig. 117). Line 1 is put to earth at the further end. Balance is obtained on the bridge in the usual way. Let the resistance required to balance be R, and let

x = resistance of the earth A
y = " " " B
L_1 = " " Line 1,

then

$$a\,R = b\,(L_1 + y + x) \qquad [1]$$

* 'Electrical Review,' April 20th, 1888.

The connections are now altered as shown by Fig. 118. The zinc pole of the battery in this case, instead of going direct to the earth A, is connected with a second line (Line 2) which runs in a different direction to Line 1 (preferably at about right angles to it), and which is put to earth at the further end. A second balance is then obtained on the bridge. Let the adjusted resistance required to do this be r, then

$$a\,(x + r) = b\,(L_1 + y)$$

that is

$$a\,x + a\,r = b\,(L_1 + y); \qquad [2]$$

and subtracting equation [2] from equation [1] we get

$$a\,R - a\,x - a\,r = b\,x,$$

or

$$x\,(a + b) = a\,(R - r);$$

that is

$$x = \frac{a\,(R - r)}{a + b};$$

or if a and b are equal

$$x = \frac{R - r}{2},$$

that is *half the difference between the two resistances.*

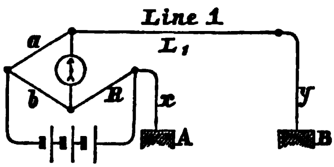


Fig. 117.

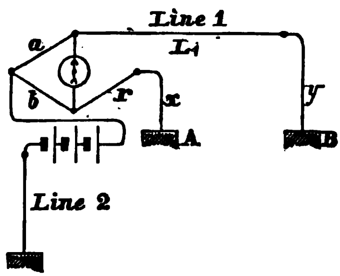


Fig. 118.

As it is desirable in making the two tests that the current passing at the earth-plate in both cases should be approximately equal, the battery power for taking the second test should be rather greater than that used in taking the first. If a simple form of galvanometer be kept in circuit with the battery, and the battery be so adjusted that the deflections of this galvanometer are about the same in both tests, then the current passing at the earth A will be about the same in both cases.

The object of joining up the battery for the second test in the way shown by Fig. 118 is to arrange that the current flowing out at the earth-plate under test (A) may be in the same direction as it is in the first test.

Measurement of the Insulation Resistance of a Telegraph Line.

274. In measuring the insulation resistance of a wire, the connections would be the same as for conductivity resistance, except that the further end of the wire, instead of being put to earth, would be insulated.

275. It sometimes happens that we require to find the insulation resistance of two sections of one wire, but we can only test from one end.

Now, if we join several wires together, one in front of the other, it is evident that the total *insulation* resistance of the combination will diminish according to the number of the wires and according to the insulation resistance of each of them. The law for the total resistance, in fact, will be the same as that for the joint *conductor* resistance of a number of wires joined up in multiple arc (page 90). That is to say, *the total insulation resistance of any number of wires joined together will be equal to the reciprocal of the sum of the reciprocals of their respective insulation resistances.* As a matter of fact, it is immaterial whether the wires be joined together one in front of the other or are all bunched together: the law of the joint insulation resistance is the same in both cases.*

A B C

Suppose, then, A C to be the wire which is required to be tested for insulation resistance from A in two sections, A B and B C. Let a be the insulation resistance of the section A B, and b the insulation resistance of the section B C; and suppose x to be the insulation resistance of the whole wire from A to C, then we have

$$x = \frac{a\,b}{a + b},$$

from which

$$b = \frac{a\,x}{a - x}.$$

* This is not the case if the insulation resistances are very low, as the resistance of the conductor then comes into question and modifies the result.

All we have to do, therefore, supposing we are testing from A, is first to get the end C insulated and to measure the insulation resistance; this gives us x. Next get the wire separated at B, and the end of the section A B insulated. Again measure the insulation resistance; this gives us a. Then from the two results b can be calculated.

For example.

The *insulation* resistance (x) of the whole wire, from A to C, was found to be 6000 ohms, and that from A to B (a), 24,000 ohms. What was the insulation resistance (b) of the section B?

$$b = \frac{24,000 \times 6000}{24,000 - 6000} = 8000 \text{ ohms.}$$

276. To obtain the *conductivity* resistance of one section of a wire when the resistance of the other section, and also of the whole wire, is known, we have only to subtract the resistance of the one section from the resistance of the whole section. The truth of this is obvious.

Measurement of the Conductivity Resistance of Wires traversed by Earth Currents.

277. When the conductivity resistance of a line of telegraph is measured by having the further end of the line put to earth, the presence of earth currents, that is to say, the currents set up by electrical disturbances over the surface of the earth, and also currents due to the polarisation of the earth plates, renders the formula $x = d\frac{a}{b}$, when equilibrium is produced, incorrect. To obtain the true value of the resistance of the wire, therefore, a different formula is necessary.

Equilibrium Method.

278. In Fig. 119 let E be the electromotive force of the testing battery, E_1 the electromotive force of the earth current, whose value will be $+$ or $-$ according to its direction, and let a, b, d, x, and r, be the resistances of the various parts of the bridge; then c', c'',

c''', c_1, and c_5 being the current strengths in the different branches, we have by Kirchoff's laws (page 178), when equilibrium is pro-

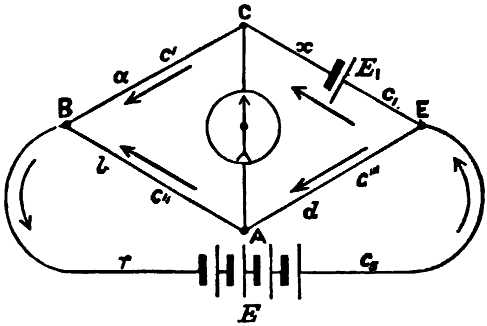


FIG. 119.

duced, the following equations connecting the resistances, current strengths, and electromotive forces:—

$$c' - c_1 = 0$$
$$c'' - c''' = 0$$
$$c_5 - c''' - c_1 = 0$$
$$c'a - c''b = 0$$
$$c_1 x - c''' d = \pm E_1$$
$$c_5 r + c''' d + c'' b = E.$$

By elimination we obtain two values of c_1, one in terms of the battery E_1, and the other in terms of E, thus

$$c_1 = \pm \frac{E_1}{x - \frac{a\,d}{b}} \qquad [A]$$

and

$$c_1 = \frac{E}{\frac{a(d+r) + b(a+r)}{b}}.$$

Equating the two values of c_1 we get

$$\frac{\pm E_1}{E} = \frac{b\,x - a\,d}{a(d+r) + b(a+r)}.*$$

* This equation gives the relation between the two electromotive forces E and E_1, and thus gives a method of determining the relative electromotive forces of the batteries.

279. From the latter equation we find

$$x = \frac{a\,d}{b} \pm \frac{E_1}{E}\left[\frac{a\,(d+r)+b\,(a+r)}{b}\right].$$

To make this equation useful it is necessary that E_1 and E be known. If, however, we reverse the testing battery and again obtain equilibrium by readjusting d to d_1, we get a second equation, viz.

$$\frac{\pm E_1}{-E} = \frac{b\,x - a\,d_1}{a\,(d_1+r)+b\,(a+r)};$$

we therefore have

$$\frac{b\,x - a\,d}{a\,(d+r)+b\,(a+r)} + \frac{b\,x - a\,d_1}{a\,(d_1+r)+b\,(a+r)} = 0;$$

by multiplying up we get

$$\begin{aligned} &b\,x\,[a\,(d_1+r)+b\,(a+r)] - a\,d\,[a\,(d_1+r)+b\,(a+r)] \\ &+ b\,x\,[a\,(d+r)+b\,(a+r)] - a\,d_1\,[a\,(d+r)+b\,(a+r)] \\ &= 0; \end{aligned}$$

that is

$$\begin{aligned} x &= \frac{a}{b}\cdot\frac{d\,[a\,(d_1+r)+b\,(a+r)]+d_1\,[a\,(d+r)+b\,(a+r)]}{a\,(d_1+r)+b\,(a+r)+a\,(d+r)+b\,(a+r)} \\ &= \frac{a}{b}\left[\frac{2\,(d+k)\,(d_1+k)}{(d+k)+(d_1+k)} - k\right] \qquad \text{[B]} \end{aligned}$$

where

$$k = \left[r\left(1+\frac{b}{a}\right)+b\right].$$

For example.

In making a conductivity resistance test of a wire in which an earth current existed, with the zinc pole of the battery to line equilibrium was obtained when d_1 was 8000 ohms. On reversing the testing current, equilibrium was obtained when d was 6000 ohms. The resistances a and b were 100 and 1000 ohms respectively, and the resistance, r, of the battery 200 ohms. What was the resistance, x, of the line?

$$k = \left[200\left(1+\frac{1000}{100}\right)+1000\right] = 3200,$$

therefore

$$x = \frac{100}{1000}\left[\frac{2\,(6000+3200)\,(8000+3200)}{(6000+3200)+(8000+3200)} - 3200\right] = 690\cdot 2 \text{ ohms}.$$

It may be pointed out that the quantity $\frac{2(d+k)(d_1+k)}{(d+k)+(d_1+k)}$ in equation (B) is the *harmonic* mean of the quantities $(d+k)$ and (d_1+k).

Various abbreviations of formula [B] have been suggested, but none of them are satisfactory except under certain conditions, and inasmuch as the formula is only required occasionally, the advantage of a simplification which at the best is only an approximation is a doubtful one.

*Mance's Method.**

280. This method, devised by Sir Henry Mance, consists in making the observations as in the last test, but without reversing the current, the first observation being made with resistances a and b in the arms B C and B A of the bridge, and the second with these resistances changed to a_1 and b_1. In the first case, then, we have

$$\frac{\pm E_1}{E} = \frac{b\,x - a\,d}{a(d+r) + b(a+r)},$$

in the second case

$$\frac{\pm E_1}{E} = \frac{b_1\,x - a_1\,d_1}{a_1(d_1+r) + b_1(a_1+r)},$$

therefore

$$\frac{b\,x - a\,d}{a(d+r) + b(a+r)} = \frac{b_1\,x - a_1\,d_1}{a_1(d_1+r) + b_1(a_1+r)}.$$

By multiplying up and extracting x, we get

$$x = \frac{a\,d\,[a_1(d_1+r) + b_1(a_1+r)] - a_1\,d_1\,[a(d+r) + b(a+r)]}{b\,[a_1(d_1+r) + b_1(a_1+r)] - b_1\,[a(d+r) + b(a+r)]}$$

$$= \frac{a\,d\,[(a_1+b_1)r + a_1\,b_1] - a\,d_1\,[(a+b)r + a\,b]}{a_1\,b\,(b_1+d_1+r) - a\,b_1\,(b+d+r)}.$$

In practice Sir Henry Mance prefers to make $b = a$ and $b_1 = a_1$, in which case the formula becomes

$$x = \frac{d(2r + a_1) - d_1(2r + a)}{(d_1 + a_1) - (d + a)}.$$

For example.

In making a conductivity test of a wire in which an earth current existed, the arms a and b of the bridge were each made

* 'Journal of the Society of Telegraph Engineers,' May 8th, 1886.

equal to 100 ohms; equilibrium was then obtained when d was adjusted to 750 ohms. On altering a and b to 1000 each, balance was again obtained by making d_1 equal to 840 ohms. The resistance, r, of the battery was 200 ohms. What was the resistance of the line?

$$x = \frac{750\,(2 \times 200 + 1000) - 840\,(2 \times 200 + 100)}{(840 + 1000) - (750 + 100)} = 636 \cdot 4 \text{ ohms}.$$

See also page 277.

281. Mance's test could with advantage be made in the following manner:—

Referring to Fig. 119, if we have a "milliampèremeter," or "current-measurer" in the branch C E, so that the strength of the current flowing can be measured, then, referring to equation [A], page 262, we have,

$$c_1 = \pm \frac{E_1}{x - \frac{a\,d}{b}}.$$

If now we change a and b to a_1 and b_1, and we again get balance by changing d to d_1, then we get a second equation

$$c_2 = \pm \frac{E_1}{x - \frac{a_1\,d_1}{b_1}};$$

therefore

$$c_1\left(x - \frac{a\,d}{b}\right) = c_2\left(x - \frac{a_1\,d_1}{b_1}\right)$$

or

$$x\,(c_1 - c_2) = c_1\frac{a\,d}{b} - c_2\frac{a_1\,d_1}{b_1},$$

that is

$$x = \frac{c_1\frac{a\,d}{b} - c_2\frac{a_1\,d_1}{b_1}}{c_1 - c_2} = \frac{\frac{a\,d}{b}\cdot\frac{c_1}{c_2} - \frac{a_1\,d_1}{b_1}}{\frac{c_1}{c_2} - 1}$$

$$= \frac{\frac{a\,d}{b}\left(\frac{c_1}{c_2} - 1\right) - \frac{a_1\,d_1}{b_1} + \frac{a\,d}{b}}{\frac{c_1}{c_2} - 1} = \frac{a\,d}{b} - \left(\frac{a_1\,d_1}{b_1} - \frac{a\,d}{b}\right)\frac{1}{\frac{c_1}{c_2} - 1};$$

or if $\frac{c_1}{c_2} = n$, then

$$x = \frac{a\,d}{b} - \left(\frac{a_1\,d_1}{b_1} - \frac{a\,d}{b}\right)\frac{1}{n - 1}.$$

If we make $a = b$, and $a_1 = b_1$, then

$$x = d - (d_1 - d)\frac{1}{n-1}.*$$

It may be pointed out that the only object of changing the ratio arms a and b to a_1 and b_1 is to alter the current strength, and as this change results in one of the readings being taken with a reduced sensibility of the galvanometer, it would be preferable to alter the current strength by altering the battery power, or by adding a resistance in the circuit of the latter.

Equal Deflection or "False Zero" Method.

282. This method is actually that described in the Note on page 145 and in § 148 of the same page. It is in fact Mance's test with a battery included in circuit with the key;† r, which in Mance's test is the resistance of the battery being measured, is of course in the present test the resistance of the line or cable.

In the practical execution of the test it would be necessary to short-circuit the galvanometer at the moment when the battery key is depressed or raised, otherwise a violent movement of the needle would be produced by the *static discharge* from the cable.

283. When the battery connections for measuring conductivity resistance are made, as shown by Fig. 96 (page 210), then in order to put the zinc current to line, we should put the cable or line to C and the earth to E. To put the copper to line we can either reverse the battery or put the cable to E and the earth to C, whichever is most convenient to the experimenter.

Measurement of the Conductivity Resistance of a Submarine Cable.

284. When we are measuring the conductivity of a submarine cable, which requires to be carefully done, the following method may be adopted:—

Put on the battery current for half a minute by pressing down the right-hand key (Fig. 96, page 210); at the expiration of that

* Values of $\frac{1}{n-1}$ corresponding to various values of n are given in Table X.

† It has been stated that it is necessary in making this test to arrange the key so that when it is raised it puts in circuit a resistance equal to that of the battery; but it must be evident from the reasoning given in the note referred to that this is altogether unnecessary. M. Emile Lacoine drew attention to this fact in the No. of the 'Journal Télégraphique' for Jan. 25th, 1888.

time, proceed to adjust the plugs, pressing down the left-hand key as required until equilibrium is produced; continue to adjust, if the needle does not remain at zero, and at the expiration of half a minute note the resistance. Now reverse the battery connections, put on the current for half a minute; again measure, again reverse and measure, and so on until about a dozen measurements with either current have been taken. It will usually be found that about half the measurements made with the negative current are the same, and also half the measurements made with the positive current; these results may be taken as the correct measurements for d and d_1.

285. In order to reverse the current through the cable, we can either reverse the battery, or the line and earth, connections (§ 283). There is an advantage in doing the latter, as by this means the galvanometer deflection due to, say, too much resistance being inserted between D and E (Fig. 96, page 210), is always on the same side of zero, although the direction of the current through the cable is reversed Thus it is easy to see at a glance in every case, and without chance of a mistake, whether balance is out in consequence of too much or too little resistance being inserted.

286. The presence of earth currents can be detected when the line, galvanometer, and earth are joined to the resistance box, by pressing down the left-hand key alone. This will cause the galvanometer needle to be deflected if there are any currents present. A line is seldom, if ever, quite neutral in this respect.

287. It is almost immaterial what battery power is used in measuring conductivity; sufficient, however, should be used to obtain a good deflection on the galvanometer needle when equilibrium is not exactly produced. About 10 or 20 cells is a convenient number to employ. There is no danger of heating the resistance coils with such a power if the battery be a Daniell charged with plain water, or even a Leclanché, as their internal resistances are considerable. It would not be advisable, however, to use a Grove or a Bunsen battery, or a Daniell charged with acidulated water, as their heating power is great in consequence of their small internal resistances.

Elimination of the Resistance of Leading Wires.

288. In order to determine the exact resistance of the conductor of a cable, or coil of cable core, for example, it is of course necessary that the resistance of the wires leading from the testing-

room to the tank in which the cable or core is placed, should be deducted from the total measured resistance. This involves a calculation which, although slight, still might be avoided with advantage, especially if a large number of measurements have to be made. At Messrs. Siemens' Works, at Charlton, a very simple device is adopted which enables the resistance of the leading wire to be eliminated, thus rendering any deduction unnecessary. For this purpose a small supplementary slide wire resistance (§ 20, page 17) is connected in the arm A E of the bridge (Fig. 96, page 210); the leading wires (when connected to the bridge) being looped together at their further ends, and all the plugs being inserted in A E, the slide resistance is adjusted till balance is obtained on the galvanometer. The leads are now connected to the cable or core to be tested, and then balance is again obtained on the galvanometer by removing plugs from A E in the usual manner. This being done, the resistance unplugged in A E (allowing for the ratio of the arms A B, B C, of the bridge, if the two are unequal) obviously gives the exact value of the resistance required, since the resistance of the leads is balanced by the slide resistance.

Measurement of Battery Resistance.

289. The resistance of a battery which consists of a large number of cells may in many cases be measured with a considerable degree of accuracy by means of the Wheatstone Bridge, in the following manner:—

Divide the battery into two equal parts, and connect the two halves together so that their electromotive forces oppose one another; under these conditions the battery may be treated as an ordinary resistance, and measured as such.

CHAPTER IX.

LOCALISATION OF FAULTS.

290. The theoretical methods of testing for the localities of faults are comparatively simple, but their practical application presents some difficulties.

Localisation of a full Earth Fault.

√ 291. The simplest kind of fault to localise is a complete fracture where the fault offers no resistance, and the conductivity resistance at once gives its position. Thus, a line which was 100 miles long, and in its complete condition had a resistance of 1350 ohms, that is to say, a resistance of $\frac{1350}{100} = 13 \cdot 5$ ohms per mile, gave a resistance of 270 ohms when broken. Then distance of fault from testing station was

$$\frac{270}{13 \cdot 5} = 20 \text{ miles.}$$

Localisation of a partial Earth Fault.

292. When the fault has a resistance, the localisation becomes somewhat difficult. The following are the theoretical methods generally adopted (Fig. 120).

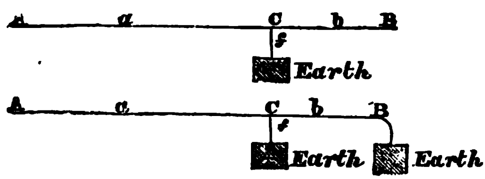


Fig. 120.

√ Blavier's Method.

293. Let A B be the line which has a fault f at C, A being the testing station. A first gets B to insulate his end of the line. He then measures the resistance, which we will call l,

then

$$a + f = l;$$

therefore

$$f = l - a. \qquad [1]$$

Next, B puts his end to earth, and A again measures. Let the new resistance be l_1, then

$$a + \frac{b f}{b + f} = l_1. \qquad [2]$$

Calling L the resistance of the line, we have also

$$a + b = L;$$

therefore

$$b = L - a. \qquad [3]$$

From these three equations we have to determine a. Substituting in [2] the values of f and b obtained from [1] and [3], we get

$$a + \frac{(L - a)(l - a)}{L + l - 2a} = l_1,$$

therefore

$$a^2 - 2 a l_1 = L l - L l_1 - l l_1;$$

from which, since a must be less than l_1 and the root consequently negative,

$$a = l_1 - \sqrt{(l - l_1)(L - l_1)}.$$

For example.

A faulty cable, whose total conductivity resistance when perfect was 450 ohms (L), gave a resistance of 350 ohms (l) when the further end was insulated, and 270 ohms (l_1) when the end was put to earth. What was the resistance of the conductor up to the fault?

$$\text{Resistance} = 270 - \sqrt{(350 - 270)(450 - 270)} = 150 \text{ ohms.}$$

If the length of the cable were 50 miles, then conductivity resistance per mile equals $\frac{450}{50} = 9$ ohms, and distance of fault from testing station consequently equals $\frac{150}{9} = 16\frac{2}{3}$ miles.

Overlap Method.

294. Two measurements are made, one by station A, and the other by station B, A and B insulating their end in turn. Thus resistance measured from A when B insulates, as before is

$$a + f = l. \qquad [1]$$

Resistance measured from B when A insulates

$$b + f = l_2, \qquad [2]$$

also

$$a + b = \text{L}. \qquad [3]$$

Subtracting [2] from [1]

$$a - b = l - l_2,$$

and adding [3]

$$2\,a = \text{L} + l - l_2;$$

therefore

$$a = \frac{\text{L} + l - l_2}{2}.$$

For example.

A faulty cable, whose total conductivity resistance when perfect was 450 ohms (L), when measured from A with the end at B insulated, gave a resistance of 350 ohms (l); and when measured from B with the end A insulated, a resistance of 500 ohms (l_2). What was the resistance of the conductor from A to the fault?

$$\text{Resistance} = \frac{450 + 350 - 500}{2} = 150 \text{ ohms.}$$

Anderson and Kennelly's Method.

295. In making the foregoing test it is often found advantageous to introduce a set of resistance coils at the end of the cable nearest the fault, and to vary this until it is found that the measurements made at the two ends give the same results. The advantage of this arrangement is that if the same amount of battery power be used at the two stations the test current flowing out at the fault will be the same in both cases, consequently the fault is likely to remain constant and more uniform results be obtained. It is obvious that if r be the added resistance, then the resistance from either end (the resistance r being taken as forming part of the cable) will be $\frac{\text{L} + r}{2}$, L being as in previous cases the total conductivity resistance of the perfect cable.

The late Mr. Dresing suggested that the most satisfactory results would be obtained if a Muirhead's artificial cable were used to introduce the resistance r, so that the testing current would (owing to the capacity) flow under practically similar conditions in the measurements made from both ends.

Practical Execution of Tests.

296. So far the testing is simple; the practical application, however, presents some difficulty. This is owing to the variation of the resistance of the fault when the testing current is put to the cable, in consequence of this current acting on the copper conductor, and through the agency of the sea water covering it with a salt, which besides increasing the resistance of the fault also sets up a current opposing the testing current. To make a proper test, then, it is necessary so to manipulate the testing apparatus and battery as to get rid of the polarisation and resistance set up by the salt formed on the fault, and to measure the resistance at the moment this is done. The following is known as:—

Lumsden's Method.

297. The further end of the cable being insulated, the conductor is cleaned at the fault by applying a zinc current from 100 cells for ten or twelve hours, the current being occasionally reversed for a few minutes. A rough resistance test is then made with a copper current.

A positive current is now applied to the cable for about one minute, using two or three cells for every 100 units of resistance which have to be measured. This coats the conductor with chloride of copper.

The cable is now again connected to the resistance coils, and the battery and galvanometer connections made as shown by Fig. 96 (page 210), the zinc pole being to terminal B′ and the copper to terminal E. The cable must be joined to C, and earth to E.

Both keys being depressed the galvanometer needle is carefully watched and plugs inserted and shifted unit by unit, so as to keep the needle at zero; for the action of the negative current is to clean off the chloride of copper, and thereby to reduce the resistance of the fault. At a certain point this decomposition becomes complete, and the needle of the galvanometer flies over with a jerk, showing that the disengagement of hydrogen has taken place at the fault, which enormously increases its resistance. The resistance in the resistance coils at that moment is the required resistance.

The fault being once cleaned by the application of the 100 cells for ten or twelve hours, it is unnecessary on repeating the

measurement (which should always be done) to apply the battery for so long a time; 10 or 20 minutes, or even less, will generally suffice.

When the measurement is made with the further end of the cable to earth, the same process of preparation can be employed.

The rate at which the decomposition of the salts at the fault takes place, depends to a very great extent upon the strength of the current flowing out at the fault; now, if the latter be very near the end at which the test is being made, the resistance between the testing battery and the fault will be so small that the changes at the latter will take place with great rapidity, and it would be a matter of great difficulty to adjust the resistance in the bridge quickly enough to follow up the change of resistance at the fault as it takes place. To avoid this difficulty the best plan is to insert a resistance between the bridge and the end of the cable; this will retard the changes by reducing the strength of the current flowing in the circuit. The value of this resistance will depend entirely upon circumstances, and will be a matter of judgment with the person making the test, but in any case it should not be out of proportion to the actual conductor resistance of the cable.

The amount of battery power used is also a matter dependent upon circumstances, but the higher the power it is found possible to use, the less will the effect of earth currents influence the accuracy of the test.

The resistances employed in the arms A B, B C of the bridge (Fig. 96, page 210), will, to some extent, modify the rate at which the changes at the fault take place, and here again discretion must be used, as no definite rule can well be laid down.

It might be imagined that a "slide resistance" (page 17) would be very advantageous for making a test of this kind, but practical experience shows that the plug resistances are preferable in many cases.

The galvanometer with which this and the following test must be made must be an ordinary astatic one (page 20) with fibre suspended, or pivoted, needles. A Thomson's reflecting galvanometer is quite useless for the purpose.

Before making the test, A must of course arrange with B, or *vice versâ*, at what time and for how long he is to insulate, put to earth, &c., his end of the cable.

Fahie's Method.

298. Mr. J. J. Fahie, in a paper read before the Society of Telegraph Engineers,* has given the results of some very careful experiments and tests which he has made, bearing upon the subject of testing for faults. His method contains many valuable points and is, in the author's words as nearly as possible, as follows:—

The cable-current is eliminated by sending into the line the current of the opposite sign to that coming from it, and arranging the strength and duration of this current to suit the strength of the one from the cable. Thus, if the latter be strong and negative put (say) sixty cells positive to line for a couple of minutes, and then note the condition of the cable current; if it be still negative, but weaker, put the battery on again for a short time, and continue to do so until the galvanometer needle indicates a weak positive current from the fault. If the latter be now left to itself and the cable put to earth through a galvanometer the needle will steadily, and as a rule leisurely, fall to zero and pass over to the other side, indicating a negative current again from the fault. While the needle is on zero the line is free and in a fit state for the subsequent test.

If the cable-current be positive, put sixty cells negative on until the fault is depolarised; the effect in this case is more brief than in the other, the needle falling quickly to zero and crossing to its original position.

Having once eliminated the current from the fault (and the operation very rarely exceeds ten minutes in the most obstinate cases) the cable can always be kept free by momentary applications of the necessary battery pole. Thus, if the needle begin to move off zero in the direction indicating a negative current from the fault, a positive current applied for a moment will bring it back, and *vice versâ*. In practice it is best to repolarise the fault slightly in the opposite direction, as a little time is thereby gained to arrange the bridge for a test.

Having shown how to prepare the cable, the test will now be described. The bridge is arranged as shown by Fig. 121 (page 275).

P is the infinity plug; when this plug is removed the connection between the branch coils *b* and the resistance *d* is severed; K_2 is an ordinary key for putting the line to earth through the

* 'Journal of the Proceedings of the Society of Telegraph-Engineers,' vol. iii. page 372.

galvanometer G_2 or to the bridge as may be required. The rest needs no explanation.

First ascertain by an ordinary test the approximate resistance of the faulty cable and leave it unplugged in *d*. Next allow the line to rest for a few minutes in order that it may recover itself from the effects of the current employed in this preliminary test, and then depress K_2, and observe the cable-current on the galvanometer G_2; let it be positive, open the key K_1, remove the plug P, and send a negative current from the testing battery of (say) sixty cells into the cable *viâ* the branch coils *a*, which should be plugged-in to avoid heating. When the cable current has been repolarised—a fact which may be ascertained by putting the cable

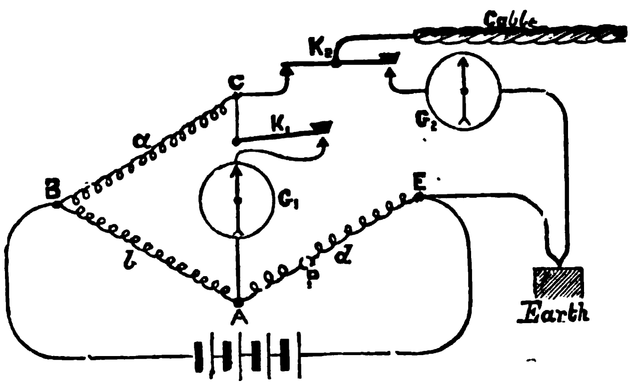


FIG. 121.

to earth at intervals through G_2—arrange the bridge, close to the key K_1, and, keeping the cable to G_2, watch till the needle comes to zero; at that moment let K_2 fly back, and send a negative current through the bridge system, observing the instantaneous effect on the galvanometer G_1. If *d* be too great the needle will be deflected in a direction (say to the right) indicative of this, but immediately after it will rush across zero and up the other side of the galvanometer (to the left), showing that the cable current has again set in. If *d* be too small the needle will pass to the left, at first slowly, but immediately after with a bound. *d* is now adjusted, resistance is inserted or removed as required, and the eliminating process begun again. As *d* more nearly resembles the resistance of the cable, the first and instantaneous deflections after battery-contact become smaller; and, when *d* and the cable

resistance are equal, the needle trembles over the zero-point for a moment, and then rushes over to the left under the influence of the cable current.

Should the current given off by the fault be negative, having arranged the bridge as before, repolarise the fault with a positive battery current, and waiting till G_2 shows the cable free, proceed to test as before, but using a positive current instead of a negative. Should d be too great the needle of G_1 will be deflected in this case, first to the left and then to the right. Should it be too small the needle will move to the right, at first slowly, but immediately after with a rush. The galvanometer G_1 must always be ready, and not short-circuited, else the first and instantaneous deflections after battery-contact will not be perceived.

In practice it is found that when the cable current is positive it is easily eliminated by a negative current, but that when it is negative the operation with a positive current is more difficult. Indeed, it is better not to employ a positive testing current at all except for a moment when it is required to eliminate a weak negative cable current. A positive current applied for a few seconds in this manner has only time to depolarise a fault, but when continued longer it seems to actually coat the exposed wire with badly conducting substances, by which the total resistance is increased.

It will be noticed that when the fault is depolarised by a positive current of any duration it does not recover itself for a long time. If a galvanometer be joined in circuit, its needle will remain at or near zero for a considerable time, occasionally oscillating feebly. The depolarisation by a negative current, on the other hand, lasts only a few moments.

The whole of the foregoing observations do not appear to be applicable to every fault. Thus, when the fault has considerable resistance in itself, or when more faults than one exist, it is not always possible to eliminate the cable current. Again, when the fault possesses resistance, the direction and strength of the cable current, when the distant end is alternately insulated and put to earth, do not always coincide. For example, a fault occurred on a six-mile piece of shore-end cable, which reduced the insulation resistance to about 2000 units absolute. Now, when the further end of this piece was to earth, a strong negative current was often obtained, but when it was insulated the cable current was slight, and positive. Again, when the fault is further off than about 150 miles, and the intervening cable perfect, the charge current interferes with the test.

299. The principal obstacle found in testing for faults is the presence of earth currents. If it were not for these there would really be but comparatively little difficulty in making satisfactory tests. But even earth currents would not create any serious difficulties, provided they kept constant in strength and direction for any length of time; this, however, is unfortunately seldom the case, and it is often only by patient watching that a few seconds can be obtained when the cable is in a quiescent condition, and a test of correct value made.

The earth current difficulty is especially met with in long cables, and it is not uncommon for days to pass without a satisfactory test being made.

Mance's Method.

300. This method, devised by Sir Henry Mance, has for its object the elimination of the effects of an earth current in a cable when making a resistance test. The general principle of this method has been described on page 264. As compared with the ordinary "*Equilibrium Method*" (page 261) it has the advantage that the polarisation current does not become changed, as it is liable to do when reversed currents are sent from the testing batteries; moreover, as the test can be made with a negative current only, the resistance of the fault does not alter materially, as it is liable to do when a positive current is applied.

In making the test practically, Sir Henry Mance considers that the simplest plan and the one giving the best results is to have the resistances a and b (Fig. 119, page 262) of equal value; the 100 and 1000 pairs of proportional coils in the ordinary bridge would be used generally for the purpose. The test is commenced by observing the resistance d, with the smaller pair of coils, *continuing the test until the resistance of the fault appears fairly steady*, when, balance being obtained by adjusting d, the galvanometer is short-circuited for an instant whilst changing the 100 coils to 1000, and then balance is again obtained by readjusting d to d_1. This operation should be several times repeated, and the pair of readings which seem most likely to be correct are then used for determining x from the formula. In working the method care should be taken that the battery is in good condition and that its resistance is not high. If the conductor is not broken *and the fault is a small one*, sufficient resistance should be added at the end nearest the fault to bring the latter near the centre (§ 295, page 271). The tests

from either side will then compare well with each other. In arranging this, the resistance of the batteries must not be overlooked, and it is therefore desirable that all stations should use similiar batteries with approximately the same internal resistance.

When testing with the 1000 to 1000 proportion coils, the observations will generally, but not invariably, be higher than when using the 100 to 100 branches. This will depend on the earth currents existing at the time. The corrected result will, however, be approximately the same, although the readings may indicate an alteration of several hundreds of units in the resistance tested. The daily variations in the tests to a fault may of course be due to alterations in the fault itself, especially if it is a small one. The application of the correction will, however, at once show how much is due to the fault, and to what extent the tests are affected by other disturbing influences. Should the alterations be caused by the latter, there will be no material change in the corrected results.

301. For the purpose of applying the test with ease and certainty Sir Henry Mance has devised a form of bridge specially adapted for the purpose. In this apparatus, which is shown by

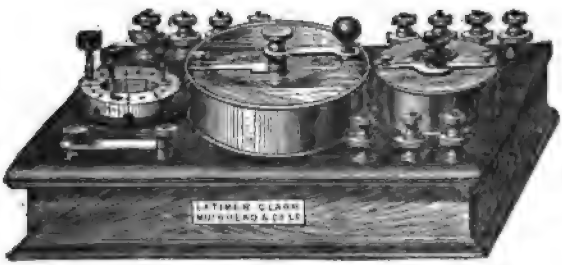


Fig. 122.

Fig. 122, a switch is provided for rapidly changing the proportion coils from 100 to 1000, and *vice versâ;* a set of single ohm slide resistances (page 17) is also added for the purpose of adjusting the main resistance (d and d_1) with rapidity.

Kennelly's Law of Fault Resistance.

302. When a cable which has become broken has its resistance measured in order to determine the locality of the break, the value of this resistance represents the resistance up to the fault

plus the resistance of the fault itself. Now although by Lumsden's method (page 272) it is often possible to nearly eliminate the resistance of the fault, yet this cannot always be done. In a paper read before the Society of Telegraph-Engineers and Electricians,* Mr. A. E. Kennelly has pointed out as the result of numerous experiments, that when the current flowing does not exceed 25 milliampères ($\frac{25}{1000}$ ampère) the resistance of the fault in a broken cable varies inversely as the *square root* of the current passing, this is to say, for example, if we *quadruple* the current we *halve* the resistance. As a consequence of this law, it is shown that it is possible to determine what is the resistance of the cable up to the break, independently of the resistance of the break itself.

Let r be the resistance of the broken cable up to the fault, and f_1 and f_2 the resistances which the fault has when the currents passing are c_1 and c_2 respectively, then by the law stated we have

$$f_1 : f_2 :: \sqrt{c_2} : \sqrt{c_1}$$

therefore

$$\frac{f_1}{f_2} = \frac{\sqrt{c_2}}{\sqrt{c_1}}. \qquad [1]$$

Let R_1 and R_2 be the total measured resistances when the currents c_1 and c_2 are passing respectively, then we have

$$R_1 = r + f_1$$
$$R_2 = r + f_2;$$

therefore

$$\frac{f_1}{f_2} = \frac{R_1 - r}{R_2 - r}, \qquad [2]$$

or

$$\frac{\sqrt{c_2}}{\sqrt{c_1}} = \frac{R_1 - r}{R_2 - r};$$

therefore

$$R_2 \sqrt{c_2} - r\sqrt{c_2} = R_1 \sqrt{c_1} - r\sqrt{c_1},$$

therefore

$$r(\sqrt{c_1} - \sqrt{c_2}) = R_1\sqrt{c_1} - R_2\sqrt{c_2},$$

* 'Proceedings of the Society of Telegraph-Engineers and Electricians,' vol. xvi. page 86.

or

$$r = \frac{R_1\sqrt{c_1} - R_2\sqrt{c_2}}{\sqrt{c_1} - \sqrt{c_2}} = \frac{R_1\sqrt{\frac{c_1}{c_2}} - R_2}{\sqrt{\frac{c_1}{c_2}} - 1}$$

$$= \frac{R_1\left(\sqrt{\frac{c_1}{c_2}} - 1\right) - R_2 + R_1}{\sqrt{\frac{c_1}{c_2}} - 1} = R_1 - (R_2 - R_1)\frac{1}{\sqrt{\frac{c_1}{c_2}} - 1},$$

or if $\frac{c_1}{c_2} = n$, then

$$r = R_1 - (R_2 - R_1)\frac{1^*}{\sqrt{n} - 1}. \qquad [A]$$

For example.

The measured resistance of a broken cable when the current passing was 23 milliampères (c_1) was 435 ohms (R_1), but when the current was reduced to 8 milliampères (c_2) the measured resistance was found to be 445 ohms (R_2); what was the resistance (r) of the cable up to the break?

We have

$$\frac{1}{\sqrt{n} - 1} = \frac{1}{\sqrt{\frac{23}{8}} - 1} = 1 \cdot 44, \text{ and, } R_2 - R_1 = 445 - 435 = 10,$$

therefore

$$r = 435 - 10 \times 1 \cdot 44 = 435 - 14 \cdot 4 = 420 \cdot 6 \text{ ohms.}$$

It is obvious that the values of c_1 and c_2 might be determined by placing a low resistance galvanometer in circuit with the cable whilst the tests are being made, and noting the deflections obtained in the two cases. The strengths of the current could be varied either by changing the battery power or by changing the resistances in the arms of the bridge, as in Mance's test. Mr. Kennelly prefers to adopt the latter method and to calculate the strengths of the current passing, instead of having a galvanometer in the cable circuit as suggested. In order to eliminate the effects of earth currents he balances to a false zero (§ 282, page 266).

* Values of $\frac{1}{\sqrt{n} - 1}$ corresponding to various values of n are given in Table X.

If the relative strengths of c_1 and c_2 are made in the proportion of 4 to 1, that is if $n = 4$, then $\frac{1}{\sqrt{n} - 1} = 1$, so that

$$r = R_1 - (R_2 - R_1) = 2 R_1 - R_2.$$

Schaefer's Law of Fault Resistance.

303. Mr. Schaefer has found, as the result of a very numerous series of experiments, that if proper account be taken of the polarisation and earth current in the cable, and also if the balance on the bridge be taken to *true* (instead of *false*, as in the Kennelly test) zero, that the resistance of the fault should be taken to vary inversely as its 1·3rd root.

Now, if e is the electromotive force of the combined polarisation and earth current, and c_1 is the strength of the current flowing through the cable, also if f_1 is the resistance of the fault with the current c_1, r the conductor resistance of the cable up to the fault, and R_1 the bridge balance resistance to true zero, then

$$R_1 = r + f_1 \pm \frac{e^*}{c_1}, \quad \text{or,} \quad f_1 = \left(R_1 \pm \frac{e}{c_1}\right) - r,$$

also if R_2 and f_2 are the respective resistances with a current c_2, then

$$f_2 = \left(R_2 \pm \frac{e}{c_2}\right) - r.$$

Since by Schaefer's law we have

$$\frac{f_1}{f_2} = \frac{\sqrt[1\cdot 3]{c_1}}{\sqrt[1\cdot 3]{c_2}}$$

* If in a circuit of resistance R a current of strength C is produced by an electromotive force E, then,

$$C = \frac{E}{R}, \quad \text{or,} \quad C R = E. \qquad [1]$$

If into the circuit an additional electromotive force $\pm e$ is introduced and also a resistance R′ is added or taken out, so that the current in the circuit still remains C, then we have

$$C = \frac{E \pm e}{R \pm R'}, \quad \text{or,} \quad C R \pm C R' = E \pm e. \qquad [2]$$

Subtracting [1] from [2], we get,

$$\pm C R' = \pm e, \quad \text{or,} \quad R' = \pm \frac{e}{C};$$

that is, the added or subtracted resistance R′ is an equivalent to the electromotive force e divided by the total current C in the circuit.

we can see by reference to equations [1], [2], and [A], pages 279 and 280, that we must have

$$r = \left(R_1 \pm \frac{e}{n\,c_2}\right) - \left[\left(R_2 \pm \frac{e}{c_2}\right) - \left(R_1 \pm \frac{e}{n\,c_2}\right)\right] \frac{1}{\sqrt[1\cdot 3]{n} - 1}, \quad [A]$$

where

$$n = \frac{c_1}{c_2}, \quad \text{or}, \quad c_1 = n\,c_2,$$

and the $+$ sign being taken for the values of $\frac{e}{c_1}$ and $\frac{e}{c_2}$ when the combined polarisation and earth current is in the same direction as the testing current, and the $-$ sign when the direction is opposed to the testing current.

Equation [A] can be written in the form

$$r = R_1 - (R_2 - R_1) \frac{1^*}{\sqrt[1\cdot 3]{n} - 1} \pm \left[\left(\frac{e}{c_2} - \frac{e}{n\,c_2}\right) \frac{1}{\sqrt[1\cdot 3]{n} - 1} - \frac{e}{n\,c_2}\right], \dagger$$

the term in the square brackets representing the correction due to the combined polarisation and earth current.

The values of c_1 and c_2 are best measured by including between the cable end and the bridge a low resistance milliampèremeter (a "Weston" instrument is very suitable for this purpose). e can be determined by noting the permanent deflection given on the bridge galvanometer, a high resistance being included in the circuit, and then comparing this deflection with one obtained from a standard cell in the place of the cable.

In order that good results may be obtained, the ratio of c_1 to c_2 should not be more than 3 to 1 or less than 2 to 1, and the currents should be strong, i.e. up to about 20 or 30 milliampères.

Rymer Jones's Method.

304. Mr. J. Rymer Jones, as the result of a large number of experiments, found that the resistance, r, up to the fault is very approximately given by the formula

$$r = 2\cdot 557\,R_1 - 1\cdot 557\,R_2,$$

* Values of $\frac{1}{\sqrt[1\cdot 3]{n} - 1}$ corresponding to various values of n are given in Table X.

† Mr. Schaefer gives this equation in the form

$$r = R_2 - (R_2 - R_1) \frac{\sqrt[1\cdot 3]{n}}{\sqrt[1\cdot 3]{n} - 1} \pm \left[\left(\frac{e}{c_2} - \frac{e}{n\,c_2}\right) \frac{\sqrt[1\cdot 3]{n}}{\sqrt[1\cdot 3]{n} - 1} - \frac{e}{n\,c_2}\right].$$

R_1 being the resistance measured with the higher current and R_2 that with the lower current, the balance on the bridge being to *true* and not to *false* zero.

Thus, to take the results given in the example to Kennelly's method, viz.

$$R_1 = 435 \quad R_2 = 425,$$

we get

$$r = 2 \cdot 557 \times 435 - 1 \cdot 557 \times 445 = 419 \cdot 4.$$

Several measurements of R_1 and R_2 should be made in quick succession and the mean of the most regular values taken.

An advantage of the test is that the *difference* between the measured values R_1 and R_2 indicates whether the result may be depended on, or whether, and what, correction is necessary. The weaker the current the greater is the *difference* between R_1 and R_2. This may be ascertained for various exposures by experiments on the particular cable cores in a bucket of sea water. After localisation tests, join up a known resistance—approximately equal to R_1—in circuit with pieces of the cable core having respectively exposures of $\frac{1}{8}$ inch, $\frac{1}{16}$ inch, and also that given by a clean cut. These should each be tested with the same two current strengths as applied to the cable for the real fault. Or a table of *differences* may be made beforehand for these exposures with current strengths of 10 and 5 milliampères. It is found that a newly made conductor exposure, even when unfingered, gives a calculated result rather higher than that obtained when it has been allowed to remain in salt water for some time, or has been already tested with a negative current so as to improve the surface contact with the salt water and approximate in this respect the condition of the broken end of the cable conductor.

In the majority of breaks with core 107 lbs. copper and 140 lbs. guttapercha, the exposure will be sufficient to ensure a true result without any correction, since for exposures of from 1 inch, or over, to $\frac{1}{8}$ inch, the calculated distance is closely the same. If then the *difference* between R_1 and R_2 be no more than that obtained with the experimental core having $\frac{1}{8}$ inch exposure, it will be evidence that the fault resistance is unimportant and that the calculated resistance may be relied on. Should the *difference* be about that obtained with $\frac{1}{16}$ inch exposure (for the before mentioned core), then subtract 5 per cent. from the result, and if the *difference* be as great as for a clean cut the calculated result will be about 30 per cent. too high. In the case of a larger difference still, comparison should be made with smaller experimental exposures.

Very small exposures will be apparent by the gas set free by the higher testing current (10 milliampères), and if the balance be very unsteady from this cause it will be well to open out and clean the fault with a strong *negative* current and then test again.

A *positive* current applied for about one minute before repeating the above test will sometimes give a steadier result. As this, however, reduces the *difference* between B and C considerably without greatly reducing the calculated result, the approximate *difference* with the negative currents, *before applying the short positive current*, should be taken as the more reliable index of the resistance to be subtracted from the calculated distance (in ohms) of a small fault.

With the core referred to (107 lbs. copper and 140 lbs. gutta-percha), and with currents (for R_1 and R_2 respectively) as high as 40 and 20 milliampères, the results obtained for exposures of from 1 to $\frac{1}{16}$ inch do not differ greatly from each other, they are, however, rather too high, whereas 10 and 5 milliampères give very close results; with currents of 10 and 5 milliampères there is, moreover, the important advantage that less gas is evolved at the fault, and balances are therefore much more reliable with the smaller exposures. For these reasons it is recommended to use 10 and 5 milliampères for *all* exposures.

The principle of testing an experimental core fault *with an exposure giving approximately the same difference between* R_1 *and* R_2 *as when testing the cable* and in circuit with coils having a *known* resistance closely equalling that measured up to the fault, and using the same two battery currents as for the cable test, is clearly a good one for obtaining a reliable standard of comparison, because the two tests are made under precisely the same conditions. If, therefore, the experimental core works out correctly the same satisfactory results may be expected for the cable tests; while if the calculated resistance in the case of the experimental fault be too high or too low a corresponding correction will be necessary for the cable fault.

Even though the two current strengths be not 10 and 5 milliampères, and have not a ratio of exactly 2 to 1, it is not material, provided we have this standard of comparison, because any error in the result obtained in the cable fault test will be the same as for the experimental fault test, since the latter being known, it can be added to or subtracted from the distance in ohms of the cable *earth*, as calculated from the formula, thus the true result can be obtained irrespective of disproportionality in the

two current strengths employed and the existence of fault resistance, if any.

The formula does not of course correct for any *actual* resistance which the fault has when the exposure is very small, and which must be distinguished from the *apparent* resistance due to polarisation; for Kennelly's law of *inverse square roots* applies only to clean surfaces and to the measured resistance between the exposed conductor and a large body of water, so that any intervening matter—whether it be a liquid confined in a contracted space or a solid incrustation on the conductor—will vitiate the result, and no formula or test has yet been suggested whereby the conductor resistance can be differentiated from any other resistance (whether fluid or solid) in circuit with it.

For exposures exceeding $\frac{1}{16}$ inch comparison with an experimental fault is unnecessary so far as the apparent fault resistance is concerned, for that is eliminated by the formula, but it is valuable as a check in case the two battery powers are not exactly in the ratio of 2 to 1.

If a preliminary test shows the resistance in circuit to be less than 1000 ohms, then put in circuit with the cable about 1000 additional ohms—to be afterwards deducted from the calculated result—so that the difference in the fault resistance may not materially alter the ratio 2 to 1 of the currents through the fault when measuring R_1 and R_2.

It is important to test first with the higher current and to keep it on until the balance is *steady*, as the resistance will sometimes creep up, especially when testing a heavy core, such as 650 lbs. copper and 400 lbs. guttapercha. A battery commutator is also desirable to change quickly from one current strength to the other.

$$\left.\begin{array}{l}\text{Number of cells for the}\\ \text{higher testing current,}\\ \text{10 milliampères}\end{array}\right\} = \frac{R + r}{100 \cdot e} \text{ cells} \quad \left[\begin{array}{c}\text{Nearest number}\\ \text{divisible by 2}\\ \text{to be used.}\end{array}\right];$$

where

e = voltage of single cell of battery;

R = approximate resistance of cable conductor and fault by preliminary measurement to *false* zero with negative current;

r = resistance in cable branch—viz. 1000 ohms, using proportional parts 1000 : 1000.

Kingsford's Method.*

305. This method is a modification of Blavier's test and its object is, that in both the measurements made, viz. one with the further end of the cable insulated and the second with the further end put to earth, the current flowing out at the fault shall be the same. This is effected by connecting a resistance to the end of the cable nearest to the fault when the first measurement is made. If this resistance be r, then formula [1], page 270, becomes

$$f = l - (a + r),$$

which makes the value of a

$$a = l_1 - \sqrt{(l - l_1 - r)(L - l_1)}. \qquad [A]$$

Now r in this equation requires to be of such a value that the current flowing through the fault in the two measurements is the same; thus if E be the electromotive force causing the current C, we must have

$$C = \frac{E}{r + a + f}$$

and

$$C = \frac{E}{a + \frac{b f}{b + f}} \times \frac{b}{b + f} = \frac{E\,b}{a\,b + a\,f + b\,f};$$

therefore

$$\frac{E}{r + a + f} = \frac{E\,b}{a\,b + a\,f + b\,f},$$

therefore

$$r\,b + a\,b + b\,f = a\,b + a\,f + b\,f$$

or

$$r\,b = a\,f,$$

that is

$$r = \frac{a\,f}{b},$$

but

$$f = l - (a + r),$$

and also (page 270, equation [3])

$$b = L - a;$$

therefore

$$r = \frac{a(l - a - r)}{L - a},$$

* 'Proceedings of the Society of Telegraph-Engineers,' vol. xiv., Dec. 10th, 1885.

therefore

$$Lr - ar = al - a^2 - ar$$

that is

$$r = \frac{a(l-a)}{L}. \qquad [B]$$

What we have then to do is first to make rough tests of l and l_1, giving r any value we like, and then having worked out the value of a from equation [A] (page 286) to insert the value in equation [B], and thus to obtain r approximately. We then make new tests of l and l_1, giving r this calculated value; this (from equation [A]) will give us a nearer its correct value. The corrected value of a thus obtained is then inserted in [B], and r again worked out, and the new value thus obtained is used to make a third test which will enable us to get the value of a closer still. Thus by making a series of tests in this way, we soon arrive at one which gives very approximate results, i.e. results such that when we work out a from formula [A] and then insert it in formula [B], we find that the value of r obtained from this formula corresponds closely with the value of r which we are using in making the test.

A continued experience of this method of testing has shown that it enables very satisfactory results to be obtained.

306. Practice is required before any of the foregoing tests can be satisfactorily made. An artificial line, however, can easily be formed with resistance coils to represent the resistance of the line up to the fault, and a short piece of cable core which has been pierced with a needle for the fault itself. This piece of core should be immersed in a vessel of sea-water, using a piece of galvanised iron plate or wire for an earth. By this means a very fair idea of *some* of the difficulties encountered in testing for faults in cables may be obtained, and good practice made.

It may be added that the accurate regulation and measurement of the strengths of the currents used when localising faults in cables is a matter of very great importance, and the practice of roughly estimating their value from the electromotive force of the testing battery and the various resistances in circuit is quite insufficient to enable accurate results to be obtained. The use of a direct reading galvanometer, or rather milliampèremeter, in the circuit through which the current whose strength should be known is flowing, is an indispensable adjunct to the apparatus used for the localisation of faults. The "Weston" milliampèremeters are very suitable for the purpose.

Jacob's Deflection Method.

307. A disadvantage in using the Wheatstone bridge for measuring the resistances in the foregoing methods is the time it takes to arrive at balance, and the difficulty of seeing what is happening in the way of earth currents, polarisation, &c.; the determination of the resistance by deflection is, however, as rapid a method as can be desired, and allows of continuous observation of the behaviour of the fault. The only requirements for the test are, the battery with a reversing switch,* a Thomson mirror galvanometer with a reversing key,* and a set of resistance coils. The battery, galvanometer, and cable are first joined up in circuit, one pole of the battery and the further end of the cable being to earth; and the galvanometer being shunted by a shunt of very low resistance (a short piece of wire answers well for this purpose). The needle of the galvanometer is turned so that it has a large inferred zero (§ 69, page 73).

The apparatus being thus joined up, the battery is switched on and one of the galvanometer reversing keys depressed so that the needle of the galvanometer turns in the direction necessary to bring the spot of light on the scale; by adjusting the shunt this deflection is brought to a convenient position. The galvanometer reversing key is now released, the battery is reversed by means of its switch, and then the second reversing key of the galvanometer is depressed so that the deflection of the galvanometer needle is in the same direction as it was in the first instance. Since in one case the battery current is in the same direction as the earth current, and in the other case it is opposing it, the two deflections will differ, but by a judicious adjustment of the shunt and of the magnitude of the inferred zero it may be arranged that both deflections come well within the range of the scale, the shunt being the same in both cases. These preliminaries being arranged, the shunt and the zero position must not be altered during the series of tests. A number of deflections are now taken with each current, and by a proper manipulation of the short-circuiting key,* the oscillations of the needle can be checked so quickly that the value of the deflections can be determined within two or three seconds or less after the battery has been switched on: thus the behaviour of the fault can be carefully observed and the reliability of the readings with either current assured without any great difficulty.

* Chapter X.

After the necessary deflections have been determined, the set of resistances is substituted in the place of the cable, and the deflections obtained are reproduced.

Let d_1 and d_2 be the deflections obtained.

Let E and e be the respective electromotive forces of the battery and of the earth or cable current.

Let x be the resistance being measured.

Let R_1 and R_2 be the resistances required to reproduce the deflections d_1 and d_2.

Lastly, let C_1 and C_2 be the currents producing the deflections d_1 and d_2; and let R be the resistance of the battery and shunted galvanometer.

Now when the deflections are taken on the cable we have

$$C_1 = \frac{E + e}{R + x} \qquad [1]$$

and

$$C_2 = \frac{E - e}{R + x}. \qquad [2]$$

When the same deflections are taken with the resistance coils in the place of the cables, then we have

$$C_1 = \frac{E}{R + R_1} \qquad [3]$$

and

$$C_2 = \frac{E}{R + R_2}; \qquad [4]$$

consequently we have

$$\frac{E + e}{R + x} = \frac{E}{R + R_1},$$

or

$$\frac{E + e}{E} = \frac{R + x}{R + R_1},$$

or

$$1 + \frac{e}{E} = \frac{R + x}{R - R_1};$$

therefore

$$\frac{e}{E} = \frac{R + x}{R + R_1} - 1.$$

We also have

$$\frac{E - e}{R + x} = \frac{E}{R + R_2},$$

or

$$\frac{e}{E} = 1 - \frac{R + x}{R + R_2};$$

therefore

$$\frac{R + x}{R + R_1} - 1 = 1 - \frac{R + x}{R + R_2},$$

therefore

$$(R + x)\left[\frac{1}{R + R_1} + \frac{1}{R + R_2}\right] = 2,$$

or

$$x = \frac{2\,(R + R_1)\,(R + R_2)}{(R + R_1) + (R + R_2)} - R \qquad [A]$$

that is to say, x equals the *harmonic mean* of $(R + R_1)$ and $(R + R_2)$, minus R. In fact we have to add R to both R_1 and R_2, take the harmonic mean of the results, and then subtract R from this mean. If R can be made so low as to be negligible, then of course the formula becomes considerably simplified, x being equal to the harmonic mean of R_1 and R_2.

Although R could be determined by a separate measurement and then inserted in the formula, there is no absolute necessity for doing this, since we have actually all the data requisite to determine x without knowing the value of R. From equation [3] we have

$$C_1\,R_1 = E - C_1\,R,$$

and from equation [2]

$$C_2\,R_2 = E - C_2\,R,$$

therefore

$$C_1\,R_1 + C_2\,R_2 = 2\,E - R\,(C_1 + C_2).$$

Also from equation [1] and [2] we have

$$C_1 + C_2 = \frac{2\,E}{R + x},$$

therefore

$$\frac{C_1\,R_1 + C_2\,R_2}{C_1 + C_2} = R + x - R = x.$$

Since the currents C_1 and C_2 are represented by the deflections d_1 and d_2 we have

$$x = \frac{d_1\,R_1 + d_2\,R_2}{d_1 + d_2}, \qquad [B]$$

an equation which is simpler than equation [A] (above) and which does not require R to be known or to be made negligible,

though in order to make the test with the greatest chance of accuracy it is advisable that R should not have a high value, for reasons which have been explained in § 96, page 103.

If, however, equation [B] is made use of it would be necessary to make the zero of the galvanometer some point on, and not off, the scale, otherwise we should not know what are the true values of the deflections d_1 and d_2. By making the zero at the extreme end of the scale the range will be 700 divisions, which will generally enable sufficiently accurate tests to be effected.

Kempe's Loss of Current Test.*

308. In this test, which is shown by Fig. 123, a battery E is permanently connected, through a galvanometer G_1, to one end A of the cable, the further B being connected to earth through a second galvanometer G.

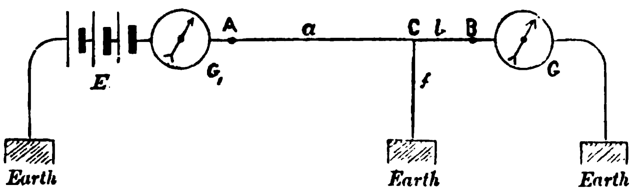


Fig. 123.

Let C_s be the current sent through the galvanometer G_1, and let C_r be the current received on the galvanometer G, then

$$C_r = C_s \frac{f}{f+b+G}, \quad \text{or,} \quad \frac{f}{f+b+G} = \frac{C_r}{C_s}.$$

Let the resistance beyond A be l_3, then

$$l_3 = a + \frac{f(b+G)}{f+b+G} = a + \frac{C_r}{C_s}(b+G);$$

also, as in the previous tests, let

$$a + b = L, \quad \text{or,} \quad b = L - a,$$

* This test was first described by the Author in the second edition of the present work in the year 1881, but it was also independently devised by M. Emile Lacoine and described by him in the 'Bulletin de la Société Internationale des Electriciens,' for April 1886.

then by substitution we get

$$l_3 = a + \frac{C_r}{C_s}(L - a + G);$$

therefore

$$C_s l_3 = C_s a + C_r (L + G) C_r a,$$

that is

$$a (C_s - C_r) = C_s l_3 - C_r (L + G),$$

or

$$a = \frac{C_s l_3 - C_r (L + G)}{C_s - C_r}.$$

If the galvanometer G is shunted, then G in the formula must be the combined resistance of the galvanometer and shunt.

L in this equation is known, it being the conductor resistance of the cable when sound. l_3 is easily determined, when the observations with the cable are completed, by joining up the galvanometer G_1 and battery E in circuit with a set of resistance coils, and then adjusting the latter until the deflection on the galvanometer G_1 is observed to be the same as it was when the cable was in circuit; the resistance in the resistance coils then gives the value of l_3.*

In order to determine C_s and C_r we must compare the deflections they produce on the respective galvanometers, with the deflections obtained on the same galvanometers from a standard current of, say, 1 milliampère.

Supposing both stations are provided with standard dry cells (page 157) of known electromotive force, then each station having noted the deflection obtained when in circuit with the cable, disconnects his galvanometer from the latter, and puts it in circuit with his standard cell, and with such a total resistance as will give a current of, say, 1 milliampère. The deflection is again noted; then this deflection, divided into the deflection obtained when the cable was in circuit, gives the value of C_s or C_r, as the case may be.

For example.

In testing a cable by the foregoing test, the connections being made as in Fig. 123, station A obtained a deflection on his galvanometer equivalent to 2800 divisions; station B obtained a deflection equivalent to 1520 divisions.

The deflection obtained by A on his galvanometer with a

* See § 3, page 1.

standard dry cell of 1·52 volts e.m.f. through a total resistance of 1520 ohms was 100 divisions, and the deflection obtained by station B with a standard dry cell of 1·50 volts e.m.f. through a total resistance of 1500 ohms was 95 divisions; then

$$C_s = \frac{2800}{100} = 28; \quad C_r = \frac{1520}{95} = 16.$$

The value of l_3 was found to be 280 ohms, and the values of L and G were known to be 345 ohms and 5 ohms respectively. What was the value of a?

$$a = \frac{(28 \times 280) - [16 \times (345 + 5)]}{28 - 16} = 186 \cdot 7 \text{ ohms.}$$

If the cable had a conductivity resistance of 10 ohms per mile, then the distance of the fault from A would be

$$\frac{186 \cdot 7}{10} = 18 \cdot 67 \text{ miles.}$$

If a shunt is used on G, then the deflection must of course be multiplied by the multiplying power of the shunt.

A great advantage which this test possesses lies in the fact, that all the necessary observations with the cable can be made simultaneously, station A arranging with station B that at a definite time the observations are to be made on the galvanometers; there is thus no chance of error from the fault changing its resistance between two independent observations, as might occur in the other tests.

It has been assumed that this test has been made with Thomson galvanometers, and it is advisable if possible to employ them; the directing magnets in the instruments would, however, have to be placed very low down and very low shunts employed, otherwise the deflections obtained would be beyond the range of the scale.

309. It will sometimes be found that the cable is traversed by an earth current. The effects of this may be eliminated (as first suggested by Mr. Latimer Clark) by means of a compensating battery of one or two *large*-sized *Daniell* cells, inserted between the end of the cable and the galvanometer. The number of these cells used should be slightly in excess of that required to counteract the earth current, exact balance being obtained by means of a shunt inserted between the terminals of the battery. To effect

this adjustment, previous to putting on the battery E, we should connect the galvanometer to earth, and then adjust the compensating battery shunt until no deflection is obtained. This being done, the battery E is connected up and the test made as if no earth current existed.

It will seldom be found that a larger compensating battery than one or two cells is required to produce a balance, and if these be of a large size their internal resistance may practically be ignored.

It is advisable to make the current from the testing battery flow in the same direction as the current which tends to flow from the compensating battery; thus, if the latter requires to be inserted so that the zinc pole is connected to one terminal of G_1 and the copper pole to the end A of the cable, then the copper pole of the testing battery should be connected to the second terminal of G_1 and the zinc pole to earth.

Best Conditions for making the Test.

The resistances of the battery E and galvanometers G and G_1 should be as low as possible.

The Loop Test.

310. When a faulty cable is lying in the tanks at a factory so that both ends of it are at hand, or when a submerged cable can be looped at the end farthest from the testing station with either a second wire, if it contains more than one wire, or with a second cable which may be lying parallel with it, as is often the case, then the simplest and most accurate test for localising the position of the fault is the loop test.

This test is independent (within certain limits) of the resistance of the fault, thus doing away with the necessity of cleaning and depolarising, as would be necessary in the ordinary tests.

There are two ways of making this test with the form of apparatus hitherto described.

Murray's Method.

311. Fig. 124 shows the theoretical and practical arrangements. p is the point where the two wires or cables are looped together at the further station, f being the fault.

Let x be the resistance from C to the fault, y the resistance

from E to the fault. Then B C being plugged up and A B (b) and E A (d) adjusted until equilibrium is produced,

$$b \times y = d \times x.$$

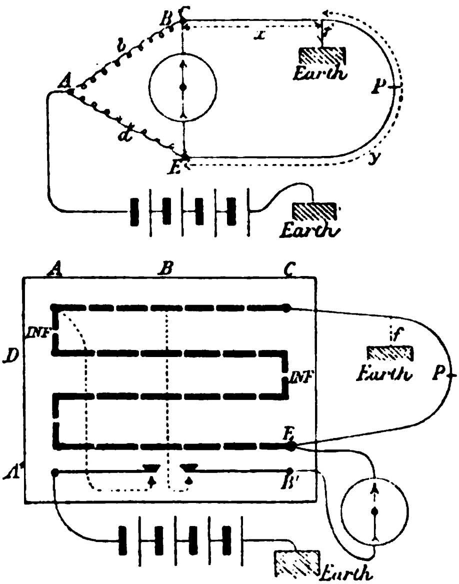


FIG. 124.

Let L be the total conductivity resistance of the whole loop, then

$$x + y = \text{L},$$

therefore

$$y = \text{L} - x;$$

substituting this value of y in the above equation, we get

$$b(\text{L} - x) = d \times x,$$

from which

$$x = \text{L}\,\frac{b}{b + d}.$$

To obtain L, we should simply join up for the ordinary conductivity test, as shown by Fig. 96 (page 210). The fault in this case has no effect upon the test, provided it is not caused by the complete fracture of the cable; in the latter case the broken ends become covered with salts, which would make the resistance

appear higher than it really is. When, however, the fault is due to a simple imperfection in the insulating sheathing, the ordinary conductivity test gives the correct result.

It is advisable to keep a record of the conductivity resistance, so that it can be ascertained without the necessity of making a measurement.

312. In the practical execution of this loop test, the connections being made as shown by the figure, all the plugs between B and C must be inserted; this is necessary, because the galvanometer connection is made on the terminal B′, which is the same as B, instead of on to C. The test could be made by placing the galvanometer on to C, but in that case we should lose the advantage of the key, which it is always best to use.

The plugs being inserted between B and C, and the other plugs being in their places, we should remove, say, the 1000 plug from between A and B, and having pressed down the left-hand key, to put the battery current on, which should be a *zinc* (or negative) one as shown, we should adjust the plugs between D and E, pressing down the right-hand key as required until equilibrium is produced. The different resistances being inserted in the formula, x is found, which being divided by the conductivity resistance per mile of the cable, gives the position of the fault.

For example.

A cable 50 miles long, whose total conductivity resistance was 450 ohms, that is, 9 ohms per mile, was looped with a second cable which had the same length and conductivity resistance as the first cable—the resistance of the loop being $450 \times 2 = 900$ ohms. The adjusting resistance in d to obtain equilibrium was 4000 ohms, b being 1000 ohms, then

$$x = 900\left(\frac{1000}{1000 + 4000}\right) = 180 \text{ ohms.}$$

Dividing this by the conductivity resistance per mile, which is 9 ohms, we get a distance of fault from testing station $= \frac{180}{9}$ $= 20$ miles.

In making a test of this kind it is advisable to use as high resistances as possible in b and d, because the greater these resistances the greater will be the range of adjustment.

313. We know that the best galvanometer to employ would be one whose resistance does not exceed 10 times the joint resistance of the resistances on either side of it.* In practice, the resistance

* Chapter XXV.

b and *d* would always be greater than the resistance of the looped cables, and the joint resistance of the two resistances would consequently never be more than one-half the resistance of the looped cables; if, therefore, we use a galvanometer with a resistance not more than, say, five times the resistance of the looped cables, we may be sure that the conditions are very favourable for making an accurate test.

The value which *d* should have depends upon the value given to *b*, and since the range of adjustment is large in proportion as *d* is large, therefore for this reason it is advantageous to make *b* as large as possible; but it is not advisable to make it higher than is requisite to obtain what may be considered to be a sufficient range of adjustment, for by making *b* and *d* large the current which passes out of the battery becomes diminished, and consequently the effect on the galvanometer will also be diminished. This can of course be compensated for by adding on extra batteries, but as the number of the latter may have to be inconveniently large, it is as well to avoid doing so, otherwise there is no limit to the values which may be given to *b* and *d*.

It is possible to avoid making *b* and *d* high by making the latter resistance adjustable to a fraction of a unit.

If the fault has a very large resistance the employment of high battery power is inevitable, as this high resistance is directly in circuit with the battery. In such a case, however, we may make *b* and *d* as high as we like, for, inasmuch as the current flowing out of the battery depends upon the *total* resistance in its circuit, the result of making *b* and *d* high is to add but very little to the *total* resistance, unless indeed they are very excessive, which in practice can hardly be the case.

To sum up, then, we have

Best Conditions for making Murray's Loop Test.

314. Make *b* as high as is necessary to obtain the required range of adjustment in *d*; if *b* and *d* would in this case require to be excessive compared with the resistance of the loop, *d* must be adjustable to a fraction of a unit.

Employ a galvanometer whose resistance is not more than about five times the resistance of the looped cables.

Employ sufficient battery power to obtain a perceptible deflection of the galvanometer needle when *d* is 1 unit, or a fraction of a unit, out of exact adjustment.

Varley's Method.

315. This is shown theoretically and practically by Fig. 125.

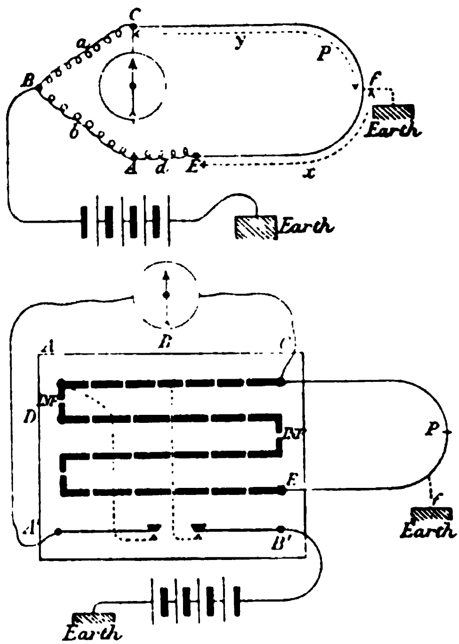


Fig. 125.

In this test B C (a) and A B (b) are fixed resistances, and E A (d) is adjusted until equilibrium is produced. Then, x and y being the resistances of the fault from E and C respectively,

$$a(d+x) = b\,y,$$

and

$$y = \mathrm{L} - x;$$

therefore

$$a(d+x) = b(\mathrm{L} - x),$$

from which

$$x = \frac{b\,\mathrm{L} - a\,d}{b+a}.$$

If $b = a$, then

$$x = \frac{\mathrm{L} - d}{2}.$$

For example.

The two cables being of the same length and conductivity resistance as in the last example, and b being equal to a, equilibrium was obtained by making $d = 600$; then

$$x = \frac{900 - 600}{2} = 150 \text{ ohms.}$$

316. It is necessary that the faulty one of the two looped cables be attached to E, or else it would be impossible to obtain equilibrium. If we were testing a looped cable, and after having joined it up we found that we could not obtain equilibrium, we may be sure that the fault lies between C and p. The cable must then be reversed, and a fresh test made.

317. The conditions for making this test with accuracy are not quite so simple as they were in Murray's test. In this case they are almost precisely similar to what they are in an ordinary bridge test, for the resistance $d + y$ takes the place of the resistance d in the latter test, and if we determine the best conditions for finding x we practically determine the best conditions for finding y, as the test is made in the same manner for determining either quantity.

It is, however, always best to have the relative positions of the battery and galvanometer as indicated in the figure. For if these were reversed the galvanometer would be affected by any earth or polarisation currents which might enter at the fault, and this would render adjustment difficult. We have, then,

Best Conditions for making Varley's Loop Test.

318. Make a as low as possible, but not lower than $\dfrac{g\,x}{g + x}$.

Make b of such a high value that when d is 1 unit out from exact adjustment a perceptible movement of the galvanometer needle is produced.

A rough test would first have to be made to ascertain approximately the values of x and y, and then if necessary the resistances must be readjusted so that the above conditions are satisfied, and then exact adjustment of E A be made.

Best General Conditions for making the Loop Test.

319. Although the loop test avoids errors due to *earth* currents it does not avoid errors due to *cable* currents, that is to say, currents set up by chemical action at the fault itself; this action causes a

current to flow in *opposite* directions through the branches of the cable on either side of the fault, in other words, it causes a current to *circulate* in the loop. This current, although comparatively weak, yet is sufficient to cause errors which it is advisable to avoid if possible. Mr. A. Jamieson states that by balancing to a "false zero" (page 266) the above cause of error may be eliminated and a very considerable increase in the accuracy of the test be obtained.

Correction for the Loop Test.

320. It sometimes happens that the resistance of the fault in a cable approaches the normal insulation resistance of the latter; then the position of the fault indicated by the loop test will not be its true position. The reason of this is, that the current flowing in a faulty cable has two paths open to it: one through the fault and the other through the whole of the insulated sheathing. The cable, in fact, possesses two faults: the actual fault, and the fault due to the conducting power of the insulating sheathing. This second or *resultant* fault, as it is called, in a homogeneous cable is equivalent to a fault in the centre of the cable of a resistance equal to the insulation resistance of the cable itself when in good condition. If the cable is not homogeneous throughout, this resultant fault will lie away from the centre. Its position can be found, however, by the ordinary loop test when the cable is sound.

We have then to determine the true position of the fault when the position and resistance of the resultant fault, the insulation resistance of the cable when imperfect, and the position of the fault indicated by the ordinary loop test, are known. The following shows how this may be done approximately.

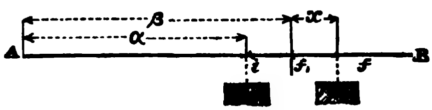


FIG. 126.

In Fig. 126 let A B be the cable joined up for the loop test, f being the actual fault, i the resultant fault, and f_1 the apparent position of the fault given by the loop test.

Let P equal the resistance of i, that is, the insulation resistance of the cable when perfect; also let I equal the insulation resistance when the cable has a fault, which resistance is due to the

joint resistances of the fault (which we will call c) and the insulation P; then

$$\therefore I = \frac{P c}{P + c}; \text{ whence } c = \frac{P I}{P - I}.$$

Now it is evident that the position of f_1 with respect to i and f will depend upon the relative values of P and c: thus if P and c were equal, then f_1 would lie midway between i and f; if P were greater than c, then f_1 would be nearer f; or again, if P were less than c, then f_1 would be nearer i. This being the case we have the proportion

$$P : \begin{pmatrix}\text{distance between} \\ f_1 \text{ and } i\end{pmatrix} :: c : \begin{pmatrix}\text{distance between} \\ f_1 \text{ and } f\end{pmatrix}.$$

Let distance $A f_1 = \beta$ and $A i = a$, therefore distance $i f_1 = \beta - a$; also let distance $f_1 f = x$, then

$$P x = c(\beta - a),$$

or

$$P x = \frac{P I}{P - I}(\beta - a);$$

therefore

$$x = \frac{I}{P - I}(\beta - a),$$

which gives us the position of the true fault beyond the apparent one.

Or the distance of the fault from A will be

$$\beta + \frac{I}{P - I}(\beta - a) = \frac{\beta P - a I}{P - I}.$$

For example.

In a looped cable, whose total length was 100 miles, and total conductivity resistance 900 ohms, the ordinary loop test showed the apparent position of a fault which existed in it to be 700 ohms from A, that is,

$$\beta = 700.$$

The position of the resultant fault given by the loop test when the cable was new was found to be 500 ohms from A, that is,

$$a = 500.$$

The insulation resistance of the cable when new was 3,000,000 ohms, and when faulty 600,000 ohms, that is,

$$P = 3{,}000{,}000.$$
$$I = 600{,}000.$$

Where was the true position of the fault?
Distance of fault from A

$$= \frac{(700 \times 3{,}000{,}000) - (500 \times 600{,}000)}{3{,}000{,}000 - 600{,}000} = 750 \text{ ohms};$$

that is to say, distance of fault beyond distance given by loop test was

$$750 - \beta = 750 - 700 = 50 \text{ ohms}.$$

Or, supposing the cable to have a resistance of 9 ohms per mile, the true distance of the fault beyond the apparent distance would be $\frac{50}{9}$, or $5\frac{5}{9}$ miles.

If the cable be homogeneous throughout, the resultant fault will appear in the middle of it. In this case a will equal $\frac{L}{2}$, where L is the total length of the loop.

If we write the equation,

$$\text{Distance of fault from A} = \frac{\beta P - a I}{P - I},$$

in the form,

$$\text{Distance of fault from A} = \frac{\beta - a\frac{I}{P}}{1 - \frac{I}{P}},$$

we can see that if P is very large then $\frac{I}{P} = 0$, in which case we get

$$\text{Distance of fault from A} = \beta,$$

as in the ordinary loop test

321. To make the foregoing test satisfactorily, it is necessary to know what are the insulation resistances of the cable when good and also when faulty, at the moment when equilibrium is obtained. Now, as will be shown in Chapter XV., the insulation resistance (P) of a sound cable alters in proportion to the time a current is kept on it; but the rate at which this alteration takes place is definite, and can be obtained by reference to previous tests

of the cable made when the latter was sound. The insulation resistance (I) of the cable when faulty cannot, however, be determined by any reference to previous tests; some plan of enabling it to be measured accurately is therefore necessary.

A method suggested by the late Mr. S. E. Phillips enables this to be done in a very satisfactory manner. The whole of the testing apparatus is carefully insulated by being placed on a sheet of ebonite, or on insulated supports; the experimenter also stands on an insulated stand or a sheet of ebonite. The battery for making the loop test, instead of being connected directly on to the terminal of the resistance coils, is connected thereto through the medium of a second galvanometer. By noting the deflection on the latter at the moment equilibrium is obtained on the first galvanometer, and comparing it afterwards with the deflection obtained through a known resistance, we obtain the value of I plus the combined resistance of the resistances in the bridge, which quantity will, however, be insignificant compared with I, and need not be taken into account.

A note should be made of the time at which the battery is connected to the instruments, and then, when the plugs are adjusted, equilibrium obtained, and the deflection on the second galvanometer observed, the time must again be noted, so that the period during which the battery current has acted may be known and the value of P correctly obtained.

The method of determining the value of P will be considered hereafter.

Individual Resistance of Two Wires by the Loop Test.

322. The late Mr. S. E. Phillips pointed out that the loop test may be made very useful for determining the individual resistance of two wires, the leads in a cable factory for instance, whose ends cannot be got at to connect to the testing apparatus.

To do this, the further ends of the leads would be joined together, and the junction put to earth. It is evident, then, that the loop test applied to the wires would give the resistance of either of them to their junction.

(For other methods of localising faults, see Chapters XVII., XXIII., and XXIV.)

CHAPTER X.

KEYS, SWITCHES, CONDENSERS, AND BATTERIES.

SHORT-CIRCUIT KEYS.

323. ALTHOUGH the short-circuit plug-hole is convenient to avoid accidental currents being sent through the galvanometer when the various resistance coils, batteries, &c., are being joined up for making a measurement, yet a key which in its normal condition short circuits the galvanometer, is extremely useful.

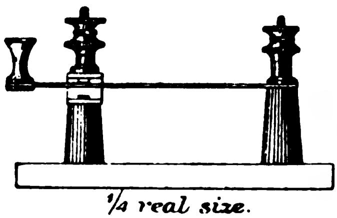


FIG. 127.

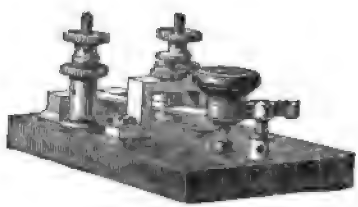


FIG. 128.

Such a key is represented by Fig. 127. In its normal condition the spring rests against a platinum contact, and, when pressed down, against an ebonite one.

The two terminals of the shunt are connected to the terminals of the key; these terminals in this and most keys are double, so as to enable the wires leading to the resistance coils, batteries, &c., to be conveniently connected to them.

If it is required to keep the key pressed down for a lengthened period, a small piece of sheet ebonite or gutta-percha can be slipped in between the contacts, so as to prevent their making connection when the finger is taken off the key. Some keys of this kind are provided with a catch (Fig. 128), which keeps the spring down when it is depressed.

The advantage of the short-circuit key over the short-circuit plug may not seem obvious, but actual practice will soon show its value.

Reversing Keys.

324. Besides the short-circuit key, a *Reversing Key* is usually inserted in the galvanometer circuit, so that the deflections of the needle may always be obtained on the same side of the scale. A form of reversing key very commonly used is shown in elevation and plan by Figs. 129 and 130, and in general view by Fig. 131.

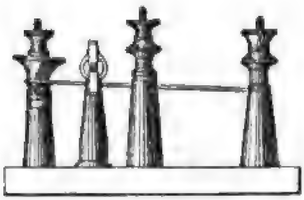
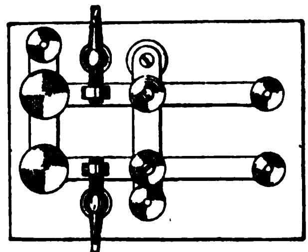


Fig. 129.



¼ real size.

Fig. 130.

The galvanometer terminals would be connected to the two end terminals of the reversing key, or, if the short-circuit key is inserted, to the terminals of the latter. By pressing down one or other of the springs, the current will pass through the galvanometer in one direction or the other. The two handles on either side of the two springs are for the purpose of clamping either of the latter down when required.

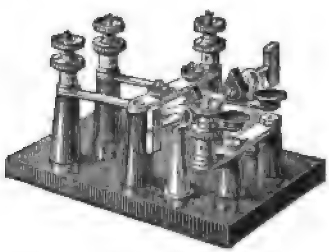


Fig. 131.

Particular care should be taken, when procuring the key, to see that the terminals, &c., are not fixed on the top of the ebonite pillars by means of bolts running right through them, as in such a case the advantage of the pillars is entirely lost, and the terminals might just as well be screwed direct into the base-board.

Care should also be taken that the contacts of the keys are clean, as when there are several contacts considerable resistance might be introduced into the circuit from their being dirty.

325. It is sometimes found in this form of reversing key that the springs fail to make the necessary contact when clamped,

owing to the loosening or wear of the cam employed to hold them down. *Pell's Patent Self-locking Key*, which is shown by Fig. 132, and which was designed by Mr. B. Pell, of the firm of Messrs. Johnson and Phillips, entirely overcomes this difficulty by dispensing with the cam altogether, and introducing a spring latch

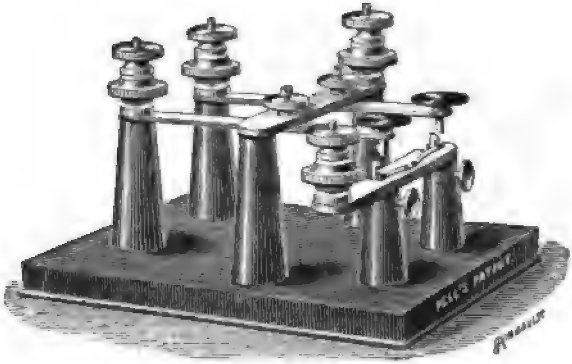


FIG. 132.

which, when the key is depressed, automatically catches and holds it with certainty in position until it is released, the movement, either in depressing the key or in releasing it, being effected with one hand. Each latch is released by pressing the corresponding ebonite knob on the insulating pillar, as shown in the figure.

The other advantages of this key over the old form, although not of so much importance, will be appreciated by all who take a pride in the appearance of their apparatus. The absence of the cams and their supporting pillars, besides improving the insulation, and allowing of the key being more easily cleaned, makes it look neater, and prevents the lacquered surface of the brasswork being disfigured, as is invariably and unavoidably the case when the cam is used.

A *Short-circuiting Key* is also made on the same principle, the spring in this case being somewhat stronger to prevent unintentional locking when the key is only gently tapped by the finger.

326. The foregoing forms of keys, although in very general use, are not in all respects satisfactory, for, unless the contacts are frequently looked to and kept clean, they fail to make proper connection and cause considerable trouble. To obviate these defects the forms of keys shown by Figs. 133 and 134 have been designed; the former by Mr. J. Rymer Jones, and the latter by Dr. Muirhead.

The great advantage of these keys lies in the fact that all the contacts are made by rubbing, thus ensuring great certainty of action, a point of very great importance in some tests.

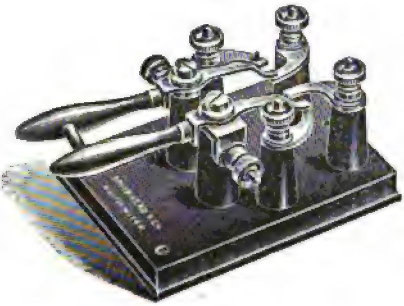


Fig. 133.

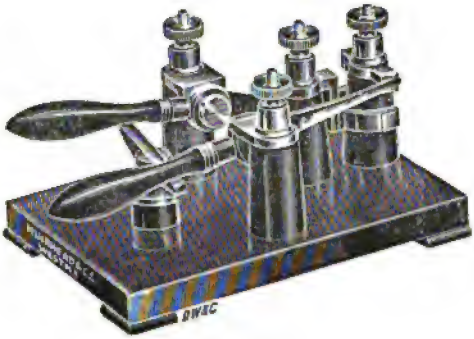


Fig. 134.

Reversing Switches.

327. In addition to the reversing key for the galvanometer, a *Reversing Switch* for the testing battery is very useful: it need not, however, be such an elaborate one as that used for the galvanometer.

Figs. 135 and 136 represent such a switch. It consists of four brass segments screwed firmly down to an ebonite base. Each segment is provided with a screw, to which to attach the testing wires.

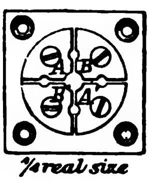


Fig. 135.

Fig. 136.

In some cases each segment is supported on an ebonite pillar, which improves its insulation very much, and, indeed, would be absolutely necessary for some tests which will be described.

The poles of the battery would be attached to two opposite terminal screws, say A and A′, and the leading wires to the two other screws, B and B′.

To make the current flow in one direction, we should place the plugs between the segments A and B, and A′ and B′, and to make it flow in the other direction, between the segments A and B′ and A′ and B. If one or both the plugs are removed the battery current will be cut off altogether. It is always best, in order to do this, to remove both the plugs in preference to one only, for if the battery is not well insulated a portion of the current may still be able to flow out of the battery and disturb the accuracy of a test.

Two other pieces of apparatus are necessary to form a very complete set, viz. a "Condenser" and a "Discharge key."

Condensers.

328. A *Condenser* is merely a Leyden jar exposing a large surface within a small space; those constructed of a standard value for testing purposes are made of sheets of tin-foil placed in layers between thin sheets of mica coated either with shellac or with shellac and paraffin wax. The alternate layers of tin-foil are connected together, so that sets are formed corresponding to the outside and inside coatings of a Leyden jar.

A very convenient and usual form of standard condenser is shown by Fig. 137. In this pattern the layers of tin-foil and

Fig. 137.

mica are placed in a round brass box with an ebonite top, on which are fixed the connecting terminals. These terminals are placed on brass blocks, the ends of which are in close proximity to one another, so that a plug can be inserted between them for the purpose of enabling the apparatus to be short-circuited. This should always be done when the condenser is not in use, so that any residual charge which may remain in it may be entirely dissipated.

Fig. 138 shows an improved form of standard condenser as arranged by Dr. Muirhead. In this pattern the capacity is divided into two halves, which can be separated by withdrawing the plug at B; each half can then be tested one against the other, and if

Fig. 138.

the *relative* values of the two are found to have remained unaltered, it is presumptive evidence that the total of the whole condenser has not changed.

These condensers have usually a fixed "electrostatic capacity" equal to $\frac{1}{3}$ microfarad, the "farad" being the unit of electrostatic

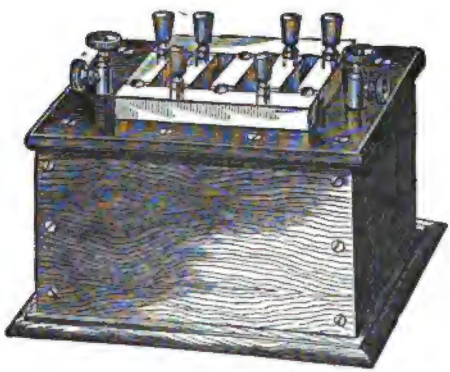


Fig. 139.

capacity. They are also made, however, so that several capacities can be obtained, by inserting plugs in different holes. Those having five different capacities (Fig. 139), viz. ·05, ·05, ·2, ·2, and ·5 microfarads, enable any value from ·05 to 1 to be obtained

by inserting one or more plugs. It is often useful to be able to vary the capacity, so that it is better to have the last form rather than that shown by Fig. 137, although it may be a little more expensive.

Fig. 140 shows another form of a divided condenser arranged in a brass box.

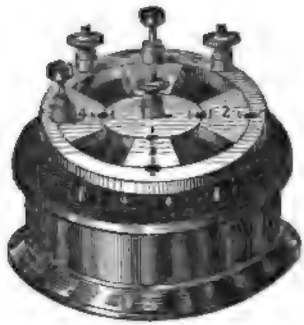


FIG. 140.

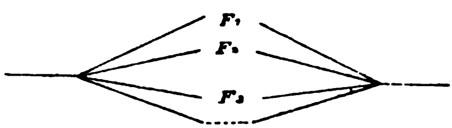
329. A good condenser should not lose, through leakage, more than 1 per cent. of its charge in one minute.

330. Condensers, like batteries, can be combined in "multiple arc" or in "series," and advantage may often be taken of this power of combination to obtain a large number of capacities from a small number of condensers.

When condensers are connected together in "multiple arc" the capacity of the combination will be equal to the sum of the respective capacities of the several condensers. Thus, if we call F_1, F_2, F_3, &c., these capacities, then the capacity of the combination will be

$$F_1 + F_2 + F_3 + \dots$$

This may be expressed symbolically thus:—



When the combination is made in "series" (corresponding to the "cascade" arrangement of Leyden jars) the joint capacity

of the series follows the law of the joint resistance of parallel circuits,* thus:—

$$\frac{\cdot\ 1}{\frac{1}{F_1} + \frac{1}{F_2} + \frac{1}{F_3} \times \ . \ . \ .}$$

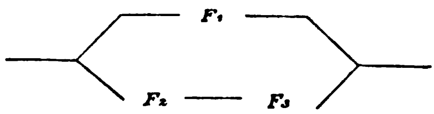
This may be symbolically expressed thus:—

$$\text{———} F_1 - F_2 - F_3 \text{———} \ . \ . \ . \ . \text{———}$$

By following out these laws, if we had two condensers, F_1 and F_2, we could obtain four different capacities, viz. F_1, F_2, $F_1 + F_2$, and $\frac{F_1 F_2}{F_1 + F_2}$.

With three condensers we could obtain no less than seventeen different capacities, viz. F_1, F_2, F_3, $F_1 + F_2$, $F_1 + F_3$, $F_2 + F_3$, $F_1 + F_2 + F_3$, $\frac{F_1 F_2}{F_1 + F_2}$, $\frac{F_1 F_3}{F_1 + F_3}$, $\frac{F_2 F_3}{F_2 + F_3}$, $F_1 + \frac{F_2 F_3}{F_2 + F_3}$, $F_2 + \frac{F_1 F_3}{F_1 + F_3}$, $F_3 + \frac{F_1 F_2}{F_1 + F_2}$, $\frac{(F_1 + F_2) \times F_3}{F_1 + F_2 + F_3}$, $\frac{(F_1 + F_3) \times F_2}{F_1 + F_2 + F_3}$, $\frac{(F_2 + F_3) \times F_1}{F_1 + F_2 + F_3}$, and $\frac{1}{\frac{1}{F_1} + \frac{1}{F_2} + \frac{1}{F_3}}$.

Any of these combinations may be expressed symbolically in the manner before shown, thus, for example, to take the $F_1 + \frac{F_2 F_3}{F_2 + F_3}$ combination, this would be shown thus:—



Discharge Keys.

331. To enable the discharge from the condenser to be read on a galvanometer a discharge key is necessary, This, like the other pieces of apparatus, is made in a variety of forms.

* See Chapter XXVII.

Webb's Discharge Key.

332. Fig. 141 shows a pattern (designed by the late Mr. F. C. Webb) which is in very general use.

It consists, primarily, of a hinged lever of solid make, pressed upwards by a spring and playing between two contacts. A vertical ebonite lever, hinged at its lower end, is fixed to the base of the instrument in the position shown. This lever has near its upper end a projecting brass tongue, which, when the lever is pressed forward (by means of a spring), hitches over the extremity of the brass lever. The end of the latter is cut away so as to form two steps; when the brass tongue on the vertical ebonite lever is hitched over the lower step then the brass lever stands intermediate between the top and bottom contacts, and is insulated from both of them, but when the tongue is hitched over the top step then the brass lever is in connection with the lower contact. Again, when the ebonite lever is drawn back the brass lever is freed and springs up against the top contact step. If we suppose the brass lever to be hitched down on the lower contact step, then by pulling back the ebonite lever a little the brass tongue unhitches from the top step and hitches on the lower one, thus allowing the brass lever to spring up from the bottom contact, but not to come in connection with the upper one; if, however (as before explained), the ebonite lever be pulled completely back, then the brass lever rises in connection with the top contact.

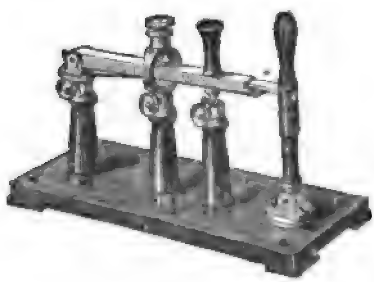


Fig. 141.

333. When using this discharge key for the purpose of measuring the charge in a condenser, the connections to the galvanometer, &c., would be made as shown by Fig. 142 (page 313). On pressing down the key K_2 the two poles of the battery are put in connection with the two terminals A and B of the condenser C, and on releasing the key so that it comes in contact with the top contact, the two terminals of the condenser are put in connection with the two terminals of the galvanometer, which thus receives the discharge current through it.

If we so arranged the connections that the top contact of the key, instead of being joined to the condenser through the galvano-

meter, is connected directly to it, and the galvanometer is placed between the back terminal of the key and the second terminal of the condenser, then on pressing down the discharge key we get the current charging the condenser through the galvanometer, whose needle will be deflected to one side of the zero point; and then, on releasing the key, we get the discharge deflection, which

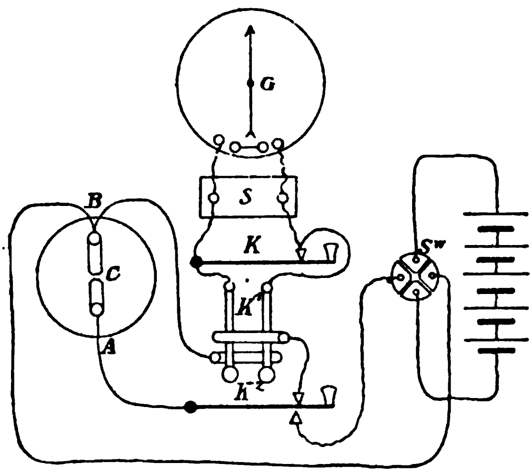


FIG. 142.

will be of the same strength as the charge deflection, but in the opposite direction to it. The first arrangement, given by the figure, is, however, the one generally employed.

The discharge deflection on the galvanometer is only momentary, the needle or spot of light immediately returning towards zero.

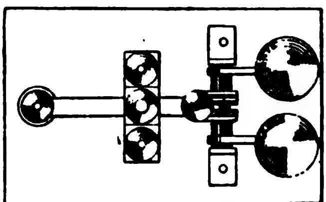
334. In using the Thomson galvanometer (which is practically the only instrument of any use for the purpose) for measuring the discharge, the adjusting magnet must be put high up if it is placed with its poles assisting the earth's magnetism, or low down if it opposes it, so as to make the needle swing slowly enough to enable the deflection to be read on the scale. It is best to avoid making the needle swing very slowly, for then the spot of light will probably not return accurately to zero, but may be three or four divisions out. A little practice will enable a comparatively quick swing to be read to half a division, or even less.

335. If a resistance of 250,000 ohms (more or less, according to circumstances) be introduced into the galvanometer circuit, the

discharge from the condenser becomes so slowed that the swing of the galvanometer needle, instead of being due to a sudden impulse, is caused by a continued but rapidly decreasing current; this causes the needle to swing with a slow movement, although the instrument is adjusted, as regards the controlling magnet, to normally swing with a comparatively rapid rate of vibration. By this device, therefore, a slow swing, which greatly facilitates ease of observation, may often be able to be obtained: unless, however, the insulation of the condenser or cable being measured is very high, the arrangement does not give satisfactory or reliable results, since the discharge, instead of wholly going through the galvanometer, is partly lost by leakage through the low insulation resistance.

Kempe's Discharge Key.

336. A form of discharge key designed by the author is shown in plan and elevation by Figs. 143 and 144, and in general view by



¼ real size

FIG. 143.

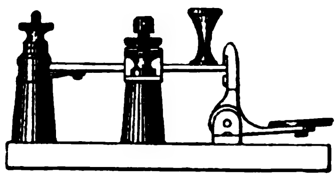


FIG. 144.

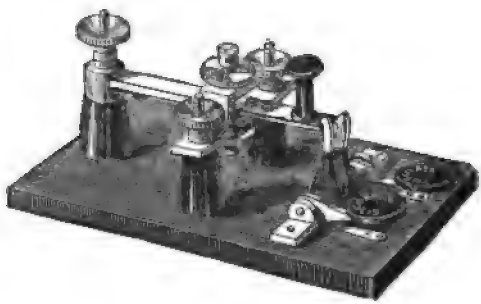


FIG. 145.

Fig. 145. It consists, like Fig. 141, of a solid lever, hinged at one end, and playing between two contacts attached to two terminals. Two finger triggers, near the other end of the lever, marked

"Discharge" and "Insulate," are connected to two ebonite hooks. The height of the hook attached to the finger trigger marked "Discharge" is a little greater than that of the other hook, so that the lever stands intermediate between the two contacts when it is hitched against it. When the lever is pressed down against the bottom contact, the shorter of the hooks hitches it down. If in this position we depress the "Insulate" trigger, the lever is freed from its hook, and springs up against the second hook, thus insulating the lever from either of the contacts. The "Discharge" trigger now being pressed down, the lever springs up against the top contact.

To the hook of the "Discharge" trigger there is a small piece of metal fixed which is broad enough to come in front of the second hook, so that if the "Discharge" trigger is depressed first it draws back both the hooks, and thereby, if the lever at starting be hitched to the bottom contact, allows the lever at once to spring up to the top contact. If, however, the "Insulate" trigger be depressed, only the hook attached to that trigger is drawn back, allowing the lever to spring up against the second hook and be thereby insulated, as at first explained.

Lambert's Discharge Key.

337. The arrangement of discharge key designed by Mr. Lambert and shown by Fig. 146, is a very good one, and possesses the advantage that the principal terminal is highly insulated when the key is in its normal condition, a point of importance in some tests. The two terminals seen at the front part of the key cor-

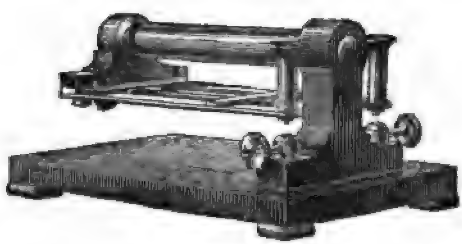


FIG. 146.

respond to the top and bottom contacts of the keys previously described. The ends of two spring levers, provided with ebonite finger-knobs, are set over these contacts; the other ends of the springs are fixed to a brass cross-piece provided with a terminal,

the cross-piece being secured to an ebonite bracket fixed at the end of a stout ebonite rod. By this arrangement the terminal connected to the spring levers is insulated by the long ebonite rod as well as by the ebonite bracket by which the rod is supported on the stand. In manipulating the key, the left-hand lever, say, is first depressed, thus putting the back terminal in connection with the contact (corresponding to the *bottom* contact of the other forms of keys) beneath it. The lever is then released, and the right-hand lever depressed, thus putting the back terminal in connection with the contact (corresponding to the *top* contact of the other keys) beneath it. The only objection to this form of key is the fact that it is possible to press both levers down at once, thus connecting together the back and the two front terminals; if this is done accidentally, then, as will be seen by reference to Fig. 142 (page 313), a direct circuit is formed by the battery through the galvanometer, which may result in the sensibility of the latter being altered through the violence of the deflection. Such an accident obviously cannot possibly occur in the other forms of keys.

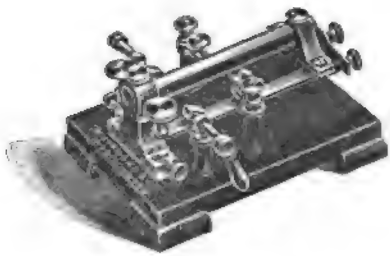


FIG. 147.

The Lambert key is often provided with cams similar to those shown in Figs. 129, 130, and 131 (page 305), so that the spring levers can be clamped down if desired. This latter pattern is shown by Fig. 147.

Rymer Jones's Discharge Key.

338. An excellent form of discharge key has been devised by Mr. J. Rymer Jones, and is manufactured by the India Rubber, Gutta Percha, and Telegraph Works Company of Silvertown. The key is so constructed that (like Lambert's key) the principal terminal is left perfectly free during the period of "insulation," as shown in Fig. 148; the leakage from this terminal is therefore confined to the ebonite support A B. The form of this support,

a vertical section of which is shown by Fig. 149 (page 318), gives a very considerable length of surface over which any leakage must pass, it being in the present case $6\frac{3}{4}$ inches in a height of only $2\frac{1}{2}$ inches; while, since the portion A screws into the outer cap B, the former may be removed, when important tests are about to be made, and scoured with glass-paper, so as to secure the advantage of a fresh surface without disfiguring the outer polished surface.

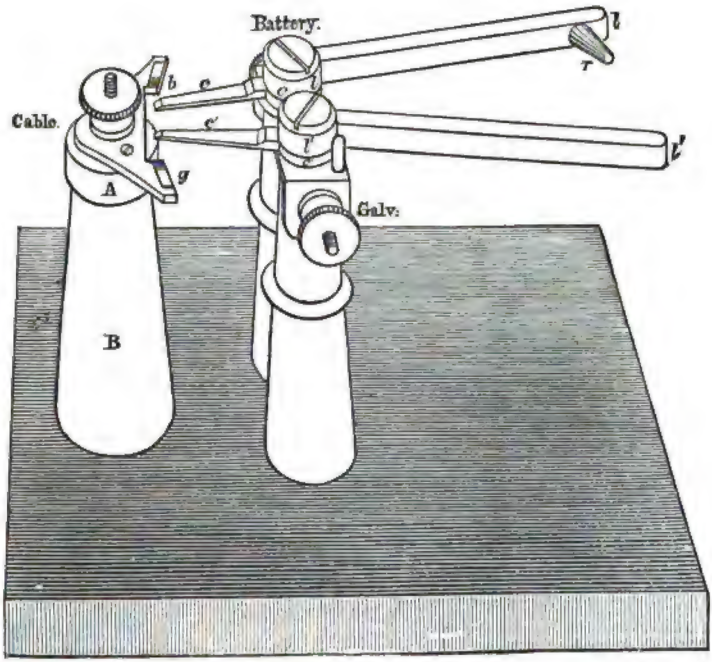
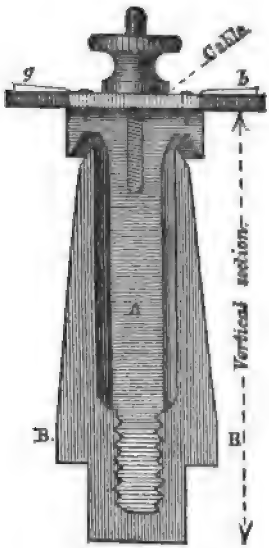


FIG. 148.

The movements for "Charge," "Insulate," and "Discharge," will be readily understood from Fig. 148. *l l'* are ebonite rods; their brass prolongations *c c'*, which move with them as one piece, have the under surfaces, where they rub against the platinum contacts *b* and *g*, tipped with platinum.

When *l* is deflected to the left, the end of the rod *r*, attached to it, presses against *l'*—should the latter happen to be turned to that side—and carries it over in the same direction, first breaking contact at *c' g*, if previously made, and afterwards making contact

at b c. Thus the "Battery" and "Cable" terminals are connected together, To "Insulate" the "Cable" terminal it is only necessary to move l back again towards the right, as in Fig. 148. To "Discharge," press l' towards the right. Should l not already be over to the right (as in the last position for "Insulate") it will be carried over with l' and the contact at b c broken before c' and g come together. The rod r in fact prevents the galvanometer and battery terminals from both being put to the cable terminal at the same time.



Length of insulating surface, 6¾ *inches.*

FIG. 149.

339. Although not perhaps absolutely necessary, it is advisable to have a second set of resistance coils (which need not, however, be of the bridge form) to act as an adjustable shunt for the galvanometer.

340. A simple form of galvanometer to enable the resistance of the Thomson to be quickly taken, is also useful. This, however, can be dispensed with, as the late Mr. S. E. Phillips pointed out that the resistance of the galvanometer can be determined by the very simple device of measuring the resistance of one of the shunts (the $\frac{1}{9}$th preferably). To do this, the shunts will have to be removed from the galvanometer and connected up to the bridge as an ordinary resistance, the galvanometer itself being used in the usual manner.

Mr. Phillips suggested that the shunts should be enclosed under the glass shade, so as to ensure that they may have the same temperature as the galvanometer coils.

As it is preferable to use a set of resistance coils as a shunt, a single resistance coil of the same wire and resistance as the galvanometer coils might be permanently fixed to the galvanometer stand under the glass shade: the resistance of this, measured by the help of the galvanometer, would at once give the resistance of the latter. If such a device were adopted, care would have to be taken that the coil is wound double on its bobbin, for otherwise it would affect the galvanometer needle when traversed by a current.

341. The form of bridge coil most generally employed with the Thomson galvanometer is that shown by Figs. 4 and 5 (page 13), the keys attached to the other form not being used.

BATTERIES.

342. Besides the foregoing instruments, a battery having an electromotive force of at least 250 volts is necessary. The form known as the Minotto has been much used for testing. It consists of an earthenware (or more frequently of a gutta-percha) jar, about 8 inches high, at the bottom of which is placed a round plate of copper, resting flat. A strip of copper about three-quarters of an inch wide, coated with gutta-percha, is fixed to this plate, and brought up the side of the jar. Over this plate a layer of coarsely powdered sulphate of copper is placed; the jar is then filled nearly to the top with damp sawdust, and resting on this is placed a thick disc of zinc, provided with a terminal at the top. A series of these cells is coupled up in the ordinary manner.

The Leclanché battery is now generally used at cable factories; it has the advantage of high electromotive force, and if care is taken that it does not become accidentally short-circuited through a low resistance it answers very satisfactorily, and requires but little attention. The Chloride of silver battery of Mr. Warren de la Rue has also been used to some extent for testing, especially on board ship, as it has the advantage of great compactness and portability.

"Dry" cells are now largely used for testing purposes : it is a mistake to have them of a very small size.

The batteries should be placed on well-insulated supports, in a dry situation, so as to avoid leakage, which interferes with their constancy and causes great trouble when an insulation test is being made.

A high resistance (of known value) may with advantage be permanently placed in circuit with the battery so as to prevent accidental short-circuiting which, if it takes place, renders the battery unsteady for a time.

343. Besides the large battery, a single cell placed in a small box, with appropriate terminals outside, may be required, whose use will be explained.

CHAPTER XI.

MEASUREMENT OF POTENTIALS.

344. LET E (Fig. 150) be a battery of which A and B are the *free* poles; then the free electricities at those poles will have equal but opposite potentials, and the difference of these potentials is the electromotive force of the battery. Thus, if V (that is the line P A) is the potential at A, then − V (that is the line B Q) will be the potential at B, and the electromotive force E of the battery will be

$$E = V - (-V) = 2\,V.$$

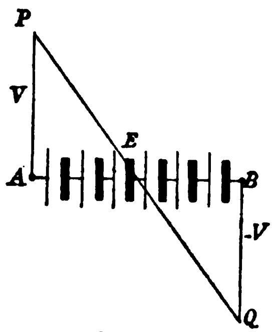


FIG. 150.

Although the expression "potentials of the *free* electricities" is, strictly speaking, more correct than "potentials" simply, yet the latter is generally used as an abbreviation of the former, and we shall so use it unless the contrary is indicated.

The potentials diminish regularly from one pole of the battery to the other, the potential at the middle of the battery being zero.

345. If the two poles are connected by a resistance A C B, as in Fig. 151 (page 321), then the potentials will diminish regularly along A C B also, as shown in the figure, the potential at the middle (F) being zero as in the case of the battery. But the potentials at A and B will be less than they were previous to the joining of the poles by A C B, the amount of the diminution being dependent upon the value of the resistance A C B, and also upon the value of the resistance of the battery. These diminished potentials may be represented by, say, the lines p A (+ v) and B q (− v), respectively.

346. If the two terminals of a condenser are connected to any two points in the resistance, the electromotive force of the charge which the condenser will take will be directly proportional to the

difference of the potentials, that is to the electromotive force, at those two points. Thus if the condenser were connected to A and B the charge it would take would have an electromotive force, E_1, of

$$E_1 = v - (-v) = 2v.$$

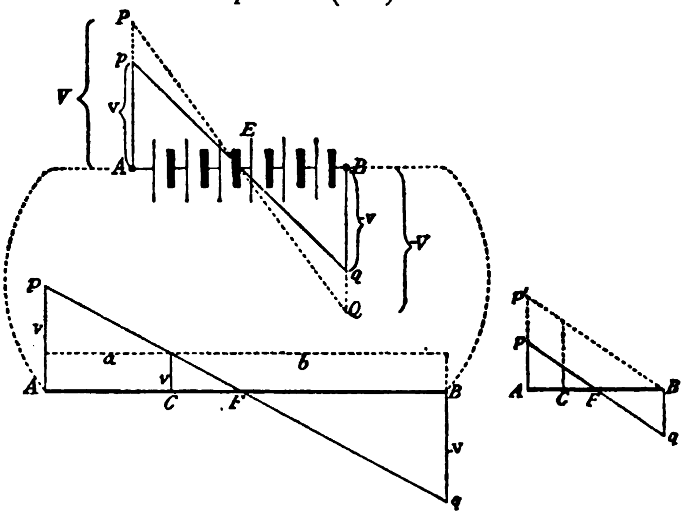


FIG. 151.

If the points to which the condenser is connected were A and C, the electromotive force, E_2, of the charge would be

$$E_2 = v - v.$$

Again, if the condenser were connected to C and B, the electromotive force, E_3, of the charge would be

$$E_3 = v - (-v) = v + v.$$

It is easy to see that

$$v - v : v + v : : a : b.$$

If, therefore, we connect two condensers between the points A and C and the points B and C respectively, and adjust the resistances a and b, we could charge the condensers to any relative electromotive forces we please.

347. Although, strictly speaking, the diminution or fall of potential along the resistance A C B is represented by the line p F q (see small figure), the zero point being at F, yet we may generally with perfect correctness assume the zero to be at B and the fall of potential to be represented by the line p' B, and

similarly with the fall from one pole of the battery to the other. In most cases it is convenient to consider the fall as taking place in this way, as we avoid having to consider the potentials as being partly + and partly − quantities, which is liable to cause confusion in making calculations.

348. We stated that if the poles of the battery are joined by a resistance, the potentials at those poles will be altered in value; they will, in fact, be reduced in proportion as the resistance is small or large. Now, when a current flows through a galvanometer, it does so in virtue of a difference of potential at its two terminals, and the strength of this current is directly proportional to the value of this difference; and conversely, if we note the difference in the strengths of currents passing through a galvanometer we shall know the relative values of the differences of potential at its terminals. It may at first sight, therefore, appear sufficient, in order to measure the relative values of the differences, simply to connect the terminals of a galvanometer to the points at which the differences are to be noted, and then to observe the deflections obtained. But by connecting up a galvanometer in this way we should reduce the resistance of the portion of the circuit between those points, and the potentials at the poles of the battery would consequently decrease, therefore the potentials at the points where the galvanometer is connected would decrease also; the current then which would produce a deflection of the galvanometer needle, would be that due to the diminished potentials. If, however, the resistance of the galvanometer is very high compared with the resistances with which it is connected, then its introduction will produce no diminution in the potentials, and consequently its deflection, that is to say, the current passing through it, will be a true index of the value of the difference. If, therefore, we wish to theoretically consider what are the relative differences of potentials at any points in any particular arrangements of batteries and resistances, we have simply to suppose these points to be connected by a galvanometer whose resistance is infinite compared with the other resistances, and then to determine the relative values of the currents which will flow through it in the several cases.

349. From what has been said we can see that, practically, if we connect a galvanometer to any two points at which a difference of potential exists, then the deflection obtained will accurately represent that difference of potential, provided the galvanometer has a total resistance in its circuit very much greater than the resistance between the two points in question.

MEASUREMENT OF ELECTROMOTIVE FORCE BY LAW'S METHOD.

350. The quantity of electricity in a condenser depends directly upon the electromotive force of the charge, and the deflection obtained upon a galvanometer depends directly upon the quantity discharged through it; the discharge deflection obtained from a condenser, therefore (§ 333, page 313), other things being constant, will represent the electromotive force of the charge in it; by successively charging a condenser, therefore, from two or more batteries, and noting the discharges on a galvanometer, we can very simply and quickly determine their comparative electromotive forces.

351. Discharge deflections on a galvanometer whose deflections are truly proportional to *constant* currents, unless they are nearly equal, are not always proportional to the currents which produce them. It is therefore very desirable, in measurements such as these, in order to ensure accuracy, to adopt the method we mentioned on page 99 (§ 89), viz. to obtain a uniform deflection by means of a variable shunt to the galvanometer. Thus, if we obtain two similar discharge deflections with two electromotive forces E_1 and E_2, using shunts of the respective resistances S_1 and S_2; then, since the deflections are the same, the electromotive forces are in the proportion

$$E_1 : E_2 :: \frac{G + S_1}{S_1} : \frac{G + S_2}{S_2},$$

or as the multiplying power of the shunts, G being the resistance of the galvanometer; for if we multiplied the deflections we obtained, by these quantities, we should get the theoretical deflections we should have had if no shunts had been used.

For example.

With an electromotive force E_1 we obtained a discharge deflection of 300 divisions on a galvanometer whose resistance G was 5000 ohms, using a shunt S_1, of 1000 ohms, and with a second electromotive force, E_2, also a deflection of 300 divisions, using a shunt, S_2, of 2500 ohms; then

$$E_1 : E_2 :: \frac{5000 + 1000}{1000} : \frac{5000 + 2500}{2500},$$

that is,

$$E_1 : E_2 :: 2 : 1.$$

It is not absolutely necessary that the same deflection be reproduced exactly, although calculation is saved by so doing: as long

as the deflections are nearly equal they approximately represent the discharges. It is necessary, of course, that these deflections be multiplied by $\frac{G + S}{S}$ to obtain the relative strengths of the currents.

For example.

With an electromotive force E_1 we obtained a discharge deflection of 300 divisions on a galvanometer whose resistance G was 5000 ohms, using a shunt S_1, of 1000 ohms, and with a second electromotive force E_2 a deflection of 292 divisions, using a shunt S_2, of 2400 ohms; then

$$E_1 : E_2 :: \frac{5000 + 1000}{1000} \times 300 : \frac{5000 + 2400}{2400} \times 292;$$

that is,

$$E_1 : E_2 :: 1800 : 900 \cdot 33,$$

or as

$$2 : 1, \text{ very nearly.}$$

This method is very often the best one to employ, not only for discharge, but also for constant deflections, as it is sometimes inconvenient to have to continually adjust until the same deflection exactly is reproduced. In certain cases, indeed, it would be impossible to do so, as will be seen hereafter.

352. It may be here mentioned that, in the case of discharge deflections, the fact that the resistance between the terminals of the galvanometer is varied by the introduction of shunts of different values, does not require to be taken into consideration.

Correction for Discharge Deflections.

353. The late Mr. Latimer Clark, in a communication addressed to the Society of Telegraph-Engineers,* pointed out an error caused by the use of shunts in measuring discharge deflections.

It was found that if a certain discharge deflection were obtained with a shunt, then on removing the latter the discharge deflection obtained was larger than that given by multiplying the original deflection by $\frac{G + S}{S}$.

After considerable research, the cause of the error was traced to the inductive action of the galvanometer needle on its coils. The movement of this needle set up a slight current, which

* 'Journal of the Society of Telegraph-Engineers,' vol. ii. p. 16.

opposed the discharge current, and consequently reduced its effect. This effect being more marked when the shunt was used, made the discharge deflection without the shunt to appear larger than it should.

The formula for finding what would be the discharge deflection obtained on the removal of the shunt, the discharge deflection without the shunt being given, may be thus arrived at:—

First suppose the shunt to be inserted.

Now in all problems in which a current from a condenser has to be considered, we may suppose the condenser to be a battery with a resistance infinitely great compared with the resistances external to it.

Let E be the force of the charge, R the resistance of the condenser circuit, G the resistance of the galvanometer, S the resistance of the shunt.

Let the movement of the needle generate an opposing force e; then calling C, a, and β the respective discharge strengths in the galvanometer, condenser, and shunt circuits, we get (by applying Kirchoff's laws, page 178) the following equations:—

$$a - C - \beta = 0, \text{ or, } \beta = a - C$$
$$a R + C G - E + e = 0$$
$$a R + \beta S - E = 0;$$

therefore

$$a R + C G - E + e = 0$$
$$a R + (a - C) S - E = 0;$$

therefore

$$a R = E - C G - e$$
$$a (R + S) = E + C S;$$

then by division

$$\frac{R}{R+S} = \frac{E - CG - e}{E + CS};$$

by multiplication and changing the signs we get

$$(R + S)(C G + e) - R E - S E = - R E - C R S;$$

therefore

$$(R + S)(C G + e) - S (E - C R) = 0.$$

Next suppose the shunt to be removed, and let the strength of the discharge be C_1, and the new force generated by the movement of the needle be e_1, then

$$C_1 = \frac{E - e_1}{R + G}; \text{ therefore } E = C_1 (R + G) + e_1;$$

substituting this value of E in the last equation, we get

$$(R + S)(C G + e) - S(C_1(R + G) + e_1 - C R) = 0.$$

Now e and e_1 will be proportional to the deflections of the needle, that is to say, to the strengths of discharge producing those deflections. They will also be proportional to the strength of the magnetism of the needle, which strength we will represent by κ.

Then

$$e = \kappa C, \qquad e_1 = \kappa C_1;$$

substituting these values we get

$$(R + S) C(G + \kappa) - S(C_1(R + G + \kappa) - C R) = 0,$$

or

$$\left(1 + \frac{S}{R}\right) C(G + \kappa) - S\left(C_1\left(1 + \frac{G + \kappa}{R}\right) - C\right) = 0.$$

Now, R is to be infinite as compared with S and G; therefore by putting $R = \infty$, we find that

$$C(G + \kappa) - S(C_1 - C) = 0,$$

therefore

$$C_1 = C\left(\frac{G + \kappa + S}{S}\right).$$

To make this formula useful, we must determine the value of κ. This can be done thus:—

Provide two condensers, one having exactly twice the capacity of the other. Charge the larger one with a sufficient battery power to obtain a discharge deflection (a_1) of, say, 200 divisions on the scale, with a shunt inserted equal in resistance to the galvanom

Now remove the shunt, and having charged the other condenser from the same battery, note the discharge deflection (a_2); let it be 204 divisions.

It will be seen that the deflection we should have obtained with the larger condenser and no shunt would have been $2\,a_2$, and this is the theoretical deflection we should obtain when a_1 is multiplied by the multiplying power of the shunt corrected by the constant κ; that is to say,

$$2\,a_2 = a_1\left(\frac{G + \kappa + G}{G}\right);$$

therefore

$$\kappa = 2\,G\left(\frac{a_2}{a_1} - 1\right).$$

To continue the example we have given, let us suppose $G = 5000$ ohms; then

$$\kappa = 2 \times 5000 \left(\frac{204}{200} - 1\right) = 200.$$

For the particular galvanometer, then, we have been considering, we say that when measuring discharges the multiplying power of any shunt (S) which may be used is

$$\frac{G + 200 + S}{S}.$$

Suppose we have given the observed deflection without the shunt and also the observed deflection with the shunt, and we require to know what this latter ought to be in order to give us the *true* deflection compared with the first. Let the true deflection be A; then by the ordinary formula

$$C_1 = A\left(\frac{G + S}{S}\right);$$

but when the error exists,

$$C_1 = C\left(\frac{G + \kappa + S}{S}\right);$$

from these two equations we get

$$C = A\left(\frac{G + S}{S}\right)\left(\frac{S}{G + \kappa + S}\right);$$

therefore

$$A = C\left(\frac{G + \kappa + S}{G + S}\right) = C\left(1 + \frac{\kappa}{G + S}\right),$$

or in words:

$$\text{True deflection} = \text{observed deflection}\left(1 + \frac{\kappa}{G + S}\right).$$

It should be clearly understood that this formula is to be applied to the correction of the deflection obtained *with* the shunt, the deflection *without* the shunt being considered as the index of the current from the condenser.

We may remark that this latter formula corresponds with that obtained by the late Mr. Charles Hockin, and given by Mr. Latimer Clark in the paper referred to.

For practical use the formula

$$C_1 = C\left(\frac{G + \kappa + S}{S}\right)$$

is the only one we should require, as by it we can at once determine from the deflection obtained *without* the shunt what the deflection *with* the shunt would be, or *vice versâ*.

Relation between the Current, the Resistance, and the Electromotive Force, between two Points in a Circuit.

354. In Fig. 152 let E be a battery of electromotive force, E, and resistance, x, joined up in a circuit with a resistance r, and let G be a galvanometer having a resistance very much greater than

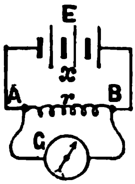


Fig. 152.

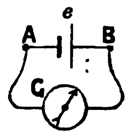


Fig. 153.

the other resistances, so that it does not affect the flow of the current in the circuit, $x + r$. Now, the current, C_1, flowing through the galvanometer will be

$$C_1 = \frac{E}{x + \frac{r\,G}{r + G}} \times \frac{r}{r + G} = \frac{E\,r}{x\,r + x\,G + r\,G}.$$

It is evident that this current must be due to the existence of an electromotive force, or a difference of potential, in some portion of the circuit in which the galvanometer is placed; and it is, moreover, evident that this electromotive force, or difference of potential, must exist between the points A and B in the portion of the circuit external to G. Let e be this electromotive force (Fig. 153), then we have (since G is very much larger than the other resistances)—

$$C_1 = \frac{e}{G},$$

but

$$C_1 = \frac{E\,r}{x\,r + x\,G + r\,G},$$

therefore

$$\frac{E\,r}{x\,r + x\,G + r\,G} = \frac{e}{G},$$

that is,

$$\frac{e}{r} = \frac{\mathrm{E\,G}}{x\,r + x\,\mathrm{G} + r\,\mathrm{G}} = \frac{\mathrm{E}}{\frac{x\,r}{\mathrm{G}} + x + r};$$

but, since G is very great compared with the other resistances,

$$\frac{x\,r}{\mathrm{G}} = 0;$$

therefore

$$\frac{e}{r} = \frac{\mathrm{E}}{x + r}.$$

But by Ohm's law (§ 2, page 1), the current C, flowing out of the battery—that is, flowing through r—is

$$\mathrm{C} = \frac{\mathrm{E}}{x + r},$$

therefore

$$\mathrm{C} = \frac{e}{r}, \quad \text{or,} \quad e = \mathrm{C}\,r;$$

that is to say,

(A) *The difference of the potential at two points in a resistance (in which no electromotive force exists) is equal to the product of the current and the resistance between the two points.*

355. We will next consider the case where an electromotive force exists in the resistance through which the current is flowing, the strength of the latter, and the potentials at the two points, being partly due to this electromotive force.

We have two cases to be considered, first, that in which the electromotive force in the resistance opposes the current actually flowing, and second, that in which the force acts in the same direction as the current.

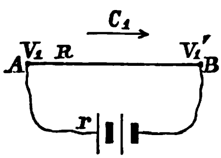


FIG. 154.

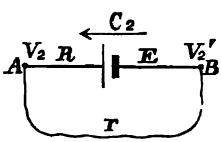


FIG. 155.

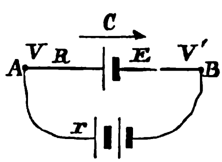


FIG. 156.

In Fig. 154, let R be a resistance between the points A and B, and let C_1 be the current flowing due to an external electromotive

force in a resistance (or combination of resistances) r; then by law (A) (page 329), we have

$$V_1 - V_1' = C_1 R. \qquad [1]$$

Next, let us first suppose, as in Fig. 155, that we have in R a current C_2 caused by an electromotive force E, then by Ohm's law we have

$$C_2 = \frac{E}{R + r}, \quad \text{or,} \quad C_2 R + C_2 r = E,$$

that is,

$$C_2 r = E - C_2 R;$$

but by the law (A) (page 329) we have

$$V_2 - V_2' = C_2 r;$$

therefore

$$V_2 - V_2' = E - C_2 R. \qquad [2]$$

Now if we take the case shown in Fig. 156 where the current C is produced by the two electromotive forces, then the respective potentials at the points A and B must be

$$V = V_1 + V_2$$

and

$$V' = V_1' + V_2'.$$

Therefore we have

$$V - V' = (V_1 - V_1') + (V_2 - V_2'),$$

and by substituting the values of $V_1 - V_1'$ and $V_2 - V_2'$ given in equations [1] and [2] we get

$$V - V' = C_1 R + E - C_2 R = R (C_1 - C_2) + E;$$

but we can see that

$$C = C_1 - C_2,$$

therefore

$$V - V' = C R + E, \qquad [3]$$

V being always greater than V'.

In the case we have taken we have supposed the electromotive force E to act *against* the current; if we take the force to act *with* the current, then we get

$$V - V' = C R - E. \qquad [4]$$

It may be remarked that this rule is true whether (*a*) C is due to E alone, (*b*) to E assisted by an external force, or (*c*) to E diminished by an external force. In cases (*a*) and (*c*) however, $V - V'$ is always positive, but in case (*b*) it is sometimes positive and sometimes negative, according to circumstances.*

356. The result, then, that we have arrived at by the foregoing investigation is, that—

(B) *The difference of the potentials at two points in a resistance in which an electromotive force exists is equal to the product of the current and the resistance between the two points, added to the electromotive force in the resistance, this electromotive force being negative if it acts with the current, and positive if it opposes it.*

Measurement of Battery Resistance by Kempe's Method.

357. Besides determining the electromotive force of a battery, we can also determine its internal resistance with great facility by means of a condenser. To do this, first charge the condenser by means of the battery, and note the discharge deflection, which we will call α; next insert a shunt, S, between the poles of the battery; again charge and discharge the condenser, and note the new deflection, which we will call β. Let e be the electromotive force between the poles of the battery when the shunt S is inserted, and let C be the current flowing, then by law (A) (page 329), we have

$$e = C\,S, \quad \text{or,} \quad C = \frac{e}{S}.$$

Also, if E be the electromotive force of the battery, and r its resistance, we have

$$C = \frac{E}{S + r};$$

therefore

$$\frac{e}{S} = \frac{E}{S + r},$$

or

$$e\,S + e\,r = E\,S$$

therefore

$$e\,r = S\,(E - e),$$

* This latter case is met with in Lumsden's Electromotive Force test, page 177.

or

$$r = \mathrm{S}\,\frac{\mathrm{E} - e}{e};$$

but we must also have

$$\mathrm{E} : e :: \alpha : \beta;$$

therefore

$$r = \mathrm{S}\,\frac{\alpha - \beta}{\beta}. \qquad [\mathrm{A}]$$

To obtain accuracy it is advisable for the value of S to be such that the deflection β is approximately equal to $\frac{\alpha}{3}$ *.

For example.

A battery whose resistance (r) was required was joined up with a galvanometer, condenser, discharge key, &c., as shown by Fig. 142, page 313. The condenser being charged and then discharged through the galvanometer (by depressing and then releasing the key K_2), a deflection of 290 divisions (α) was produced. A resistance of 20 ohms (S) was then joined between the terminals of the battery, and the condenser again charged and discharged through the galvanometer, the value of the deflection obtained being 105 divisions (β). What was the resistance of the battery?

$$r = 20\,\frac{290 - 105}{105} = 35 \cdot 2 \text{ ohms.}$$

It is evident that if S be adjusted till $\beta = \frac{\alpha}{2}$, then

$$r = \mathrm{S}. \qquad [\mathrm{B}]$$

An error in the foregoing kind of test may possibly arise from one measurement being made with the poles of the battery free, when no action goes on in it, and the second being made with it shunted, which may cause a falling off in its electromotive force, as action would then be taking place; the accuracy of the test depends upon the electromotive force being constant in both cases. If the shunt S be connected to the battery by means of a key, then the second discharge deflection β is best obtained by first pressing down the key K_2 (Fig. 142, page 313), then pressing down the key which connects the shunt to the battery, and then *immediately* afterwards releasing the key K_2, and noting the deflection. Thus as little time as possible is allowed for polarisation to take place.

* The reason of this will be obvious from a consideration of the investigations given in § 123, page 124, and § 128, page 123.

MEASUREMENT OF BATTERY RESISTANCE BY MUIRHEAD'S METHOD.

358. A very excellent modification of the foregoing method has been devised by Dr. A. Muirhead; it possesses the great advantage of being free from the source of error just mentioned.

In this test (Fig. 157) the battery, galvanometer and condenser are joined up in circuit with a key K_2. The condenser C being short-circuited for a moment, so as to dissipate any charge which may have been accidentally left in it, key K_2 is depressed; this causes a charge to rush into the condenser through the galvanometer, producing the same deflection as would be produced if the condenser, when charged from the battery direct, were discharged through the galvanometer.

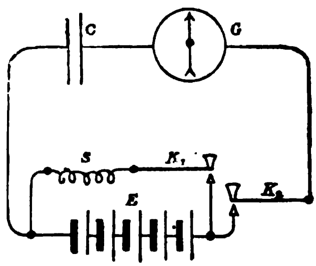


FIG. 157.

The charge deflection (α) being noted, the key K_2 is kept permanently down, so as to keep the condenser charged. By means of key K_1, a shunt (S) is now connected between the poles of the battery; at the moment this takes place the potential at the poles of the latter falls, and a reverse deflection of the needle of the galvanometer is produced. If we suppose this deflection to be due to an alteration of the potential from α to β (the latter being the same quantity as that given in the previous test), its value, ζ, will be

$$\zeta = \alpha - \beta, \quad \text{or,} \quad \beta = \alpha - \zeta.$$

If, then, we substitute this value of β in equation [A] (page 332) of the previous test, we get

$$r = \mathrm{S}\,\frac{\zeta}{\alpha - \zeta}.$$

For example.

The shunt S having a resistance of 10 ohms, the deflection produced on depressing key K_2, was 310 divisions (α). K_2 being held down, K_1 was depressed, when a deflection, of 100 divisions (ζ)—in the reverse direction to α—was obtained. What was the resistance of the battery?

$$r = 10 \frac{100}{310 - 100} = 4 \cdot 76 \text{ ohms.}$$

As in the previous test, it is advisable to give S such a value that ζ is approximately equal to $\frac{a}{3}$.

As no polarisation of any extent takes place in the battery till some seconds after the shunt has been connected to the former by the key, and as the deflection takes place immediately the key is depressed, it follows that very accurate results will be obtained by this test. It may be remarked, however, that Professor Garnett has found that polarisation takes place in a battery in an extremely short space of time—in even the $\frac{1}{1000}$th part of a second; the amount, is however, of course very small. In Muirhead's test the time during which polarisation would tend to affect the accuracy of the test would be that occupied by the galvanometer needle in swinging from zero to the deflection, ζ, consequently the quicker the swing (consistent with accurate reading) the better.

Measurement of Battery Resistance by Munro's Method.

359. A modification of Muirhead's method has been suggested by Mr. J. Munro, which simplifies calculation, inasmuch as it gives the value of $a - \zeta$ by a single deflection.

Key K_1 is first depressed, and then immediately afterwards key K_2 is also depressed; this gives a deflection θ, which is equivalent to the difference between the deflections a and ζ in the last test. Key K_1 is now raised, leaving key K_2 down; and as soon as the galvanometer needle becomes steady, K_1 is depressed again, and the deflection ζ read, then we have

$$R = S \frac{\zeta}{\theta}.$$

As a slight interval of time may elapse between the depression of key K_1 and key K_2, when obtaining the deflection θ (during which time the battery would be partially short-circuited), it would be preferable to make the test in the following manner:— Make the connections so that the front contact of key K_1 is joined on to the lever of key K_2 instead of on to the front contact of the latter, as in Fig. 157 (page 333); then in order to obtain θ, depress K_1 and keep it down, and immediately afterwards depress K_2; the deflection observed in this case will be θ. Now raise key K_1, keeping key K_2 down, and when the galvanometer needle has become steady, depress K_1, then the deflection obtained will be ζ.

Measurement of Polarisation in Batteries.

360. The amount of polarisation which takes place in a battery when the latter is short-circuited may, if required, be easily ascertained in the following manner:—In Fig. 157 (page 333) let S be a short piece of wire of practically no resistance, then having short-circuited the condenser C for a moment, depress key K_2, and note the deflection d_1. Keeping K_2 down, depress K_1, and hold it down for a definite time, say one minute; at the end of the interval release K_1, and note the deflection d_2; then the percentage of polarisation in the one-minute interval will obviously be

$$\frac{100\,(d_1 - d_2)}{d_1}.$$

Measurement of the Resistance of Batteries of Low Resistance.

361. In cases where it is required to measure the total resistance of a number of cells of extremely low resistance (secondary batteries, or accumulators, for example) by any of the foregoing methods, the heating effect produced by the current passing through the shunt S when the latter is connected to the battery by means of the key K_2, would be liable to heat and damage the coils of which the shunt is composed. In such cases the cells should be divided into two sets, one set having, say, one more cell than the other; the two sets should then be joined together so that their electromotive forces oppose one another. By this arrangement we practically obtain a battery whose electromotive force is equal to one cell only, but whose resistance is equal to that of all the cells; consequently the current generated can be but comparatively small, and would have but little heating effect. The contact in key K_2 should be made by means of a mercury cup.

362. When comparing large electromotive forces with small ones—as, for instance, 100 cells with one cell—by the condenser discharge method, the smaller force should be taken first; for a large charge usually leaves a residuum in the condenser, which may be greater than the small force, and which can only be thoroughly dissipated by leaving the condenser short-circuited for some time. If the smaller force is measured first, then any residuum it may leave becomes entirely swamped by the larger force, and no increase of charge is added to the condenser beyond that due to the larger force itself.

363. Although the condenser practically becomes charged instantaneously, it is usual to keep the current on for a definite time; twenty seconds is the period very generally adopted; this ensures that the charging is complete.

364. When discharge currents are being measured, especial care must be taken to insert a shunt of small resistance in the galvanometer at first, as momentary currents are very liable to weaken the magnetism of the needles when these currents are strong. If this precaution is not taken, a set of measurements for one test may be rendered useless, as a comparison of measurements made before the magnets become weakened, with measurements taken after, would be obviously impossible, and much loss of time would result.

CHAPTER XII.

MEASUREMENT OF CURRENT STRENGTH.

365. If we have a simple circuit, as shown by Fig. 158, then if we know the total resistance, $R + G + r$, of the same, and also the electromotive force, E, of the battery, we can at once determine the strength of the current flowing, for by Ohm's law we have

$$C = \frac{E}{R + G + r}.$$

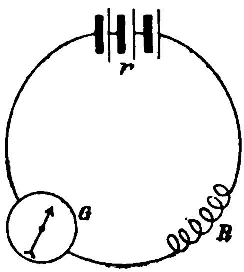


Fig. 158.

If the resistances are in ohms and the electromotive force of the battery in volts, then the resulting current will be in ampères.

For example.

The electromotive force, E, of a battery which produced a current, C, in a circuit whose total resistance, R_1, was 500 ohms, was found by comparison with a Daniell cell (1·08 volts approximately) to be 3·5 times as strong as the latter; what was the strength of the current, C, flowing in the circuit?

$$E = 3{\cdot}5 \times 1{\cdot}08 = 3{\cdot}78 \text{ volts}.$$

$$C = \frac{3{\cdot}78}{500} = {\cdot}00756 \text{ ampères} = 7{\cdot}56 \text{ milliampères}.$$

366. In the foregoing method of measurement, in order to determine the strength of the current, it was necessary to know both the resistance of the circuit and also the electromotive force producing the current. A direct determination of the latter can only be made by comparing it with a current of a known strength as follows:—

Direct Deflection Method.

367. In this method a galvanometer is inserted in the circuit through which flows the current whose strength is to be measured. The resistance of the galvanometer should be sufficiently low not to appreciably increase the total resistance of the circuit in which it is inserted. The deflection produced by the current being noted, the galvanometer is removed and joined up in a circuit with a standard battery (page 157) and a resistance; the latter is then adjusted until the deflection which was obtained in the first instance is reproduced; in this case, then, the current flowing must be equal to the current whose strength is required. If therefore we divide the electromotive force (in volts) of the standard battery, by the *total* resistance (in ohms) in its circuit, we get at once the required strength of the current, in ampères. The resistance of the standard battery requires of course to be included in the total resistance unless it is so small that it can be neglected.

368. As a standard cell of the Clark type (§ 169, page 159) cannot be used for tests of this description, owing to the fact that its electromotive force would run down, a Daniell cell or an accumulator should be used, the electromotive force of the same being compared, previous to and after the test, with a Clark standard cell by means of one of the methods described in Chapter VII.

369. The degree of accuracy attainable in a test of this kind is directly proportional to one-half the degree of accuracy with which the galvanometer deflection can be read, for, since two measurements have to be made, one with the current whose strength is required, and the other with the standard cell, there may be errors made in both of these. If the current to be measured is a strong one, so that it is necessary to shunt the galvanometer when obtaining a deflection, this shunt not being used when the deflection is reproduced with the standard cell, then in this case the result obtained by dividing the electromotive force of the standard cell by the total resistance in its circuit, must be multiplied by the multiplying power of the shunt (page 89) in order that the correct strength may be obtained.

For example.

In measuring the strength of a current, the deflection produced on the galvanometer shunted with the $\frac{1}{10}$th shunt, was 50°. The galvanometer (without the shunt) being connected up with a

Daniell cell of 10 ohms resistance, and a set of resistance coils, it was found necessary to adjust the latter to 560 ohms in order to bring the needle to 50°; what was the strength, C, of the current to be measured?

$$C = \frac{1 \cdot 08}{560 + 10} \times 10 = \cdot 019 \text{ ampères} = 19 \text{ milliampères.}$$

370. When a galvanometer is inserted in a circuit through which a current is flowing whose strength it is required to measure, it is very necessary that the resistance of the instrument be very low compared with the resistance of the circuit itself, otherwise the introduction of the galvanometer will reduce the current flowing, and the result obtained will not be the one required. Before making the test it would of course be necessary to ascertain whether the galvanometer available for use would meet the desired conditions.

To make the test as accurately as possible it would be necessary that the galvanometer needle when deflected be as near to the *angle of maximum sensitiveness* (page 25) as possible. If the strength of current necessary to give this angle be found by joining up a standard cell and a set of resistances, and varying the latter until the required deflection is obtained, then we can always tell whether the instrument would be suitable for a particular purpose. Thus, for example, suppose the galvanometer had a resistance of 1000 ohms, and the angle of maximum sensitiveness were approximately equal to 60°, and suppose further that this deflection were obtained by 1 Daniell cell through a total resistance of 8000 ohms, that is, with a current of $\frac{1 \cdot 08}{8000} = \cdot 000135$ ampères ($\cdot 135$ milliampères); then to measure such a current we must have the whole resistance of the galvanometer, viz. 1000 ohms, in circuit. If the instrument were shunted with the $\frac{1}{10}$th shunt, then the resistance would be reduced to 100 ohms, and the current corresponding to 60° deflection would be $\cdot 00135$ ampères; and again, if the $\frac{1}{1000}$th shunt were employed the resistance would be reduced to 1 ohm, and the current corresponding to 60° would be $\cdot 135$ ampères; thus we see that if it were required to measure a current of about $\cdot 0135$ ampères and it were necessary that a resistance no greater than one ohm should be inserted in the circuit, then it is evident that the galvanometer in question would not answer the purpose required, since a good deflection with $\cdot 0135$ ampères of current would not be obtained if a lower shunt than

$\frac{1}{100}$th were employed, which latter shunt would reduce the galvanometer resistance down to 10 ohms only.

It is preferable, when possible, to employ a galvanometer of high resistance shunted down, rather than one of low resistance not shunted down, since with such a galvanometer it is easier to measure the "constant" of the instrument accurately; for the high resistance of the latter, together with the high resistance which it would be necessary to place in circuit in order to get a readable deflection with one standard cell only, would swamp, as it were, the resistance of the cell, which resistance need not then be taken into account, or at least need only to be known approximately. With a galvanometer of low resistance, however, where a comparatively small resistance only would have to be introduced into the circuit in order to get the required deflection, the resistance of the cell would be required to be known accurately, as it would form an important item in the total resistance of the circuit.

371. The foregoing test has the advantage that it can be made with almost any form of galvanometer, for as only one deflection has to be obtained it is not necessary to know what proportions the various degrees of deflection which it is possible to have, bear to the currents which produce them. If, however, a *tangent* galvanometer is employed to make the test, then it is unnecessary to reproduce the same deflection exactly, though it is advisable to make it approximately near to it.

372. Suppose that in the last example the test had been made with a tangent galvanometer, and the deflection obtained with the standard cell had been 54° instead of 60° (the deflection given by the current whose strength was required), then in this case the actual strength, C_1, of the current would be

$$C_1 = \frac{1 \cdot 08}{560 + 10} \times 10 \times \frac{\tan 60^\circ}{\tan 54^\circ} =$$

$$\frac{1 \cdot 08}{560 + 10} \times 10 \times \frac{1 \cdot 7321}{1 \cdot 3764} = \cdot 0239 \text{ ampères} = 23 \cdot 9 \text{ milliampères.}$$

373. If we use no shunt, or the same shunt when taking both the deflections, and, further, if we make the total resistance in circuit with the Daniell cell equal to 1080 ohms, then calling d° the deflection given by the current, and $d_1{}^\circ$ the deflection given by the standard cell, we have

$$C_1 = \frac{1 \cdot 08}{1080} \times \frac{\tan d^\circ}{\tan d_1{}^\circ} = \frac{\tan d^\circ}{1000 \tan d_1{}^\circ} \text{ ampères} = \frac{\tan d^\circ}{\tan d_1{}^\circ} \text{ milliampères;}$$

or, further still, if by means of an adjusting magnet we can arrange that the deflection given by the standard cell through 1080 ohms equals 45°, then since tan 45° = 1, we must have

$$C_1 = \frac{\tan d^\circ}{1000} \text{ ampères} = \tan d^\circ \text{ milliampères.}$$

Cardew's Differential Method.*

374. This method, devised by Major Cardew, R.E., is a very satisfactory and useful one; its theory is shown by Fig. 159.

The galvanometer G is wound with two wires, g and g_1; the current C_1 whose strength it is required to measure is passed through the coil g_1 (which has a low resistance), and a standard

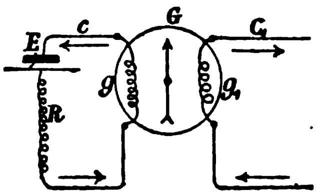


Fig. 159.

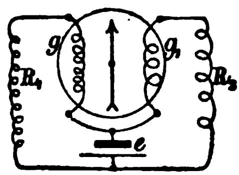


Fig. 160.

battery E is connected in circuit with the second coil g and with an adjustable resistance R. The current being passed through the coil g_1, the resistance R is adjusted until the needle comes to zero.

If we call n and n_1 the relative deflective effects, for the same current, of the two coils g and g_1, and if C_1 and c be the currents flowing through g_1 and g, respectively, then in order to produce equilibrium we must have

$$c : C_1 :: n_1 : n,$$

or

$$C_1 = \frac{c\,n}{n_1}.$$

Now the current through c will evidently be

$$c = \frac{E}{R + g},$$

the resistance of the battery being included in R; therefore

$$C_1 = \frac{E}{R + g} \times \frac{n}{n_1}.$$

* 'Journal of the Society of Telegraph-Engineers,' vol. xi. p. 301.

For example.

The relative deflective effects of the coils g and g_1 were as 1000 to 1; the resistance of g was 100 ohms. The battery E was a 1-cell Daniell. In order to obtain equilibrium the resistance R had to be adjusted to 4800 ohms. What was the strength of the current C_1?

$$C_1 = \frac{1 \cdot 08}{4800 + 100} \times \frac{1000}{1} = \cdot 221 \text{ ampères} = 221 \text{ milliampères.}$$

375. The relative deflective effects of g and g_1 are easily ascertained by joining up a battery e and two resistances R_1 and R_2, as shown by Fig. 160, and then adjusting until equilibrium is produced; in this case we have

$$n : n_1 :: R_1 + g : R_2 + g_1,$$

or

$$\frac{n}{n_1} = \frac{R_1 + g}{R_2 + g_1}.$$

376. As the accuracy with which a test can be made depends, amongst other things, upon the accuracy with which $\frac{n}{n_1}$ is known, the higher the battery power, e, it is possible to use, the better, since in this case the higher will be the values which can be given to R_1 and R_2, and the higher consequently will be their range of adjustment; thus if we use sufficient battery power to enable a change of $\cdot 1$ per cent., that is to say, 1 ohm in 1000 ohms, or $\frac{1}{10}$th of an ohm in 100 ohms in $R_1 + g$, to produce a perceptible movement of the needle, then we can obtain the value of $\frac{n}{n_1}$ to an accuracy of $\cdot 1$ or $\frac{1}{10}$th per cent.

The resistance of g_1 would have to be very small compared with the resistance of g, so that it would not add appreciably to the resistance of any circuit in which it is inserted.

377. As regards the ***Best conditions for making the test***, this will be directly proportional to the relative values, of the figure of merit (page 85), of the coil g_1, and the current to be measured, for it is evident that no matter whether equilibrium exists owing to there being no current flowing through the coils g and g_1, or to equal currents flowing, still the current which will deflect the needle 1 division will be the same in both cases; hence if the figure of merit of the coil g_1, be, say, $\frac{1}{10,000}$th of an ampère, then an increase of $\frac{1}{10,000}$th of an ampère ($\frac{1}{10}$th milliampère) in the current C_1, no matter what the strength of

the latter may be, will produce a deflection of 1 degree. It is evident, therefore, that the greater the strength of the current the greater is the degree of accuracy with which its value can be determined; thus if c' be the figure of merit of the coil g_1 and C_1 be the current to be measured, then the *Percentage of accuracy attainable* will be the percentage which c' is of C_1.

To enable this percentage to be obtained, however, it would be necessary that the total resistance of the circuit of g be adjustable to a degree of accuracy similar to that in the case of g_1; in order that this may be so, E must be of such a value that the number of units in $R + g$ is not less than that which satisfies the equation

$$\frac{1}{R+g} = \frac{c'}{C_1}.$$

Now,

$$C_1 = \frac{E}{R+g} \times \frac{n}{n_1}.$$

therefore

$$C_1 = \frac{E\,c'}{C_1} \times \frac{n}{n_1},$$

or

$$E = \frac{C_1^2}{c'} \times \frac{n_1}{n}.$$

For example.

The figure of merit of coil g_1 of the galvanometer was ·0001 ampères (c'); the current to be measured was approximately ·5 ampères (C_1); and the value of $\frac{n_1}{n}$ was ·001. What was the possible degree of accuracy attainable in making the test, and what would have been the lowest value which should have been given to E in order that this degree of accuracy might be attained?

$$\text{Percentage of accuracy} = \frac{100 \times \cdot 0001}{\cdot 5} = \cdot 02 \text{ per cent.};$$

also

$$E = \frac{\cdot 5 \times \cdot 5}{\cdot 0001} \times \cdot 001 = 2 \cdot 5 \text{ volts.}$$

If, therefore, E consisted of 3 Daniell cells, the required value of $R + g$ would have been obtained.

To sum up, then, we have

Best Conditions for making the Test.

378. Make E not less than $\frac{C_1^2}{c'} \times \frac{n_1}{n}$, c' being the figure of merit of the coil through which the current to be measured is passed.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{100\,c'}{C_1}.$$

KEMPE'S BRIDGE METHOD.

379. This method is a modification of the foregoing, and it has the advantage that it does not require a special form of galvanometer for its execution. It is shown in principle by Fig. 161.

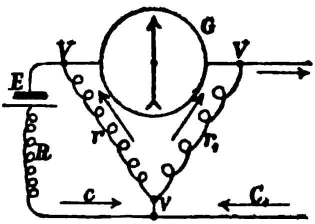


Fig. 161.

In making the test the resistance R is adjusted until no deflection is observed on the galvanometer; when this is the case the current c from the battery must also be the current flowing through r, and, again, the current C_1 must also be the current flowing through r_1. Now since no current flows through the galvanometer, the potentials V, V, on either side of it must be the same, hence if v be the potential at the junction of r and r_1, then by law (A) (page 329) we have

$$V - v = c\,r,$$

and

$$V - v = C_1\,r_1;$$

therefore

$$C_1\,r_1 = c\,r,$$

or

$$C_1 = \frac{c\,r}{r_1};$$

but

$$c = \frac{E}{R + r},$$

therefore

$$C_1 = \frac{E\,r}{r_1\,(R + r)}. \qquad [A]$$

For example.

The battery E consisted of a single Daniell cell. The resistances r and r_1 were 100 ohms and 1 ohm, respectively. Equilibrium was obtained on the galvanometer G when R was adjusted to 4000 ohms; what was the strength of the current C_1?

$$C_1 = \frac{1{\cdot}08 \times 100}{1\,(4000 + 100)} = {\cdot}0264 \text{ ampères} = 26{\cdot}4 \text{ milliampères.}$$

380. Let us now consider the *Best Conditions for making the Test.*

What we have to determine is,—1st, what should be the values of E and r? and 2nd, what should be the value of R?

The values which E and r should have should be such that the deflection of the galvanometer needle is as large as possible when equilibrium is very nearly, though not quite, produced. Now if we regard R as a constant quantity, then the value which E must have will depend upon the value given to r, consequently we have to determine what the latter quantity should be.

Practically the resistance r_1 would in all cases have to be of a very low value, and if we consider it to be so the problem to be solved becomes a comparatively simple one. We may regard the current c' producing the deflection of the galvanometer needle as due to a difference of two currents, C_1 being one, and the current produced by the electromotive force E, being the other. Let, then, c_1 and c_2 be the portions of these currents which flow in opposite directions through the galvanometer G, then if we suppose the deflection to be due to R being incorrectly adjusted to $R + \delta$, we have (supposing r_1 to be very small),

$$c_1 = \frac{C_1 r_1}{G + \frac{(R + \delta) r}{R + \delta + r}} = \frac{C_1 r_1 (R + \delta + r)}{G R + G \delta + G r + R r + r \delta},$$

and

$$c_2 = \frac{E}{R + \delta + \frac{r G}{R + G}} \times \frac{r}{r + G} = \frac{E r}{G R + G \delta + G r + R r + r \delta};$$

but since from equation [A] (page 344) we have

$$C_1 = \frac{E r}{r_1 (R + r)}, \quad \text{or,} \quad E r = C_1 r_1 (R + r),$$

therefore

$$c_2 = \frac{C_1 r_1 (R + r)}{G R + G \delta + G r + R r + r \delta}.$$

Now,

$$c' = c_1 - c_2,$$

therefore

$$c' = \frac{C_1 r_1 (R + \delta + r)}{G R + G \delta + G r + R r + r \delta} - \frac{C_1 r_1 (R + r)}{G R + G \delta + G r + R r + r \delta}$$

$$= \frac{C_1 r_1 \delta}{G R + G \delta + G r + R r + r \delta};$$

or, since δ is very small, we may say,

$$c' = \frac{C_1 r_1 \delta}{G R + G r + R r} = \frac{C_1 r_1 \delta}{R (G + r) + G r}.$$

From this equation we can see that r should be made small, but we can also see that there is but little advantage in making it much smaller than G. In fact, there is an actual disadvantage in making r extremely small, for this would necessitate E being made very large, which would be inconvenient.

We have next to determine what is the best value to give to R. Now, the larger we make the latter, the greater will be its range of adjustment, consequently, as in previous tests, we should give it the *highest value such that a*

change of 1 unit from its correct resistance produces a perceptible deflection of the galvanometer needle.

We have

$$c' = \frac{C_1 r_1 \delta}{R(G + r) + G r},$$

and if in this equation we put $\delta = 1$ we shall get the current corresponding to a change of 1 unit from the correct value of R, that is

$$c' = \frac{C_1 r_1}{R(G + r) + G r};$$

or, since r must be small, we may practically say

$$c' = \frac{C_1 R_1}{R G},$$

from which

$$R = \frac{C_1 r_1}{c' G}. \qquad [B]$$

If then we make c' the figure of merit (page 85) of the galvanometer, the value of R worked out from the equation will show the highest value that the latter quantity should have. But the value of R depends upon the value given to E; we must therefore determine what the latter should be.

We have

$$C_1 = \frac{E r}{r_1 (R + r)},$$

or

$$E = \frac{C_1 r_1 (R + r)}{r};$$

and substituting the value of R obtained from equation [B], we get

$$E = \frac{C_1 r_1 \left(\frac{C_1 r_1}{c' G} + r\right)}{r};$$

or, as r is small compared with R, that is with $\frac{C_1 r_1}{c' G}$, we may say

$$E = \frac{(C_1 r_1)^2}{c' G r}.$$

For example.

It was required to measure the exact strength, C_1, of a current whose approximate strength was known to be ·03 ampères. A Thomson galvanometer of 5000 ohms resistance (G) was employed for the purpose, its figure of merit being ·000,000,001 (c'). The resistances of r and r_1 were 100 ohms and 1 ohm respectively. What should be the value of E in order that R may be as high as possible?

$$E = \frac{(\cdot 03 \times 1)^2}{\cdot 000,000,001 \times 5000 \times 100} = 1 \cdot 8 \text{ volts};$$

that is to say, practically, E should consist of 2 Daniell cells.

Assuming E to be equal to 2 volts approximately, then (from equation [B] page 346) the value which R would have in order to obtain balance would be

$$R = \frac{\cdot 03 \times 1}{\cdot 000{,}000{,}001 \times 5000} = 6000 \text{ ohms}$$

approximately.

381. In order to determine the *Possible degree of accuracy attainable*, let us suppose R to be 1 unit out of adjustment, and let λ be the corresponding error produced in C_1, then we have

$$C_1 + \lambda = \frac{E\,r}{r_1\,(R - 1 + r)},$$

or

$$\lambda = \frac{E\,r}{r_1\,(R - 1 + r)} - C_1 = \frac{E\,r}{r_1\,(R - 1 + r)} - \frac{E\,r}{r_1\,(R + r)}$$

$$= \frac{E\,r}{r_1\,(R - 1 + r)\,(R + r)};$$

or, since R is large, we may say

$$\lambda = \frac{E\,r}{r_1\,(R + r)^2};$$

but

$$C_1 = \frac{E\,r}{r_1\,(R + r)}, \quad \text{or,} \quad (R + r)^2 = \left(\frac{E\,r}{C_1\,r_1}\right)^2;$$

therefore

$$\lambda = \frac{E\,r}{r_1} \cdot \left(\frac{C_1\,r_1}{E\,r}\right)^2 = \frac{C_1^2\,r_1}{E\,r}.$$

If we call λ' the *percentage* of accuracy, then

$$\lambda' = \frac{\lambda'}{100} \text{ of } C_1, \quad \text{or,} \quad \lambda' = \frac{100\,\lambda}{C_1} = \frac{100\,C_1\,r_1}{E\,r}.$$

If we take the values given in the foregoing example we have approximately

$$\lambda' = \frac{100 \times \cdot 03 \times 1}{2 \times 100} = \cdot 015 \text{ per cent.}$$

To sum up, then, we have

Best Conditions for making the Test.

382. Make E the nearest possible value above $\dfrac{(C_1\,r_1)^2}{c'\,G\,r}$, where c' is the figure of merit of the galvanometer, and C_1 is the approximate strength of the current to be measured.

The value which R will require to have will be

$$R = \frac{C_1\,r_1}{c'\,G}.$$

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{100\,C_1\,r_1}{E\,r}.$$

Difference of Potential Deflection Method.

383. Fig. 162 shows the general principle of this method.

A B is a low resistance through which the current, C_1, to be measured passes. A galvanometer, G, in circuit with a high

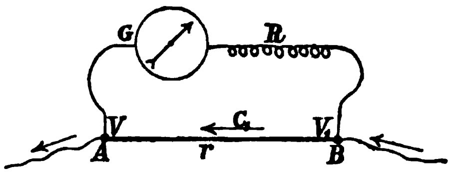


Fig. 162.

resistance, R, is connected between the ends of A B as shown, then, calling V and V_1 the potentials at A and B respectively, we have by law (A) (page 329)

$$C_1 = \frac{V - V_1}{r}.$$

To determine $V - V_1$ all we have to do is to note the deflection d on the galvanometer G, and then, having disconnected the latter, together with the resistance R, from A B, to join them in circuit with a standard cell of known electromotive force, E, and to obtain a new deflection d_1; we then have

$$V - V_1 : E :: d : d_1,$$

or

$$V - V_1 = \frac{E\,d}{d_1},$$

so that

$$C_1 = \frac{E\,d}{r\,d_1}.$$

384. In order that the test may be a satisfactory one the resistance G + R should be very high compared with the resistance r, so that the strength of C_1 is practically the same whether G + R is connected to A B or not; also r should be as low as possible, so that it may not appreciably add to the resistance of the circuit in which it is placed. In order, therefore, that a good deflection may be obtained, the galvanometer G should be one with a high figure of merit (page 85); a Thomson galvanometer answers the purpose very satisfactorily.

For example.

In making a measurement according to the foregoing test the resistance r was $\frac{1}{10}$th of an ohm, and the deflection obtained on G was 250 divisions (d). When G and R were connected to a Daniell cell in the place of being joined to A B, a deflection of 230 divisions (d_1) was obtained; what was the strength of the current C_1?

$$C_1 = \frac{1\cdot08 \times 250}{\frac{1}{10} \times 230} = 11\cdot7 \text{ ampères.}$$

As it is obviously advisable that the deflections obtained should both be as high as possible, the standard electromotive force E may have to be adjusted for the purpose, that is to say, it may have to consist of several cells. Instead of adjusting E only we may make the latter of any convenient high value, and then adjust R so that the required deflection is obtained; in this case if R_1 be the resistance when E is in circuit, we must have

$$C_1 = \frac{E\,d\,(R + G)}{r\,d_1\,(R_1 + G)}.$$

For example.

In making a measurement according to the foregoing test the resistance of r was $\frac{1}{10}$th of an ohm and the deflection obtained on G was 270 divisions (d); the resistances of G and R were 5000 ohms and 1000 ohms respectively. When G and R were connected to a Daniell cell, R had to be adjusted to 7000 ohms (R_1) in order to obtain a deflection of 300 divisions (d_1); what was the strength of the current C_1?

$$C_1 = \frac{1\cdot08 \times 270 \times (1000 + 5000)}{\frac{1}{10} \times 300 \times (7000 + 5000)} = 4\cdot86 \text{ ampères.}$$

Of course if the value of R_1 is made such that the deflections d and d_1 are equal, then

$$C_1 = \frac{E\,(R + G)}{r\,(R_1 + G)}.$$

385. From the extreme simplicity of the test it must be obvious that the "Best conditions for making the test" and the "Possible degree of accuracy attainable" must be as follows:—

Best Conditions for making the Test.

Make R and R_1 of such values that the deflections obtained are as high as possible.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = 100\frac{1}{m}\left(\frac{1}{d}+\frac{1}{d_1}\right)$$

where $\frac{1}{m}$ is the fraction of a division to which each of the deflections can be read.

Difference of Potential Equilibrium Method.

386. Fig. 163 shows the general principle of this method.

A B is a slide wire resistance, *s* being the slider. A galvanometer, G, and a standard battery, E, are joined up as shown, so that the latter tends to send a current through r_1 in a direction opposing

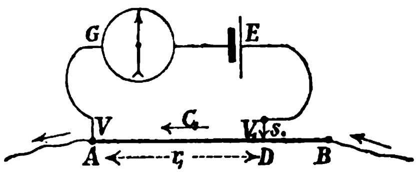


Fig. 163.

the current C_1; *s* is then slid along A B until the point is reached at which no deflection of the galvanometer needle is observed; when this is the case, then by law (A) (page 329) we have

$$V - V_1 = C_1 r_1;$$

and by law (B) (page 330), since no current is flowing through the galvanometer,

$$V - V_1 = E;$$

therefore

$$C_1 r_1 = E,$$

or

$$C_1 = \frac{E}{r_1}.$$

If the resistance of the whole length of wire A B be r ohms, and if it be divided into n divisions, then if the number of divisions between A and D be n_1, the resistance r_1 will be

$$r_1 = \frac{n_1 r}{n};$$

consequently we must have

$$C_1 = \frac{E n}{r n_1}.$$

For example.

The electromotive force E consisted of 1 Daniell cell; the wire A B had a resistance of 1 ohm (r), and was divided into 1000 parts (n). Equilibrium was obtained when the slider was set at the 750th division (n_1); what was the strength of the current C_1?

$$C_1 = \frac{1 \cdot 08 \times 1000}{1 \times 750} = 1 \cdot 44 \text{ ampères.}$$

387. The conditions for making the test in the most satisfactory manner are comparatively simple. The nearer we have the slider to B, that is to say, the larger we make n_1, the smaller will be the percentage of error in the latter due to the slider being, say, 1 division out of position. As the position of the slider for equilibrium depends upon the value of E, the latter must be sufficiently great to enable n_1 to be as large as possible. The greatest theoretical value which E could have must be that which it would possess when $n_1 = n$, in which case we get

$$C_1 = \frac{E}{r}, \quad \text{or,} \quad E = C_1 r.$$

As it is only possible to adjust E by variations of 1 cell, we must take care that its actual value is less rather than greater than $C_1 r$, otherwise it would be impossible to obtain equilibrium.

It is also necessary that the figure of merit (page 85) of the galvanometer be sufficiently high to enable a perceptible movement of the needle to be obtained when the slider is moved a readable distance, δ, from the position of exact balance. If we suppose the slider to be at D when equilibrium is produced, then the electromotive force which would tend to send a current through the galvanometer, supposing the slider to be displaced a distance δ, would be

$$E \times \frac{\delta}{n_1},$$

consequently the current c', passing through the galvanometer, will be

$$c' = \frac{E\,\delta}{G\,n_1} = \frac{C_1\,r\,\delta}{G\,n};$$

if, therefore, we require to adjust the slider to an accuracy of δ, the figure of merit c' of the galvanometer must not be less than $\frac{E\,\delta}{G\,n_1}$.

The percentage of accuracy, λ', with which C_1 can be obtained must obviously be

$$\lambda' = \frac{100\,\delta}{n_1},$$

or since

$$C_1 = \frac{E\,n}{r\,n_1}, \quad \text{or,} \quad n_1 = \frac{E\,n}{C_1\,r},$$

therefore

$$\lambda' = \frac{100\,C_1\,r\,\delta}{E\,n}.$$

For example.

It being required to measure the strength, C_1, of a current whose approximate value is 1·5 ampères, a galvanometer of 500 ohms resistance (G), whose figure of merit is ·000001 (c'), is proposed to be employed for the purpose. The resistance of the whole length of the slide wire, which is divided into 1000 divisions (n), is 1 ohm (r): the position of the slider can be read to an accuracy of $\frac{1}{2}$ a division (δ). What is the highest value that could be given to E? also to what percentage of accuracy could C_1 be determined, and what should be the figure of merit of the galvanometer in order that this percentage of accuracy may be attained?

$$E = 1 \cdot 5 \times 1 = 1 \cdot 5 \text{ volts};$$

therefore we cannot make E greater than, say, 1 Daniell cell (1 volt approximately).

$$\text{Percentage of accuracy} = \frac{100 \times 1 \cdot 5 \times 1 \times \frac{1}{2}}{1 \times 1000} = \cdot 075 \text{ per cent.}$$

To enable this percentage of accuracy to be obtained, the figure of merit (c) of the galvanometer must not be less than

$$c' = \frac{1 \cdot 5 \times 1 \times \frac{1}{2}}{500 \times 1000} = \cdot 0000015;$$

the figure of merit, therefore, of the galvanometer in question is sufficient for the required purpose.

To sum up, then, we have

Best Conditions for making the Test.

388. Make E the nearest possible value below $C_1 r$.

The figure of merit of the galvanometer should not be less than $\frac{C_1 r \delta}{G n}$.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{100\, C_1 r \delta}{E n}.$$

SIEMENS' ELECTRO-DYNAMOMETER.

389. This apparatus, although it can be used for measuring ordinary powerful currents, yet has the special advantage that it enables rapidly *alternating* currents to be measured; such currents would give no indications on an ordinary galvanometer.

The principle of the electro-dynamometer is based upon the mutual action of currents upon one another, i.e. upon the fact that currents in the same direction attract, and in opposite directions repel, one another. Fig. 164 shows how the principle is applied.

A B C D is a fixed wire rectangle, and *a b c d* a smaller one,

suspended by a thread, *t*, within the larger, so that it can turn freely about its axis; the planes of the two are at right angles to each other. Now, if the two rectangles be connected together in the way shown, then a current entering at W_1, and passing out at W_2, will traverse the two, and the current passing from B to C will attract the current passing from *a* to *d*, and will repel the current passing from *c* to *b*. A similar action takes place with reference to the current passing from D to A, consequently the smaller rectangle, under the influence of the forces, will tend to turn about its axis, in the direction in which the hands of a watch rotate. If the current enters at W_2, and leaves at W_1, then, inasmuch as the directions of all the currents in the wires are reversed, the small rectangle must still tend to turn in the direction indicated. If one or both of the rectangles consist of several turns of wire, the turning effect for a given current will be proportionately increased.

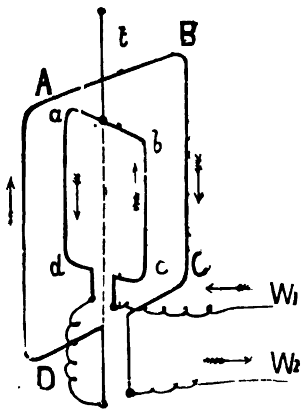


FIG. 164.

As the turning effect on the coil is produced by the action of the current through the fixed coil acting on the current through the movable coil, and as the two coils are in the same circuit, it follows that if the current passing through the fixed coil is doubled, then the current passing through the movable coil is also doubled, consequently we have one doubled current acting upon another doubled current, and therefore we must have a quadruple deflective effect—in other words, the deflective force tending to turn the movable coil will vary as the *square* of the current. The way in which this principle is utilised will be best understood by reference to Fig. 165 (page 354) which shows a general view of the Siemens Dynamometer.

The apparatus consists of a rectangle of wire hung from a fibre whose upper end is fixed to a thumb-screw; the latter is provided with a pointer which can be moved round a graduated dial; one end of a spiral spring is also attached to the rectangle, the other end being fixed to the thumb-screw. In this arrangement the number of degrees to which the pointer is directed evidently indicates the amount of torsion given to the spiral spring. To the rectangle also is fixed a pointer, the end of which just laps over

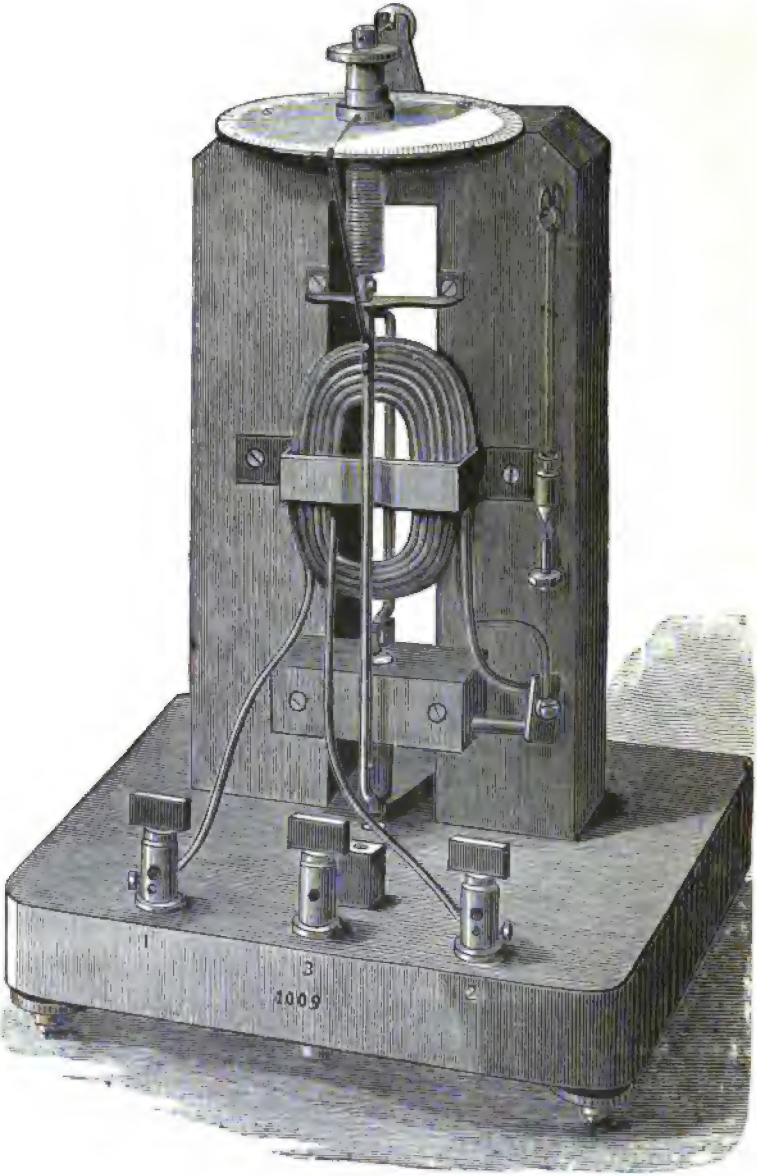


FIG. 165.

the edge of the graduated dial. The rectangle encircles a coil consisting of several turns of thick, and a larger number of turns of thin, wire; the two ends of the thick wire are connected to terminals 2 and 3, and the two ends of the thin wire to terminals 1 and 3. Connection is made between the rectangle and the wire coils by mercury cups, into which dip the ends of the wire forming the rectangle. The base-board has three levelling screws; the level consists simply of a small pointed weight hung at the end of a rod (seen on the right of the figure), the pointed end hangs exactly over a fixed point when the instrument is level.

390. The method of using the instrument is as follows:—The wires leading the current whose strength is to be determined are connected to terminals 1 and 3, or 2 and 3, according as a strong or weak current has to be measured. The current deflects the rectangle; the thumb-screw is now turned in the reverse direction to that in which the rectangle has turned, and torsion being thereby put on the spiral spring the rectangle is forcibly brought back towards its normal position—that is, at right angles to the coils, or to the position at which the pointer attached to the rectangle stands at zero on the scale. The number of degrees of torsion given to the spiral spring being then read off, the strength of the current is found by reference to a table supplied with each instrument. To construct this table a current of a known strength is sent through the instrument, and then the degree of torsion required to bring the rectangle back to zero is carefully noted. This being done, the currents corresponding to other degrees of torsion are easily calculated. The force of torsion varies directly as the number of degrees through which the spiral spring is twisted, whilst, as has been before explained, the deflective effect of the current varies directly as the *square* of the latter. In other words, if ϕ° be the number of degrees of torsion required to bring the rectangle back to zero when it is traversed by a current of C ampères, then if C_1 be the current which will correspond to any other degree of torsion ϕ_1°, we have

$$\phi^\circ : \phi_1^\circ :: C^2 : C_1^2;$$

or

$$C_1 = \sqrt{\frac{\phi_1^\circ C^2}{\phi^\circ}}.$$

For example.

If 180° (ϕ°) of torsion were required to bring the rectangle back to zero when it was traversed by 47·57 (C) ampères of

current, what current (C_1) would be represented by 80° ($\phi_1°$) of torsion?

$$C_1 = \sqrt{\frac{80 \times 47 \cdot 5 \times 47 \cdot 5}{180}} = 31 \cdot 7 \text{ ampères.}$$

391. Like galvanometers, the Siemens electro-dynamometer is not susceptible of great accuracy when the readings are very low; in fact, the higher the readings are, the more accurate are the results obtainable. Thus, for example, 5° of torsion of the spring represents a current (in the instrument (No. 1009) shown by Fig. 165) of 7·93 ampères, whilst 5° more, that is 10° in all, represents a current of 11·23 ampères. In other words, a range of 5° of torsion only, represents a difference in the current of

$$\frac{(11 \cdot 23 - 7 \cdot 93)\,100}{7 \cdot 93} \text{ per cent.} = 42 \text{ per cent.}$$

If, however, the current had been 66·38 ampères, which corresponds to a torsion of 350°, then 5° more of torsion, or 355° in all, represents a current of 66·86 ampères, consequently the range of 5° of torsion in this case represents a difference in the current of

$$\frac{(66 \cdot 86 - 66 \cdot 38)\,100}{66 \cdot 38} \text{ per cent.} = \cdot 72 \text{ per cent.};$$

and a greater degree of torsion would have rendered the error still less.

Every instrument is supplied with a table which shows the current strengths corresponding to various angles of torsion; practically this table is different for every instrument, as it is almost impossible (nor is it necessary) to make two dynamometers alike. The table supplied with the instrument shown by Fig. 165 (No. 1009) is calculated so that the latter can theoretically be used for measuring currents varying from 1·05 to 66·86 ampères in strength. The thin wire coil is to be employed when currents of from 1·05 to 19·87 ampères are to be measured, and the thick wire coil for currents of from 3·54 to 66·86. The numbers of degrees of torsion representing various currents are all multiples of 5; thus the first calculation on the table (thick wire coil) is 1°, which represents 3·5 ampères of current; the next is 5°, representing 7·93 ampères; the next, 10°, representing 11·28 ampères; and so on. Practically the instrument cannot well be adjusted to a closer degree of accuracy than 5°.

The thin wire coil, having about three times the magnetic

effect of the thick one, requires, for a definite current, that the number of degrees of torsion to bring the needle back to zero be about three times that which is required in the case of the thick coil; in other words, with the thin wire coil we can practically measure currents to about three times the degree of accuracy which is possible with the thick coil; but, on the other hand, the highest current which we can practically measure with the thin coil is about one-third only of the highest current which can be measured with the thick coil.

The lowest current which can be measured consistent with a degree of accuracy equal to 10 per cent. is $5 \cdot 76$, for the next current below this on the table is $5 \cdot 25$, and therefore we have

$$\frac{(5 \cdot 76 - 5 \cdot 25)\,100}{5 \cdot 25} \text{ per cent.} = 10 \text{ per cent. nearly.}$$

If we require to be accurate within 1 per cent., then the lowest current we could measure would be $16 \cdot 77$, as the next current below this on the table is $16 \cdot 20$, and we therefore have

$$\frac{(16 \cdot 77 - 16 \cdot 60)\,100}{16 \cdot 70} \text{ per cent.} = 1 \text{ per cent. nearly.}$$

Since the percentage of accuracy is equal to

$$\frac{(C - C_1)\,100}{C_1} = \left(\frac{C}{C_1} - 1\right) 100$$

where C is a particular current, and C_1, the current next below it on the table, and since

$$C^2 : C_1^2 :: \phi^\circ : \phi_1^\circ$$

where ϕ° and ϕ_1° are the degrees of torsion corresponding to the currents C, C_1, therefore

$$\text{Percentage of accuracy} = \left(\sqrt{\frac{\phi^\circ}{\phi_1^\circ}} - 1\right) 100\,;$$

and as the smallest difference to which we can practically read is 5°, therefore

$$\text{Percentage of accuracy} = \left(\sqrt{\frac{\phi_1^\circ + 5^\circ}{\phi_1^\circ}} - 1\right) 100 = \lambda' \text{ say.}$$

Therefore

$$\sqrt{1 + \frac{5^\circ}{\phi_1^\circ}} = \frac{\lambda'}{100} + 1\,;$$

therefore

$$1 + \frac{5^\circ}{\phi_1{}^\circ} = \frac{\lambda'^2}{10,000} + 1 + \frac{\lambda'}{50};$$

therefore

$$\frac{5^\circ}{\phi_1{}^\circ} = \frac{\lambda'^2}{10,000} + \frac{\lambda'}{50};$$

or,

$$\phi_1{}^\circ = \frac{50,000}{\lambda'^2 + 200\,\lambda'},$$

which shows the smallest number of degrees of torsion which must be given to the spiral spring when measuring a current, in order that the latter may be measured to an accuracy of λ' per cent.

For example.

It was required to be able to measure currents of 10 ampères and upwards to an accuracy of 1 per cent., by means of an electro-dynamometer; how many degrees of torsion would the spiral spring be required to make?

$$\phi_1{}^\circ = \frac{50.000}{1 + 200} = 248^\circ;$$

showing that the electro-dynamometer must be so constructed that when currents of 10 ampères and upwards have to be measured, not less than 248° of torsion have to be given to the spiral spring in order to bring the needle back to zero.

392. From the construction and principle of the electro-dynamometer it must be evident that the accuracy of the absolute results obtained by its means must depend entirely upon the torsion of the spiral spring remaining constant. It seems possible that change of temperature and frequent use might alter the value of the torsion, but this is stated not to be the case. The instrument might probably be made of more value if its coil were composed of a large number of turns of thin wire, shunted by a thick wire shunt. The latter would be used when measuring the strong currents, whilst the correctness of the instrument could be verified by sending a comparatively weak current through the unshunted coil. It is not often that powerful currents of an accurately known value can be had for the purpose of verifying the correctness of an instrument, though weaker currents are almost always obtainable.

CHAPTER XIII.

MEASUREMENT OF ELECTROSTATIC CAPACITY.

DIRECT DEFLECTION METHOD.

393. THE simplest way of measuring electrostatic or inductive capacities is, with the same battery power, to compare the discharges from the unknown capacities with the discharge from a condenser of a known capacity; thus we note the discharge deflection a given by the standard condenser F, and then the discharges a_1, a_2, &c., given by the cables or condensers whose capacities F_1, F_2, &c., are required, in which case

$$F : F_1 : F_2 :: a : a_1 : a_2.$$

For example.

A standard condenser had a capacity of $\frac{1}{3}$ microfarad, and gave a discharge deflection of 300, and two other cables or condensers, F_1, F_2, gave discharge deflections of 225 and 180 respectively, then

$$\tfrac{1}{3} : F_1 : F_2 :: 300 : 225 : 180;$$

that is,

$$F_1 = \tfrac{1}{3} \cdot \frac{225}{300} = \tfrac{1}{4} \text{ microfarad,}$$

and

$$F_2 = \tfrac{1}{3} \cdot \frac{180}{300} = \tfrac{1}{5} \text{ microfarad.}$$

If we use shunts and obtain the *same* deflection, then

$$\tfrac{1}{3} : F_1 : F_2 :: \frac{G+S}{S} : \frac{G+S_1}{S_1} : \frac{G+S_2}{S_2}.$$

394. In measuring the electrostatic capacity of a cable by this method, the connections for measuring the discharge from the cable would be made in the manner shown by Fig. 166 (page 360). The arrangements for measuring the discharge from the condenser would be those indicated by Fig. 142 (page 313).

Then, as before, the capacity of the cable will be to the capacity of the condenser, as the discharge deflection of the one is to the discharge deflection of the other, or obtaining the same deflection by means of shunts, as the multiplying power of the shunts.

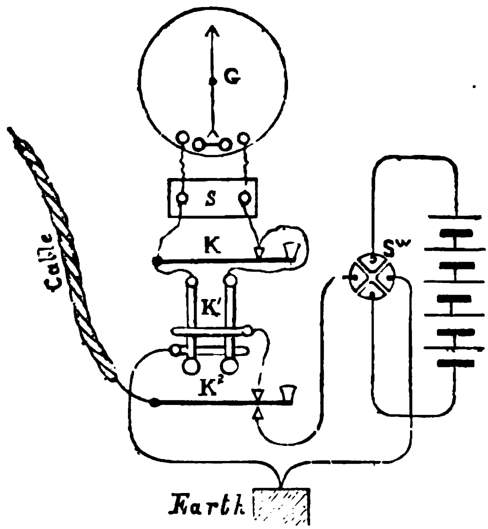


FIG. 166.

395. The capacity per mile will be the result divided by the mileage of the cable.

396. When a number of capacities of about the same value have to be measured, as, for instance, the capacities of two-knot lengths of cable core, a device may be adopted which considerably simplifies the operation. Let F be the capacity of the standard condenser whose discharge is D divisions, and let f be the capacity of one of the lengths of cable, and d the discharge from the same. Then we have

$$F : f :: D : d,$$

or

$$f = \frac{F\,d}{D}.$$

Now if we make $\frac{F}{D}$ a submultiple of 10, then the value of d read off from the scale will give at once the value of f. Thus if

F were a condenser of $\frac{1}{3}$ microfarad capacity, and we so adjusted the galvanometer that this capacity gave a discharge deflection of a little over 333 divisions, then we should have

$$f = \frac{\frac{1}{3}\,d}{333\frac{1}{3}} = \frac{d}{1000};$$

so that if the discharge deflection reading from the cable consisted of three figures, a decimal point put before the latter would give at once the capacity of the cable; or if the reading consisted of two figures, then we must put a decimal point and a cypher. In the same way, if we had a condenser of 1 microfarad capacity, we should adjust the galvanometer so as to obtain a deflection of 100 divisions, for then

$$f = \frac{1\,d}{100} = \frac{d}{100}.$$

Siemens' Loss of Charge Discharge Method.

397. The principle of this method of measurement is that of observing the rate at which the charged condenser or cable, whose capacity is required, discharges itself through a known resistance, and calculating the capacity from a formula which we will now consider.

The elements with which we have to deal are: capacity (farad), resistance (ohm), quantity (coulomb), time (second), and potential (volt).

Let us suppose the cable or condenser has an electrostatic capacity of F farads, and is charged to a potential of V volts, so that it contains Q coulombs (equal to V F) of electricity, and is discharging itself through a resistance of R ohms during one second.

The quantity of electricity in the condenser or cable at starting is Q coulombs.

If now we take a very short interval of time t, we may consider the discharge, which really varies continually, to flow throughout that time t, at the same rate as it had at the commencement; and the smaller t is taken, the more accurate will be the result.

Thus, since the quantity escaping is directly proportional to the potential driving it out, and to the time during which the escape occurs, and inversely proportional to the resistance through

which the escape takes place, the quantity escaping will vary as

$$\frac{V\,t}{R};\ \text{that is it equals}\ \frac{V\,t}{R}\,K,$$

where K is a constant to be determined.

Now the units are so made that a condenser of 1 farad electrostatic capacity charged to a potential of 1 volt, that is, containing 1 coulomb of electricity, will commence to discharge itself through a resistance of 1 ohm, at the rate of 1 coulomb per second. That is to say,

$$1 = \frac{1 \times 1}{1}\,K, \quad \text{therefore,} \quad K = 1.$$

The quantity escaping during the interval of time t in our problem is therefore

$$\frac{V\,t}{R}.$$

The quantity remaining in the condenser will be

$$Q - \frac{V\,t}{R} = Q - \frac{V\,F\,t}{F\,R} = Q\left(1 - \frac{t}{F\,R}\right).$$

Again, since this is the quantity at the commencement of the second interval, that at the end will be

$$\left[Q\left(1 - \frac{t}{F\,R}\right)\right]\left[\left(1 - \frac{t}{F\,R}\right)\right] = Q\left(1 - \frac{t}{F\,R}\right)^2,$$

and that at the end of the nth interval will be

$$Q\left(1 - \frac{t}{F\,R}\right)^n = q.$$

Let these n intervals of t seconds equal T, so that $n\,t = T$.

Now we have seen that the smaller t is, the more accurate will our results be. Let us therefore make t infinitely small, and n infinitely great, so that $n\,t$ still $= T$, we shall then get a perfectly accurate result, and the amount remaining at the end of time T will be

$$q = Q\left(1 - \frac{T}{n\,F\,R}\right)^n$$

where $n = \infty$.

To evaluate q put

$$\frac{T}{n\,F\,R} = -\frac{1}{x},$$

so that

$$x = \infty \text{ when } n = \infty ;$$

then

$$q = Q\left[\left(1 + \frac{1}{x}\right)^x\right]^{-\frac{T}{FR}}$$

when $x = \infty$; but when this is the case the expression within the square brackets is known to be equal to e,* thus

$$\frac{q}{Q} = e^{-\frac{T}{FR}},$$

therefore

$$\frac{T}{FR} = \log_e \frac{Q}{q},$$

therefore

$$F = \frac{T}{R \log_e \frac{Q}{q}};$$

but

$$\frac{Q}{q} = \frac{VF}{vF} = \frac{V}{v},$$

where v is the value of the potential corresponding to the value q of the quantity, thus

$$F = \frac{T}{R \log_e \frac{V}{v}} = \frac{T}{2 \cdot 303 \; R \log \frac{V}{v}},$$

where, as stated at first, T is measured in seconds, F in farads, and R in ohms.

Since V and v now appear in the form of a proportion, the unit in which they are measured is immaterial, although they were measured at the outset in volts.

In practice R is usually measured in megohms (1,000,000 ohms), and consequently F will, in such a case, be measured in microfarads ($\frac{1}{1,000,000}$ farad).

For example.

A fully charged condenser gave a discharge deflection of 300 divisions (V); after being recharged and allowed to discharge

* Todhunter's Algebra, fifth edition, Chapter XXXIX.

itself through a resistance of 500 megohms for 60 seconds (T), the discharge deflection obtained was 200 divisions (v). What was the capacity of the condenser?

$$F = \frac{60}{2 \cdot 303 \times 500 \log \frac{300}{200}} = \cdot 295 \text{ microfarads.}$$

398. In executing this test it is advantageous to make V and v bear a certain proportion to one another, for this will cause any small error in reading the value of v to produce as small an error as possible in the value of F when the latter is worked out from the formula. This may be proved thus:—

Let us assume R to be constant, and let there be an error λ in F caused by an error δ_1 in v and an error δ_2 in V, the error δ_1 being plus and δ_2 minus, so that the total resulting error is as great as possible; we then have

$$F + \lambda = \frac{T}{R \log_e \frac{V - \delta}{v + \delta_1}}, \text{ or, } \lambda = \frac{T}{R \log_e \frac{V - \delta_2}{v + \delta_1}} - F$$

but

$$F = \frac{T}{R \log_e \frac{V}{v}}, \text{ or, } \frac{T}{R} = F \log_e \frac{V}{v},$$

therefore

$$\lambda = F \frac{\log_e \frac{V}{v}}{\log_e \frac{V - \delta_2}{v + \delta_1}} - F = F \frac{\log_e \frac{V}{v} - \log_e \frac{V - \delta_2}{v + \delta_1}}{\log_e \frac{V - \delta_2}{v + \delta_1}}$$

$$= F \frac{\log_e \left(1 + \frac{\delta_1}{v}\right) - \log_e \left(1 - \frac{\delta_2}{V}\right)}{\log_e \frac{V - \delta_2}{v + \delta_1}};$$

but if δ_1 and δ_2 are very small,* we get

$$\lambda = F \frac{\frac{\delta_1}{v} - \left(- \frac{\delta_2}{V}\right)}{\log^e \frac{V}{v}}.$$

If the deflections are taken on a Thomson galvanometer (as would practically be always the case), then $\delta_1 = \delta_2$, so that we get

$$\lambda = F \frac{\delta_1 \left(\frac{1}{v} + \frac{1}{V}\right)}{\log_e \frac{V}{v}}.$$

* Todhunter's Trigonometry, third edition, Chapter XII.

The value of v, which makes λ a minimum, may be determined in the following manner:—

To make λ a minimum we must make

$$\frac{\log_e \frac{V}{v}}{\frac{1}{v} + \frac{1}{V}}$$

a maximum.

Let the above expression equal u, and let

$$v = \frac{V}{n},$$

we then get

$$u = V \frac{\log n}{n + 1},$$

then

$$\frac{du}{dn} = \left(\frac{1}{n+1}\right)^2 \left\{ (n + 1)\frac{1}{n} - \log_e n \right\} = 0$$

at a maximum; therefore

$$\log_e n = \frac{n+1}{n},$$

or

$$\log n = \left(\frac{n+1}{n}\right) \cdot 4343.$$

The solution of this equation is best effected by the "trial" method, viz. by giving n various values until one is found which approximately satisfies the equation. If we make $n = 3 \cdot 59$, we get

$$\cdot 55509 = \left(\frac{3 \cdot 59 + 1}{3 \cdot 59}\right) \cdot 4343 = \cdot 55527,$$

which is sufficiently close for the purpose.

We have, therefore,

$$V = v\,n, \text{ or, } v = \frac{V}{n} = \frac{V}{3 \cdot 59},$$

so that practically we may say—make $v = \frac{V}{3 \cdot 5}$.

We need not be particular, however, about making v *exactly* equal to $\frac{V}{3 \cdot 5}$, as we could make it 50 per cent. greater or less than this value without materially increasing λ. If the rate of fall were comparatively quick, there would be a positive advantage in making v less than $\frac{V}{3 \cdot 5}$, as the greater we make T the less will any small error in its value affect the correctness of F, as must be self-evident.

Now, if R is adjustable, it is clear that by making it large enough, we could make T large without reducing v too much. In the case of a cable, R, being the insulation resistance, is of course a fixed quantity; but when the measurement is being made with a condenser, any value may be given to R that is considered convenient. We therefore have

Best Conditions for making the Test.

399. Make v as nearly as possible equal to $\frac{V}{3 \cdot 5}$. When it is possible to adjust R, make the latter as high as convenient.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = F \frac{100\,(\delta_1 + \delta_2)}{2 \cdot 303\, v \log \frac{V}{v}}.$$

If the deflections are read on a Thomson galvanometer (as would usually be the case) then

$$\text{Percentage of accuracy} = F \frac{200\,\delta}{2 \cdot 303\, v \log \frac{V}{v}},$$

where δ is the fraction of a division to which each of the deflections V and v can be read.

400. When it is an ordinary condenser (whose insulation resistance would practically be infinite) that is to be measured, the connections would be the same as those given in Fig. 142, page 313, with the addition of the resistance, which would be inserted between the terminals of the condenser.

The instantaneous discharge (V) can be taken without removing the resistance; for, since the latter would be extremely high, there would be no time for any of the charge to have leaked out through it during the small interval occupied by the lever of the key in passing from the bottom to the top contact. To take the discharge after the interval of time, having charged the condenser by pressing down the lever of the discharge key (Fig. 142, page 313), we should depress the "Insulate" trigger, which would take the battery off but not discharge the condenser; then, after the noted interval of time, we should depress the "Discharge" trigger, which would allow the charge remaining to flow out, the deflection obtained from which gives us v.

401. To measure the capacity of a cable by this method, the connections would have to be those given in Fig. 166, page 360, and the way of making the test would be the same as has just been explained. R in this case would be the insulation resistance of the cable, which in this and the following method would have to be determined beforehand in the manner described in Chapter XV.

Inasmuch as R in a cable is a variable quantity and is dependent upon the time a charge is kept in the cable, a mean value only can be given to it, and therefore this and the following test can only give the value of F approximately.

SIEMENS' LOSS OF CHARGE DEFLECTION METHOD.

402. If the two terminals of a condenser are connected by a high resistance in the circuit of which a galvanometer is placed, and if the two terminals be also connected to a battery, then the condenser will become charged up, and the permanent deflection obtained on the galvanometer will represent the potential of the charge. If now the battery be taken off, a current will flow from the condenser through the resistance and the galvanometer, which current will continually decrease in strength as the condenser empties itself. But the current flowing at any particular moment will be represented by the deflection obtained at that moment, and this deflection will be the same as that which would be obtained if the condenser were kept continuously charged to the potential it had at that moment.

The deflection obtained therefore on the galvanometer when the battery is connected to the condenser, indicates the potential which the latter has when fully charged, and the deflection after any interval of time after the battery has been taken off, indicates the potential of the charge remaining; the capacity therefore is given by the formula

$$F = \frac{T}{2 \cdot 303\ R \log \frac{D}{d}} \text{ m.f.,} \qquad [A]$$

in which D is the deflection obtained when the battery is on, and d the deflection obtained after T seconds, the battery being off during that time. R is the resistance in megohms through which the charge flows.

It may be remarked that the deflection obtained when the battery is on is not affected by the presence of the condenser; it would be the same whether the condenser were connected or not.

403. The connections for making a test of this kind would be as follows:—Referring to Fig. 166, page 360, the terminal of K_1 which is connected to the top contact of K_2, should in the present case be connected through the resistance R to terminal A of the condenser; the other connections remain the same.

404. In the case of a cable where the flowing out of the charge

takes place through the insulating sheathing, a galvanometer cannot be put in the circuit of the flow. To enable the fall of charge to be observed, therefore, a high resistance in circuit with the galvanometer is connected to the cable, and through this resistance a part of the charge passes. As it is only the *rate* at which the fall takes place that is required, it is quite sufficient, in order to observe this fall, that a part only of the charge be allowed to flow through the galvanometer.

If we call R_1 the insulation resistance of the cable, and R_2 the resistance connected to it, then the total resistance through which the charge flows will be

$$\frac{R_1 R_2}{R_1 + R_2}.$$

This quantity must be substituted in the place of R in equation [A], so that we have

$$F = \frac{T}{2 \cdot 303 \frac{R_1 R_2}{R_1 + R_2} \log \frac{D}{d}} \text{ m.f.}$$

The resistance R_2, it may be remarked, includes the resistance of the galvanometer.

As in the first test, it is necessary that R_2, through which the discharge has to pass, be sufficiently great to prevent the flow from being too rapid.

For example.

A cable 30 knots in length being connected up, for making the test just described, with a galvanometer, and a resistance R_2, of 4 megohms, the deflection obtained was 300 divisions (D). On taking off the battery the deflection after 30 seconds (T) fell to 100 divisions (d); the mean insulation resistance R_1 of the cable was 10 megohms. What was the electrostatic capacity (F) of the cable?

$$F = \frac{30}{2 \cdot 303 \times \frac{10 \times 4}{10 + 4} \log \frac{300}{100}} = 9 \cdot 55 \text{ m.f.}$$

or

$$\frac{9 \cdot 55}{30} = \cdot 318 \text{ m.f. per knot.}$$

405. The connections for making this test would be as follows:—Referring to Fig. 166, page 360, the terminal of key

K_1, instead of being connected to the top contact of the discharge key, would in the present case be connected to the cable through the resistance R_2.

406. A great advantage which this test possesses over the first method (page 359) lies in the fact that it is correct either for long or short cables. Discharge deflections from long cables do not correctly represent their capacity, in consequence of a retardation which takes place in them and which causes the deflection of the galvanometer needle to be less than it would be if this retardation did not exist. By adopting the fall of deflection plan we avoid this cause of error; but as we pointed out at the conclusion of the last test, since R_1 can only have a mean value, the value of F obtained from the formula will only be approximate.

THOMSON'S METHOD.

407. This is a very good method, and it can be applied to long cables, &c., with very accurate results.

The following is its principle:—

If we have two condensers containing equal charges of opposite potentials, and we connect the two together, the two charges will combine and annul one another, and if we then connect the two condensers, so joined, to a galvanometer, no deflection will be produced, there being no charge left in either of the two. If, however, the charge in one condenser exceeds that in the other, then the union of the two condensers will not entirely annul their charges, but an amount will remain equal to the difference of the two quantities. This quantity will deflect the needle if the joined condensers be now connected to the galvanometer, the deflection being to the right or left, according as the charge in the one or other of the condensers had the preponderance in the first instance.

If then we know the capacity of one condenser, and we so adjust the potentials of the two that no charge remains when they are joined together, we can determine the capacity of the other condenser.

Let Q_1 and Q_2 be the charges in each; then

$$Q_1 : Q_2 : : V_1 F_1 : V_2 F_2,$$

where F_1 and F_2 are the capacities of the two, and V_1 and V_2 the potential of their charges.

When $Q_1 = Q_2$ then

$$V_1 F_1 = V_2 F_2,$$

or

$$F_1 = \frac{V_2}{V_1} F_2.$$

408. An important element in this test is the adjustment of the potentials V_1 and V_2. Fig. 167 shows a method of making the test when it is a cable whose capacity has to be measured.

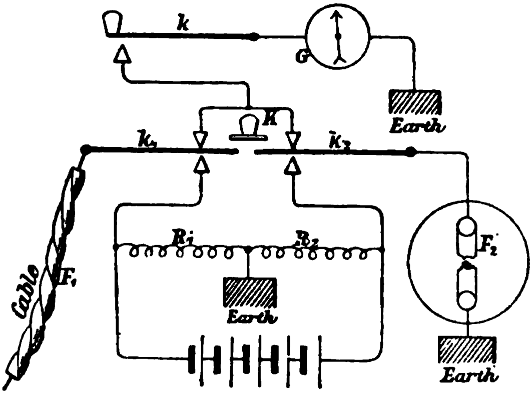


FIG. 167.

The poles of the battery are joined together by two resistances, R_1 and R_2, connected to earth as shown. Then the potentials at the points of junction of the battery with the resistances will be in the proportion

$$V_1 : V_2 :: R_1 : R_2;^*$$

and since

$$F_1 = \frac{V_2}{V_1} F_2,$$

therefore

$$F_1 = \frac{R_2}{R_1} F_2. \qquad [A]$$

409. In making the test practically, R_1 and R_2 are first adjusted as nearly as can be guessed in the proportion of F_2 to F_1, keys k_1 and k_2 are then depressed by means of the knob K; this charges the cable and the condenser.

K is now released so as to allow k_1 and k_2 to come in contact with their upper stops; as the two latter are joined together, the cable and condenser become connected to each other.

* Page 321.

Key k is now pressed, which allows any charge which may remain uncancelled to be discharged through the galvanometer G. If no deflection is produced, then R_1 and R_2 are correctly adjusted, but if not they must be readjusted until no discharge is obtained; F_1 is then calculated from the formula.

For example.

A cable 500 knots long was joined up with a condenser of 20 microfarads capacity, and with resistance coils, according to Thomson's method of measuring electrostatic capacities. When R_1 and R_2 were adjusted to 500 and 4400 ohms respectively, no charge remained in the cable and condenser when the two were connected together. What was the capacity of the cable?

$$F_1 = \frac{4400}{500} \times 20 = 176 \text{ m.f.},$$

or

$$\frac{176}{500} = \cdot 352 \text{ m.f. per knot.}$$

410. Fig. 168 shows a very convenient form of key, designed by Mr. Lambert, which enables the test to be made with the greatest facility. By pushing forward key button K the two keys

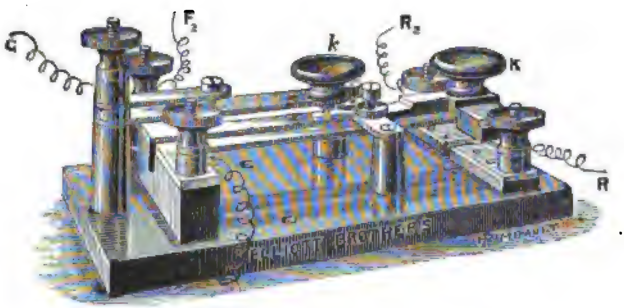


FIG. 168.

k_1, k_2 (Fig. 168) are depressed, so that F_1 and F_2 become charged, and upon drawing K back, k_1 and k_2 are allowed to rise, thus causing the charges to mix; finally, by depressing k the galvanometer is brought into circuit.

In the most recent form of this piece of apparatus, on the depression of key k the cable F_1 becomes disconnected, so that

only the condenser F_2 becomes connected to the galvanometer. By this arrangement any disturbing force which may cause the charge in the cable to vary slightly, and consequently to affect the galvanometer, is prevented from acting.

411. If it were the capacity of a condenser that was to be measured, then the connections would be similar to those in Fig. 167, with the exception that the points there put to earth would in the present case be connected to the second terminal of the condenser.

The resistances R_1 and R_2 may be formed of a slide resistance, the slider being to earth in the case of a cable test, or connected to the second terminal of the condenser in the case of a condenser test.

412. As in the "Direct deflection method" (page 359), the test can be considerably simplified if we make $\frac{F_2}{R_1}$ (equation [A], page 370) a submultiple of 10, for then the value of R_2 read off from the resistance box will at once give the value of F_1. Thus if F_2 were a condenser of, say, ·5 of a microfarad, and if R_1 were 5000 ohms, then the capacity of F_2 can be read off directly from R_2 to four places of decimals.

413. When a long cable has to be tested by this method, Mr. A. Jamieson recommends that K be depressed for five minutes to charge, and then raised for ten seconds for mixing previous to depressing *k*. It is also advisable to take the mean of several tests made alternately with zinc to line and copper to line.

414. With regard to the "Best conditions for making the test" it is advisable that the capacity of the condenser F_2 be as nearly equal to F_1 as possible, so that the potentials to which the two have to be charged may not differ to any very great extent. For if a long cable has to be tested, then inasmuch as the latter would have to be charged to a potential of at least 5 Daniells so as to swamp, as it were, any local charge, the potential to which the condenser (if small) would have to be charged would be very great; this would be liable to cause an error, from the fact that with a very high potential a certain amount of the charge becomes absorbed, and this charge would cause a deflection of the galvanometer needle over and above that due to the simple inequality between the actual free quantities in the two capacities. This abnormal deflection might of course be mistaken as being due to an incorrect adjustment of R_1 and R_2. If F_2 is about a fifth of F_1 it will not be too small for the purpose of the test.

The values given to R_1 and R_2 should be as high as possible so

that their range of adjustment may be sufficiently wide. The battery power should be sufficiently high to enable a perceptible discharge deflection to be obtained when R_2 (the larger of the two resistances) is 1 unit out of exact adjustment; this is best determined by experiment.

We have therefore

Best Conditions for making the Test.

415. Make F_2 as nearly equal to F_1 as possible.

Make R_1 and R_2 as high as possible.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{100}{R_2}.$$

GOTT'S METHOD.*

416. This method is shown by Fig. 169; it is executed as follows:—

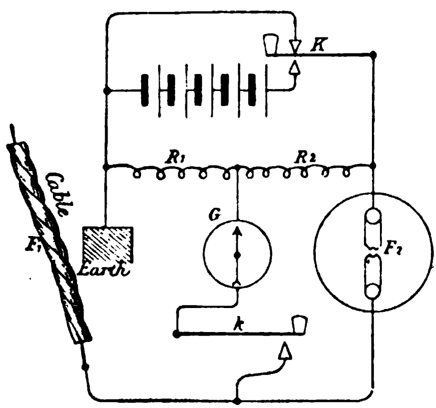


FIG. 169.

R_1 and R_2 are first adjusted as nearly as can be estimated in the proportion of F_2 to F_1. The key K is then depressed and clamped down; this causes both the cable and condenser to

* 'Journal of the Society of Telegraph-Engineers,' vol. x. p. 278. This method, although independently devised by Mr J. Gott, is practically identical with that of Sir William Thomson (Lord Kelvin) described in vol. i. p. 397, of the same Journal.

become charged, since they are connected together in "cascade," or series. After an interval of five seconds key k is depressed, and if a deflection is observed on the galvanometer G, this key is raised, key K is unclamped so that the latter is put to earth, and the condenser is short-circuited by means of its plug for a few seconds. R_1 or R_2 is now readjusted, and the foregoing operations again gone through. When finally it is found that no deflection on the galvanometer is observed on depressing key k, then

$$F_1 : F_2 :: R_2 : R_1,$$

or

$$F_1 = \frac{R_2}{R_1} F_2.$$

It is obvious that we must have

Best Conditions for making the Test.

417. Make F_2 as nearly equal to F_1 as possible.
Make R_1 and R_2 as high as possible.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{100}{R_1}.$$

Gott's method is a very satisfactory one, and it possesses the advantage over that of Thomson (page 369) of not requiring a well-insulated battery. The method is almost exclusively employed in the Cable Department of Messrs. Siemens' Telegraph Works, Charlton, a slide resistance of 10,000 ohms adjustable to 1 ohm being employed to give the ratio $\frac{R_1}{R_2}$.

Muirhead's Correction for Gott's Method.

418. Dr. A. Muirhead, who has had considerable experience of the Gott method of measuring capacities, especially with reference to the measurement of large condensers, points out that when a condenser or cable which is subject to absorption or *electrification* (page 409) is being measured, any delay in obtaining a balance or reading on the slides, causes incorrect results to be obtained,

since the effect of the electrification is apparently to increase the capacity, according to the time the electrification is allowed to continue.

In order therefore to obtain the true capacity of a condenser or cable which is liable to this absorption or electrification, a correction is required. To arrive at this correction it is assumed (and probably correctly so) that the actual capacity of the condenser or cable changes as the current is kept on, but that when the charging battery is taken off the capacity immediately springs back, as it were, to its true value, the potential of the charge remaining unaltered through a corresponding portion of the charge being reabsorbed, this reabsorbed charge immediately afterwards oozing out.

Assuming the foregoing assumption to be correct, the correction required is arrived at as follows:—

Balance being obtained on the galvanometer (Fig. 169), let v_1 be the potential at the junction of F_1 and F_2, that is at the key (k) contact, and also at the junction of R_1 and R_2; then the quantity of charge in the condenser F_2 (in which no absorption must take place*) is $(V - v_1)\ F_2$, V being the potential at the junction of R_2 and the condenser.

Now if at the moment balance is obtained on the galvanometer we disconnect the condenser from R_2 and connect it to earth, then the charge in F_2 will combine with the charge in F_1, and if these two charges were equal, as would be the case if there were no absorption, that is, if F_1 were the true capacity of the cable, the combination would clear both cable and condenser; but inasmuch as there is absorption in the cable, then at the moment F_2 is disconnected from R_2, and the battery is consequently also disconnected, the capacity of the cable springs back to its true value F_1 and, a corresponding part of its charge being reabsorbed, the total charge in F_1 is less than it was before. When therefore the condenser and cable become connected together as explained, the combination of the charges does not produce neutrality, but leaves a small charge in the combined capacities, which charge is of course equal to that reabsorbed by F_1, being the excess of the charge in F_2 over that in the cable. The combined capacities of F_1 and F_2 being $F_1 + F_2$, let v_2 be the potential of the charge remaining, then the quantity of this charge will be

$$v_2\,(F_1 + F_2),$$

* A mica condenser (§ 328, page 308) is necessary for this to be the case.

and this must be equal to the difference between the charge in F_2, that is $(V - v_1) F_2$, and the charge in F_1, that is $v_1 F_1$. We have then

$$v_2 (F_1 + F_2) = (V - v_1) F_2 - v_1 F_1,$$

therefore

$$v_2 F_1 + v_2 F_2 = V F_2 - v_1 F_2 - v_1 F_1,$$

therefore

$$F_1 (v_1 + v_2) = F_2 (V - (v_1 + v_2)),$$

that is

$$F_1 = F_2 \frac{V - (v_1 + v_2)}{v_1 + v_2}.$$

In order to be able to make use of the formula, we require to know the value of v_2, in terms of the slide resistance $R_1 + R_2$, since V and v_1 are respectively represented by R_1 and R_2. The determination of v_2 may be effected as follows:—

Immediately after balance has been obtained on the galvanometer, the connection of the latter on to the resistances is shifted to the extreme left of R_1, the condenser connection to R_2 is disconnected from the latter, and put to earth, and the galvanometer key, k, is depressed; this causes the residual charge in the condenser and cable to discharge through the galvanometer, producing a throw of a divisions; the key being raised again, the galvanometer connection is now shifted from the end of R_1 to a distance of, say, r ohms to the right; the key k is again depressed, causing a charge of a potential corresponding to the resistance r, to charge the cable and condenser, and to produce a throw of β divisions. The required value of v_2 will then obviously be

$$r \frac{a}{\beta}.$$

Hence, since $R_1 + R_2$ corresponds to V, R_1 to v_1, and $r \frac{a}{\beta}$ to v_2, we get

$$F_1 = F_2 \frac{R_1 + R_2 - \left(R_1 + r \frac{a}{\beta}\right)}{R_1 + r \frac{a}{\beta}} = F_2 \frac{R_2 - r \frac{a}{\beta}}{R_1 + r \frac{a}{\beta}}.$$

For example.

Balance was obtained by making $R_1 = 2000$, and $R_2 = 8000$. On shifting the galvanometer connection to the end of R_1 and

putting the condenser connection to earth, as explained, a throw of 80 divisions (a) was observed when k was depressed. k being raised again, the galvanometer connection was moved a distance of 10 ohms (r) to the right; on depressing k a deflection of 200 divisions, β, was observed. The capacity of F_2 being 10 microfarads, what was the apparent and the corrected capacity of F_1?

$$\text{Apparent capacity} = 10\,\frac{8000}{2000} = 40 \text{ microfarads.}$$

$$\text{Corrected capacity} = 10\,\frac{8000 - 10\,\frac{80}{200}}{2000 + 10\,\frac{80}{200}}$$

$$= 10\,\frac{8000 - 4}{2000 + 4} = \frac{79960}{2004} = 39 \cdot 9 \text{ microfarads.}$$

419. In order that the correction may be properly effected, it is very necessary that all the operations mentioned be carried out one after the other as quickly as possible, otherwise the absorbed charge will ooze out and falsify the results; a special form of key is therefore required for the test. With the key designed by Mr. Saunders, and shown in general view by Fig. 170, all the requirements are satisfied. The way in which the connections are made for the test is shown by Fig. 171.

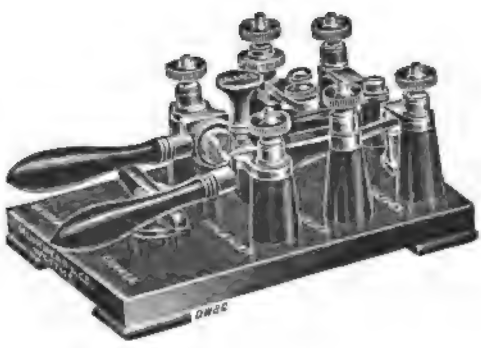


FIG. 170.

The manipulations for carrying out the test are as follows:—

1st. K_1 is pressed down; this by means of a rocking piece b raises key K_2 to its top contact. Under these conditions both the

cable and condenser are put to earth (through the galvanometer), thereby rendering both neutral.

2nd. K_2 is pressed down; this causes K_1 to be raised to its top contact. The cable and condenser are now joined up as in Fig. 169; the slider of the resistances R_1 R_2 is then adjusted until the equilibrium position is obtained, i.e. until no deflection is observed when key k is depressed.

3rd. Immediately after equilibrium is obtained the slider s is moved to A, and K_2 is raised; this causes the charges in the cable and condenser to combine. After a few seconds K_1 is pressed

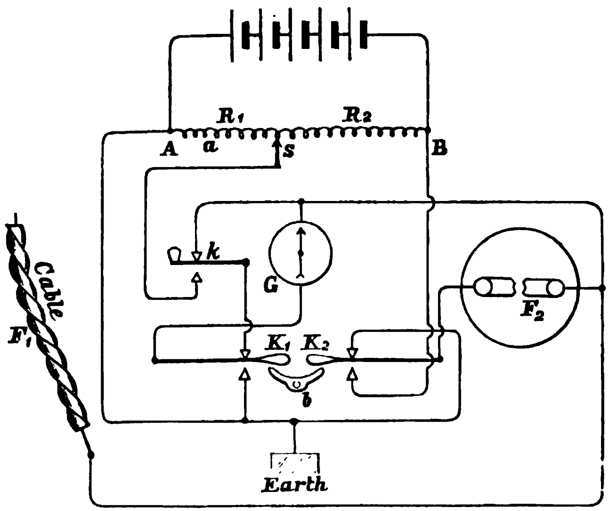


FIG. 171.

down; this causes the residual charge in the cable and condenser to discharge through the galvanometer giving α.

4th. The slider s is moved back, say, a distance of 10 ohms, that is to a, and K_1 being raised k is depressed; this causes a back rush into the cable and condenser through the galvanometer, producing a throw which gives β.

When applied to measurements of 1000 or 2000 knots of cable, Muirhead's method ceases to yield correct results, apparently on account of the extra retardation of discharge by the resistance of the conductor.

DIVIDED CHARGE METHOD.

420. If a charged condenser has its two terminals connected to the two terminals of a second condenser, which contains no charge, then the charge will become distributed over the two; and if the condensers be then separated, the quantities held by them will be directly proportional to their respective capacities. Thus, if Q_2 be the charge contained in a condenser whose capacity is F_2, then if it is connected to a condenser or cable whose capacity is F_1, the quantity Q which will remain in F_2 will be

$$Q = Q_2 \frac{F_2}{F_1 + F_2}.$$

From this we get

$$F_1 = F_2 \frac{Q_2 - Q}{Q}. \qquad [A]$$

If therefore Q_2 be the discharge obtained from a condenser F_2 when full, and Q the discharge obtained from it when, after being charged from the same battery, it is connected for a few seconds to F_1, then the capacity of F_1 is given by the above formula.

For example.

A condenser of $\frac{1}{3}$ microfarad capacity (F_2), when fully charged gave a discharge of 300 (Q_2). After being recharged and connected a few seconds to a piece of cable whose capacity F_1 was required, the quantity of charge remaining gave a discharge of 140 (Q). What was the capacity of the piece of cable?

$$F_1 = \frac{1}{3} \times \frac{300 - 140}{140} = \cdot 381 \text{ m.f.}$$

421. The capacity which the condenser F_2 should have in order that the test may be made as accurately as possible, may be thus arrived at:—

Let there be an error λ in F_1 caused by an error $-\delta$ in Q and an error $+\delta$ in Q_2, so that λ is as great as possible; we then have

$$F_1 + \lambda = F_2 \frac{Q_1 + \delta - (Q - \delta)}{Q - \delta} = F_2 \frac{Q_2 - Q + 2\delta}{Q - \delta};$$

but we know that

$$F_1 = F_2 \frac{Q_2 - Q}{Q}, \quad \text{or,} \quad F_2 = F_1 \frac{Q}{Q_2 - Q};$$

therefore

$$F_1 + \lambda = F_1 \frac{Q}{Q_2 - Q} \times \frac{Q_1 - Q + 2\delta}{Q - \delta};$$

that is

$$\lambda = F_1 \left\{ \frac{Q}{Q_2 - Q} \times \frac{Q_2 - Q + 2\delta}{Q - \delta} - 1 \right\} = F_1 \frac{(Q_2 + Q)\delta}{(Q_2 - Q)(Q - \delta)};$$

or, since δ is a very small quantity, we may say

$$\lambda = \frac{(Q_2 + Q)\,\delta}{(Q_2 - Q)\,Q}. \qquad \text{[B]}$$

We have then to find the value of Q which makes λ as small as possible. Now

$$\frac{(Q_2 + Q)\,\delta}{(Q_2 - Q)\,Q} = \frac{\delta}{Q_2}\left\{\frac{Q_2 - Q}{Q} + \frac{2\,Q}{Q_2 - Q} + 3\right\}$$

$$= \frac{\delta}{Q_2}\left\{\frac{Q_2 - Q}{Q}\left[1 - \frac{Q\sqrt{2}}{Q_2 - Q}\right] + 2\sqrt{2} + 3\right\},$$

and to make the latter expression as small as possible we must make

$$\left[1 - \frac{Q\sqrt{2}}{Q_2 - Q}\right]$$

as small as possible; that is to say, we must make

$$1 - \frac{Q\sqrt{2}}{Q_2 - Q} = 0,$$

or

$$\frac{Q\sqrt{2}}{Q_2 - Q} = 1;$$

therefore

$$Q\sqrt{2} = Q_2 - Q,$$

or

$$Q(\sqrt{2} + 1) = Q_2;$$

that is

$$Q = \frac{Q_2}{\sqrt{2} + 1} = \frac{Q_2}{2\cdot4142}.$$

It was pointed out, however, in a similar investigation which we made in § 123, page 124, that practically we may say, make—

$$Q = \frac{Q_2}{3},$$

or, in other words, the capacity of F_2 should be such that when it is connected to F_1 it should lose two-thirds of its charge. This is obtained, of course, by making F_2 equal to $\frac{F_1}{2}$.

422. The connections for the practical execution of the test would be very similar to those shown in Fig. 142, page 313, but the condenser or cable under trial would be substituted in the place of the battery. When it is a cable whose capacity is being measured, then terminal B would be put to earth, and the wire shown as leading from B to the battery would be removed. The test would then be made in the following manner:—

Key K_2 being pressed down so as to hitch on the "Insulate" trigger (Fig. 142, page 313), the condenser C would be charged

by touching the terminals A and B with the wires from the two poles of the battery. The "Discharge" trigger of the key then being depressed, the discharge Q_2 is noted. The key then being again placed at "Insulate," and the condenser again charged up by the battery, the key would be pressed down on to its bottom contact; this puts the condenser C in connection with the trial condenser or cable. The "Discharge" trigger then being pressed, the discharge Q is noted.

Best Conditions for making the Test.

423. Make F_2 as nearly equal to $\frac{F_1}{2}$ as possible.

Possible Degree of Accuracy attainable.

From equation [B] (page 380) it follows that

$$\text{Percentage of accuracy} = \frac{100\,(Q_2 + Q)\,\delta}{F_1\,(Q_2 - Q)\,Q},$$

where δ is the fraction of a division to which each of the deflections Q and Q_2 can be read.

424. By a modification of the foregoing method, due to the late Dr. Siemens, a comparatively small condenser may be used for measuring large capacities. It may be called

SIEMENS' DIMINISHED CHARGE METHOD.

If we connect a condenser to a charged cable, the latter loses the amount which the condenser takes up, and if the condenser be discharged and then again connected to the cable, and again discharged, and this process be repeated several times, the quantity in the cable can be definitely diminished as much as we like. The quantity removed each time, however, is not the same, but becomes less and less after each discharge.

Let Q_2 be the quantity contained in the condenser, and Q_1 the quantity contained in the cable, when the two are charged full from the same battery. Then

$$Q_2 : Q_1 : : F_2 : F_1,$$

or

$$Q_1 = Q_2 \frac{F_1}{F_2}.$$

Supposing now the cable to be completely charged, and the battery taken off, and the condenser to be empty, then on connecting the condenser to the cable, the charge the former will take will be

$$Q_1 \frac{F_2}{F_1 + F_2} = Q_2 \frac{F_1}{F_2} \times \frac{F_2}{F_1 + F_2} = Q_2 \frac{F_1}{F_1 + F_2},$$

whilst the quantity remaining in the cable will be

$$Q_1 \frac{F_1}{F_1 + F_2}.$$

On discharging the condenser and connecting it a *second* time to the cable, the charge it will take will be

$$Q_1 \frac{F_1}{F_1 + F_2} \times \frac{F_2}{F_1 + F_2} = Q_2 \frac{F_1}{F_2} \times \frac{F_1}{F_1 + F_2} \times \frac{F_2}{F_1 + F_2} = Q_2 \left(\frac{F_1}{F_1 + F_2}\right)^2;$$

consequently, after the nth application, the charge Q it will take will be

$$Q = Q_2 \left(\frac{F_1}{F_1 + F_2}\right)^n;$$

therefore

$$\frac{F_1}{F_1 + F_2} = \frac{\sqrt[n]{Q}}{\sqrt[n]{Q_2}};$$

from which

$$F_1 = F_2 \frac{\sqrt[n]{Q}}{\sqrt[n]{Q_2} - \sqrt[n]{Q}} = \frac{F_2}{\sqrt[n]{\frac{Q_2}{Q}} - 1}.$$

For example.

A condenser of 1·0 microfarad capacity (F_2), when full, gave a discharge equal to 300 (Q_2). A cable whose capacity was required was charged from the same battery which was employed to charge the condenser. The latter was then alternately connected to the cable, removed and discharged 16 times (n); on the sixteenth occasion the discharge was noted, and it was found equal to 83 (Q). What was the capacity of the cable?

$$F_1 = \frac{1 \cdot 0}{\sqrt[16]{\frac{300}{83}} - 1} = 11 \cdot 97 \text{ m.f.}$$

425. In order to make this test as accurately as possible when it is applied to a cable, the repeated charges and discharges must be made with as little loss of time as possible, as during that time leakage of the charge will be going on through the insulating sheathing of the cable, and absorption will be taking place; the accuracy of the test depends upon this leakage and absorption being nothing, or at least very small.

426. The connections for making the test would be similar to those employed in the foregoing one, and the practical execution would be the same, with the exception that the trial condenser or cable, and not the standard condenser, would be charged from the battery, and in taking the repeated discharges the galvanometer would have to be short-circuited.

Best Conditions for making the Test.

427. Make n equal to $1 \cdot 278 \frac{F_1}{F_2}$, approximately.

This may be proved as follows:—

In order to determine F_1 as accurately as possible from the equation

$$F_1 = \frac{F_1}{\sqrt[n]{\frac{Q_2}{Q_1}} - 1}, \quad \text{that is,} \quad F_1 = \frac{F_2}{\left(\frac{Q_2}{Q}\right)^{\frac{1}{n}} - 1},$$

we must determine $\left(\frac{Q_2}{Q}\right)^{\frac{1}{n}}$ as accurately as possible.

Let $\left(\frac{Q_2}{Q}\right)^{\frac{1}{n}}$ equal $\frac{1}{k}$, and let there be a small plus error δ in Q_2, and a small minus error δ in Q, and let there be a corresponding error λ_1 in k, that is, let

$$k + \lambda_1 = \left(\frac{Q_1 - \delta}{Q_2 + \delta}\right)^{\frac{1}{n}};$$

therefore

$$\lambda_1 = \left(\frac{Q_1 - \delta}{Q_2 + \delta}\right)^{\frac{1}{n}} - k.$$

Now

$$\left(\frac{Q_2}{Q_1}\right)^{\frac{1}{n}} = \frac{1}{k}, \text{ or, } \frac{Q_1}{Q_2} = k^n, \text{ or, } Q_1 = Q_2\, k_n;$$

therefore

$$\lambda_1 = \left(\frac{Q_2\, k^n - \delta}{Q_2 + \delta}\right)^{\frac{1}{n}} - k = k\left[\left(\frac{Q_2 - \frac{\delta}{k_n}}{Q_2 + \delta}\right)^{\frac{1}{n}} - 1\right],$$

but since δ is very small, we get

$$\lambda_1 = k\,\frac{Q_2^{\frac{1}{n}} - \frac{\delta}{n\,k^n} Q_2^{\frac{1}{n} - 1} - Q_2^{\frac{1}{n}} - \frac{\delta}{n} Q_2^{\frac{1}{n} - 1}}{Q_2^{\frac{1}{n}}} = -\frac{\delta\, k}{Q_2}\left(\frac{k^{-n} + 1}{n}\right).$$

To make λ_1 a minimum, we must make $\frac{\text{“}k^{-n}+1\text{”}}{n}$ a minimum.

Let

$$u = \frac{k^{-n}+1}{n},$$

then

$$\frac{d\,u}{d\,n} = \frac{1}{n^2}\left[-n\,k^{-n}\log_e k - (k^{-n}+1)\right] = 0$$

at a minimum; therefore

$$n \log_e k + 1 + k^n = 0,$$

or

$$\log_e k^n + 1 + k^n = 0,$$

or

$$\log k^n + (1 + k^n)\cdot 4343 = 0.$$

The solution of this equation may be obtained by the "trial" method, i.e. giving k^n various values until one is found which approximately satisfies the equation. If we make k^n equal to $\cdot 27846$ the equation will be very nearly satisfied, for

$$\log \cdot 27846 = \bar{1}\cdot 4447628 = -\cdot 5552372$$

and

$$(1 + \cdot 27846)\cdot 4343 = 5552352.$$

Now

$$F_1 = \frac{F_2}{\left(\frac{Q_2}{Q}\right)^{\frac{1}{n}} - 1}, \quad \text{or,} \quad \frac{F_1+F_2}{F_1} = \left(\frac{Q_2}{Q}\right)^{\frac{1}{n}} = \frac{1}{k};$$

therefore

$$\left(\frac{F_1}{F_1+F_2}\right)^n = k^n = \cdot 27846;$$

hence

$$n = \frac{\log_e \cdot 27846}{\log_e \frac{F_1}{F_1+F_2}} = \frac{-\cdot 5552372 \times 2\cdot 303}{\log_e \frac{F_1}{F_1+F_2}} = \frac{1\cdot 278}{\log_e \frac{F_1+F_2}{F_1}}.$$

Expanding by Taylor's theorem, we get

$$n = \frac{1\cdot 278}{\frac{F_2}{F_1} - \frac{1}{2}\left(\frac{F_2}{F_1}\right)^2 + \frac{1}{3}\left(\frac{F_2}{F_1}\right)^3 - \&c.};$$

and neglecting squares and higher powers of $\frac{F_2}{F_1}$, which we may do for an approximation, we get

$$n = \frac{1\cdot 278}{\frac{F_2}{F_1}} = 1\cdot 278\,\frac{F_1}{F_2}.$$

For example.

It being required to measure the exact electrostatic capacity of a cable whose capacity was 12 microfarads (F_1) approximately, a condenser of 1 microfarad (F_2) was used for the purpose. How many times should the condenser be applied to the cable in order that the test may be made with the greatest chance of obtaining an accurate result?

$$n = 1\cdot 278\,\frac{12}{1} = 15.$$

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{\delta}{n} \cdot \frac{(Q_2 + Q)\, Q_2^{\frac{1}{n} - 1}}{Q\,(Q_2^{\frac{1}{n}} - Q^{\frac{1}{n}})}\, 100,$$

where δ is the fraction of a division to which each of the deflections Q and Q_2 can be read.

Capacity Measurements of Long Submarine Cables.

428. With reference to the exact measurement of the capacity of long submarine cables, Mr. J. Elton Young points out * that there are several sources of error which require to be guarded against, and for which corrections require to be made if accuracy is to be ensured. In balancing a cable against a paraffin condenser, if they both have the same rate of *absorption* there will be no apparent variation of results with the time of charging. This, however, is not the case in general, for not only do the specific absorptive capacities of paraffin and gutta-percha differ, but the conductor resistance of a long cable greatly modifies its rate of absorption, at all events near the beginning of charge or discharge. Thus, assuming that comparisons are made by the Thomson or Gott methods, there will be a divergence of results with different times of charge—quite independently of that due to true leakage effects, which also require to be taken into account.

Besides this effect of time, there is, moreover, one of temperature to be noticed. The absorptive capacity of both gutta-percha and paraffin is augmented by fall of temperature, thus causing an increase in their apparent capacities. This variation is of a smaller order of magnitude than the former, being in fact the increment of an increment. Thus in the case of paraffin the temperature effect is to increase the capacity by only 0·025 per cent. per fall of 1° C. The rate at which it affects that of gutta-percha, whilst in the same direction, is still smaller, and has eluded determination altogether hitherto. Still it cannot be regarded as negligible when we are seeking the maximum accuracy attainable, and dealing with cables of many hundreds of microfarads submerged at depths such that their temperature falls by some 35° below the standard 75° of the factories. In the

* 'Proceedings of the Institution of Electrical Engineers,' vol. xxviii.

case of ordinary paraffin condensers when employed for accurate measurement it is a factor requiring careful allowance to be made for it; though in the compensated standards constructed by Dr. Muirhead, partly of paraffin and partly of shellac, the opposite behaviour of the shellac neutralises their temperature variation.

Mr. Young finds that a modified Thomson test (page 369) gives very satisfactory results. In this method both leakages and absorptions are balanced *by observation and adjustment* during charging, absorption in *this* case being treated as apparent improvement of Insulation Resistance. That is to say, the condensers are shunted by a resistance which is adjusted till it approximately balances the *apparent* Insulation Resistance of the cable at any time of charge. The connections for charging are those of Gott's method (page 373), with this important difference, that the key k (Fig. 169) is closed during the charge. The effect of this is to keep the potential of the point of junction of the condenser and cable constant, as in the Thomson balance, while at the same time the deflection on the galvanometer shows whether or not there is a balance of (apparent) insulation resistances in the bridge system. Since this balance is required to be such that $\frac{R}{r} = \frac{F_1}{F_2}$, R being the shunt resistance across the condenser, and r the insulation resistance of the cable, the balance must be observed when $\frac{R_2}{R_1} = \frac{F_1}{F_2}$ nearly; that is to say, we must commence with an approximately correct position of the slide resistance slider; then R, the leakage shunt across the condensers, is adjusted till the galvanometer shows a resistance balance, as nearly as possible, at the end of any desired time of charging. For this purpose a set of plumbago resistances is useful, or it can be done roughly by the application of a pocket-handkerchief across the terminals of the condenser more or less closely as required; but it is preferable to estimate approximately the proper resistance and to arrange it beforehand.

Having thus charged, and balanced, the apparent Insulation Resistances by observation, the test proceeds in the same way as in the case of the Thomson, viz. by short-circuiting and mixing the charges, and observing if any outstanding quantity remains on earthing the junction of the two capacities through the galvanometer. An additional advantage over the original Thomson arrangement is that there is no serious vitiation of results by battery leakage. Moreover, it admits of allowance being made for "cable current" by false zero observation, a most important

point on long submarine lines. It should be remarked that in the modified, as in the original, method the *mixing* period must be sufficiently prolonged to discharge the cable, or variable results will ensue. On a section of 1850 knots Mr. Young found 15 seconds' mixing sufficiently long, within the limits of appreciation, even after a full minute's charging—i. e. there was no difference between a 15 and a 25 seconds' mixing interval.

By means of a measurement thus carried out, it is possible to arrive at the same result for all times of charge, and for any values of the dielectric resistance, and to dispense with all calculated allowances for leakage. Thus the desired uniformity of results is ensured, taking the constant values of the balancing condensers, determined by Muirhead's method, as a basis. The condensers should be marked with these values, as well as with the temperatures at which they were measured. Care must then be taken to correct the condensers for their temperature changes, by means of the coefficients furnished by Muirhead. It only remains to determine the very small coefficient of temperature variation applicable to the specific induction capacity of gutta-percha; this has yet to be done.

429. The importance of being able to determine accurately the capacity of a cable will be evident if it be remembered that on ocean cables a microfarad means some 3 nautical miles; so that a mistake of 1 per cent. in the distance to a sealed break, on a section of, say, 700 microfarads—if for any reason localisation from the other end were impracticable, as sometimes happens—would amount to an error of 20 knots.

CHAPTER XIV.

THE THOMSON QUADRANT ELECTROMETER.

430. This is a most valuable and useful instrument for accurately measuring potentials.

Description.

Fig. 173 (page 388) gives a general view of the instrument.

In the small figure to the right, *n n* is a thin needle of sheet aluminium, shaped like a double canoe-paddle. It is rigidly fixed at its centre to an axis of stiff platinum wire *k* (Fig. 172) in a plane perpendicular to it. At the top end of the wire a small cross-piece *i* is fixed, to the extremities of which single cocoon fibres are attached. These fibres are fixed to small screws *c* and *d*, by the turning of which the length of the former can be altered. The small screws *a* and *b* enable the screws *c* and *d* to be shifted either to the right or left. Finally, by turning *e*, the screws *a* and *b* can be parted more or less, thereby separating the threads of suspension, and rendering the tendency of the needle to lie in its normal position more or less powerful.

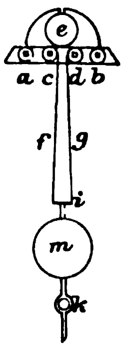


Fig. 172.

A little below the cross-piece *i* is fixed the mirror *m*, whose movements are reflected on a scale, as in a Thomson galvanometer (page 48). The platinum wire below the mirror passes through a guard tube, *t* (Fig. 173) to prevent any great lateral deviation of the needle and its appendages, which might cause damage should the instrument receive any rough usage. The guard tube itself is fixed to the framework from which the needle is suspended.

It will be seen in the figure that the needle is suspended, apparently, beneath four quadrants (*q*), A, B, C, and D. There are, however, four quadrants also below the needle, united to the top ones at their circumferences. The arrangement is in fact a round, flat, shallow box, cut into four segments.

The alternate segments are connected together by wires as shown in the figure.

Now, if the needle is electrified and the quadrants are in their normal unelectrified condition, and are placed symmetrically with reference to it, no effect will be produced on the needle. That is to say, the spot of light on the scale will be stationary exactly at the centre line.

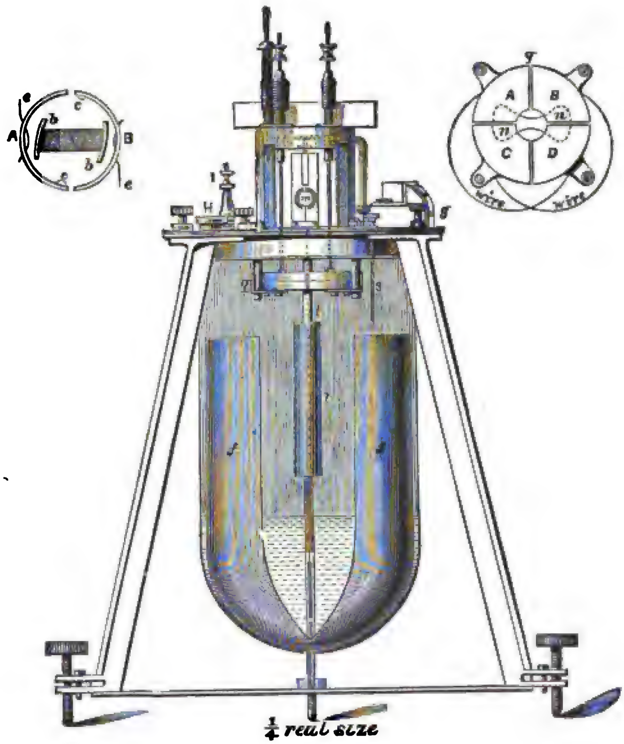


FIG. 173.

But if the quadrant D, and consequently A, be electrified, then an attraction or repulsion will be exerted on the needle, causing it to turn through an angle proportional to the potential of the electricity.

As the angular movements are very small, the number of divisions of deflection on the scale will directly represent the degree of potential which the quadrants possess.

We can also connect another electrified body to C and B; the needle will then move under the influence of both forces.

To render the instrument of practical value, several conditions must be assured.

Let us suppose the needle to be electrified.

We stated that, at starting, the ray of light should point to the centre line on the scale. To ensure this, the quadrants must be symmetrically placed. This can be roughly done by hand, as means are provided for enabling the quadrants to slide backwards or forwards, and to be fixed by means of small screws, shown in the large figure. For obtaining the final positions, one of the quadrants (B) is provided with a micrometer screw (*g*), which enables a fine adjustment to be given to it.

We must also have means of keeping the needle at one uniform potential for a considerable time.

The needle itself could only contain a very small charge of electricity, and a slight escape of this would seriously lower the potential, and make comparative measurement useless; for it is evident that the whole principle of the instrument depends upon the potential of the needle remaining constant during the time a set of experiments are being made.

To get over this difficulty a large glass jar, like an inverted shade, is provided, partially coated with strips of tin-foil (*f*) outside. Inside the jar, to about a third of its height, strong sulphuric acid is placed. This answers a threefold purpose. It enables the air inside it to be kept quite dry, thereby very perfectly keeping those parts insulated which require to be so; secondly, it holds a charge of electricity (acting as the inner coating of the jar); and thirdly, it allows the charge to be communicated to the needle without impeding its movements. This latter is effected by means of a fine platinum wire, which is attached to the lower end of the thick wire which supports the needle and mirror.

The fine wire dips into the acid, a portion of the charge of which is thereby communicated to the needle.

To keep the wire from curling up out of the acid, and also to steady the movements of the needle, a small plummet of platinum is attached to the end of the wire, as will be seen in the figure.

A thick platinum wire, fixed to the lower extremity of the guard tube *t*, and reaching nearly to the bottom of the jar, is for the purpose of enabling the latter to be charged, in a manner to be explained.

So far, the jar answers the purpose of keeping the needle supplied with electricity; but although this may prevent the potential from falling very rapidly, it will not prevent its doing so entirely.

The Replenisher.

431. As the instrument is extremely sensitive to very slight changes of potential, some means are requisite by which any small loss can be easily supplied without there being any fear of putting in too much.

This is effected by means of the "*replenisher*," whose principle may be explained by the help of the small cut to the left, in Fig. 173.

A and B are two curved metal shields, one of which (say A) is connected to the acid in the jar, and the other, B, to the framework of the instrument, and through it to the foil outside the jar.

b and *b* are two metal wings insulated from one another by a small bar of ebonite, which is centred at *s*, so that it turns in a plane represented by the paper. The spindle is represented in the large figure by *s*, other parts being omitted for simplicity.

It will be observed that the wings curve outwards. This is done in order that they may make a short contact in their revolution with springs *c c* and *e e*. *c* and *c* are connected together permanently, but are insulated from the rest of the apparatus. *e* and *e* are connected to the shields A and B respectively.

Now let us suppose the wings to be rotated in the reverse direction to that in which the hands of a watch turn.

As soon as the left-hand wing comes in contact with the spring *c*, at the lower part of the figure, the right-hand wing comes in contact with the other spring. The two wings being thus connected together, and under the influence of the shields, the electricity in A, which we will call positive, draws negative electricity to the wing close to it, and drives the positive to the other wing.

On being rotated a little farther the wings clear the springs, and being thus disconnected, each wing retains its charge.

Continuing the rotation, the right-hand wing, which had the positive charge communicated to it, comes in contact with the spring *e* of shield A, and the charge is communicated to the jar, the negative electricity in like manner on the other wing running to the outer coating of the jar. The shields are now in a neutral condition, as at first, and on continuing the rotation the process is repeated.

Thus every turn increases the potential of the charge in the jar, and by continuing the rotation we can augment this as much as we please.

By reversing the motion we can diminish the charge, if we require to do so.

The axis of the replenisher projects above the main cover, and is easily turned by the finger.

The Gauge.

432. But we still require some arrangement by which we can see whether we have kept the potential constant. This is done by means of a small "*gauge.*"

The gauge consists of two metallic discs having their planes parallel and close to each other. The lower of these planes which will be seen dotted at the upper part of the figure, is in electrical connection with the acid of the jar from which it takes its potential. The upper disc is perforated with a square hole immediately over the centre of the lower disc.

A light piece of aluminium, shaped like a spade, has the part corresponding to the blade fitting in this square hole. At the point where the handle would be joined to the blade this spade is hinged, by having a tense platinum wire fixed to it, which runs at right angles on each side of the handle and blade, and lies in the same plane as the latter.

When the lower plate is electrified, it would attract the blade, thereby raising the end of the handle. So that if we notice the position of the end of the handle with respect to a mark, and see that it moves above or below it, we know that the electricity of the lower plate is either overcoming the tendency of the light platinum wire to keep it up, or is unable to do so.

If then we charge our jar to such a potential that the handle is situated close to the mark, and we keep it so, we know that the potential of the jar is constant. When we notice the handle sinking below the mark, we know that the potential of the electricity in the jar is falling; but a few turns of the replenisher will bring it up again.

In the actual arrangement, the rung of the handle is formed of a fine black hair.

Inside the handle there rises a small pillar, with two black dots on it. The sign of division ÷ represents this, the line being the hair which, by the movement of the spade blade, rises above or below the two dots, which of course would be almost quite close together.

To enable the hair and spots to be seen distinctly, a plano-convex lens is placed a little distance off. Care must be taken, in order to avoid parallax error, to keep the line of sight a normal to the centre of the lens.

We spoke of the lower disc, which becomes electrified by the jar, and which acts on the spade blade. Now it is evident that if the distance between the plates be always the same, and the elasticity of the platinum axial wire be also the same, to get the hair between the two spots is to obtain the jar at a particular fixed potential.

But we may require to get this potential, although the same whilst a certain set of experiments are being made, yet different for different series of experiments. This is provided for by enabling the lower disc to be lowered by screwing it round.

The Induction Plate.

433. To enable high potentials to be measured, an "*induction plate*" is added. It consists of a thin brass plate, smaller in area than the top of the quadrant beneath it, and supported from the main cover by a glass stem. It is provided with an insulated terminal I. The use of the plate will be explained later on.

434. A flat brass plate covers the mouth of the jar, and is secured to it so as to be air-tight and prevent the entrance of moisture.

A kind of lantern rises from the middle, which covers the mirror and its suspending arrangements, and above this a box with a glass lid protects the gauge.

The front of the lantern is of glass, which allows the ray of light to fall on the mirror and be reflected back on the scale.

Terminal rods or electrodes, in connection with each set of quadrants, pass through ebonite columns to the outside of the case, and have terminals attached to them. These electrodes can be pulled up and disconnected from the quadrants if necessary.

A charging rod (seen in Fig. 173, page 389, to the left of the left-hand quadrant terminal) also is provided, which can be turned round on its axis. It has at its lower end a small spring, fixed at right angles to it. By turning this terminal rod round, the spring can be brought in contact with the framework from which the needle is suspended, and thereby, through the medium of the guard tube and the platinum wire attached to it, the acid in the jar can be charged. When this is done, the spring is moved away, so that no accidental leakage can take place through it.

Various insulating supports are provided inside the jar and lantern. One supports the guard tubes and the adjusting screws of the needle; others support the quadrants.

The whole arrangement is supported by a kind of tripod on a metal base, to keep it steady. There are also levelling screws, and a level on the brass cover, to enable the instrument to be properly levelled, so that the axis of the needle may swing clear of the guard tube.


Fig. 174.

H is a screw-capped opening through which acid can be introduced into the glass jar.

Reversing Key.

435. Fig. 174 represents a reversing key which is specially adapted for use with the instrument.

To Set up the Electrometer.

436. In setting up the instrument for use the following instructions* should be followed.

The cover being unscrewed and lifted off and supported about 18 inches above the table, it will be observed that the stiff platinum wire to which the needle is attached just appears below the narrow guard tube enclosing it in the centre of the quadrants, and terminates in a small hook. The loop at the end of the fine platinum wire is to be slipped over this hook, so that the fine wire and plummet may hang from it. The wide guard tube, when in its proper position, forms a continuation of the upper guard tube, so as to enclose the fine platinum wire just suspended. It must therefore be passed upwards over the suspended wire, and neck foremost, until the neck embraces the lower part of the upper guard tube, where it must be fixed by the screw pin provided for the purpose; this pin is screwed in by means of one of the square-pointed keys, supplied with the instrument, fitting the square hole in its head. This being done, replace and fasten the cover, place the instrument on a sheet of ebonite or block of paraffin wax so as to insulate it, and level up by means of the circular spirit level on the cover.

Next unscrew and lift off the lantern and, if necessary, adjust the four quadrants so that they hang properly in their places, with their upper surfaces in one horizontal plane. The needle

* From instructions drawn up by the late Mr. W. Leitch.

and mirror which have been secured during transit by a pin passing through the ring in the platinum wire just above the guard tube, and screwed into the brass plate behind, must now be released by unscrewing this pin with the long steel square-pointed key, and placing it in the hole made for it in the cover just behind the main glass stem to prevent its being lost. The needle will now hang by the fibres.

The two quadrants in front of the mirror should now be drawn outwards from the centre as far as the slots allow, by sliding outwards the screws from which they hang, and which project above the cover of the jar with their nuts resting upon flat oblong washers; a better view will thus be obtained of the needle. The surfaces of the latter ought to be parallel to the upper and under surfaces of the quadrants, and midway between them. This will be best observed by looking through the glass of the jar just below the rim. If the needle requires to be raised or lowered, it is done by winding up or letting down the suspending fibres, that is, by turning the proper way the small pins *c*, *d* (Fig. 172, page 388). The suspending wire which passes through the centre of the needle should also be in the centre of the quadrants. This is best observed when the quadrants have been moved to their closest position. The fourth quadrant is moved out or in by the micrometer screw *g* (Fig. 173, page 389), whose graduated disc overhangs the edge of the cover. A deviation of the suspending wire from its proper central position, as was explained at the beginning of the chapter, may be corrected by means of the small screws *a*, *b*, *c*, and *d* (Fig. 172, page 388). When proper adjustment is attained the black line on the top of the needle should be parallel to the transverse slit made by the edges of the quadrants when these are symmetrically arranged.

The sulphuric acid may now be put into the jar. For this purpose, the strongest sulphuric acid of commerce is to be boiled with some crystals of sulphate of ammonia, in a florence-flask supported on a retort-stand over a jet of gas or other convenient source of heat. It is recommended to boil under a chimney, so that the noxious fumes rising from the acid may escape. To guard against the destructive effects of the acid in the event of the flask breaking by the heat, there should be placed beneath it a broad pan filled with ashes, or it should stand above a fire-place containing a sufficient quantity of cold ashes. A little sand put into the flask will lessen the risk of breaking. The object of boiling the acid is to expel the volatile acid impurities which will

otherwise impregnate the air inside of the jar and tarnish the works. When cool, the acid may be best poured into the jar through a glass filler with a long stem inserted through the screw opening H (Fig. 173, page 389) provided for the purpose. The stem of the filler should reach the bottom of the jar, to avoid splashing upon its sides or upon the works, and in removing it care should be taken that it is drawn out without its end touching any of the brasswork. The acid may be poured in till the surface is about an inch below the lower end of the wide brass tube which hangs down the middle of the jar. It must at least reach the three platinum wires hanging from the works.

437. The instrument thus adjusted and charged with acid should be allowed to rest for some little time so that any films of moisture on the insulating portions of the apparatus may become absorbed.

The scale should now be placed at the proper distance so that the reflected image is sharply defined and stands at the middle of the scale, that is, at 360; for the electrometer scale (unlike that of a galvanometer) is graduated from 0 to 720, 360 being the middle point. Care must be taken that the two ends of the scale are equidistant from the centre of the mirror.

Next connect together the two electrodes of the quadrants and the induction plate electrode, by means of a piece of thin wire joined to the cover of the jar; also turn the charging rod so that it touches the framework of the platinum wire of the needle.

Now charge the jar positively by means of a few sparks from a small electrophorus, the frame of the instrument being put to earth for the purpose, and afterwards disconnected. When the proper potential is reached, it is indicated by the lever of the aluminium balance rising; the charging rod should then be turned so as to disconnect the latter from the needle. The replenisher must now be used to adjust the charge exactly, so that the hair may stand between the black spots when observed through the lens. When the lever carrying the hair is at either extremity of its range, it is apt to adhere to the stop; in using the replenisher to bring it from either limit, therefore, it is necessary to free it from the stop by tapping the cover of the jar with the fingers.

If the charge has caused the reflected image to be deflected from the middle of the scale, it may be brought back to that position by turning the micrometer screw which moves the fourth quadrant, and, if necessary, sliding out or in one or more of the other quadrants.

The small percentage of the charge lost from day to day may

be recovered by using the replenisher. Under ordinary conditions this loss will not amount to more than ½ per cent. per day.

The charge may suffer loss from several causes, the most prevalent being the presence of dust on portions of the apparatus inside the jar. Every portion should be carefully dusted with a camel-hair brush, and especially the round induction plate beneath the aluminium balance.

Loss may occur by shreds inside of the quadrants drawing the charge from the needle. It should be ascertained whether this takes place. Insulate alternately each pair of quadrants by raising the corresponding electrode, while the other pair are connected through their electrode with the cover. If the reflected image in either case keeps moving slowly along the scale, for instance over 20 scale divisions in half an hour, the charge in the jar being at the same time kept constant by the use of the replenisher if necessary, the insulated pair of quadrants is receiving a charge from the needle. In that case the inside of the quadrants may be brushed with a light feather, or camel-hair brush, after sliding them outwards as far as the slots allow, and securing the needle in the position in which it was fixed during transit; care being taken not to press upon the needle so as to bend it or the suspending wire. Without securing the needle, each quadrant may be drawn outwards and brushed, while the needle is deflected away from it by the screws *a*, *b* (Fig. 172, page 388), or by any obvious means of keeping the needle deflected, care being taken not to strain the fibres.

Another possible source of loss of charge is want of insulation over the portion of the glass jar above the acid. If the percentage of the charge lost from day to day be so considerable as to require much use of the replenisher to recover it, the glass should be cleaned with a wet sponge, rubbed with soap at first, or with a piece of hard silk ribbon, wet and soaped at first, then simply wet with clean water, which may be drawn round the glass to clean every part of it. The ribbon being dried before a fire, may be used in the same manner to dry the glass.

If everything fails to make the apparatus keep its charge, the cause is probably due to a defective glass jar, and this can only be remedied by the manufacturers.

438. The good insulation of the instrument being satisfactorily accomplished, the symmetrical suspension of the needle by the fibres should be tested. The conditions sought to be realised are, that in the level position of the instrument the needle may hang with equal strain on the two fibres, and in a symmetrical position

with regard to the four quadrants. It is plain that if these conditions be fulfilled the deflection produced by the same electric force in the level position of the instrument, will be less than it will be in any position of the instrument which throws the greater part of the weight on one fibre, or brings the needle nearer to any part of the inner surface of the quadrants than it is in its symmetrical position, which is its position of greatest distance from all the quadrants. To make the test, the two quadrant terminals should be connected to the two poles of a single-cell battery, and the deflections produced upon the scale compared, while the instrument is set at different levels, by screwing one or more of the three feet on which it is supported. At each observation the extreme range, or difference of readings got by reversing the battery, should be noted. If the range diminishes as one side of the instrument is raised, the suspending fibre on that side must be drawn up, by turning very slightly the small pin *c* or *d* (Fig. 172, page 388), round which it is wound, and another series of observations taken in the same manner, beginning with the instrument levelled. Instead of drawing up one fibre, the other may be let down, to keep the needle midway between the upper and under surfaces of the quadrants, and after each alteration of the suspension it will be necessary to readjust the screws *a*, *b* (Fig. 172, page 388), to make the black line on the needle hang exactly midway between the quadrants when the needle is undisturbed by electricity. It will be observed also that the charge of the jar is lost by touching these screws, unless the insulated key is used. They are reached without taking off the lantern by screwing out a vulcanite plug in the glass window in front of them.

In deflecting the instrument much from its level position, the guard tube may be brought into contact with the wire hanging from the needle, and the movements of the latter be thus interfered with by friction. When the needle vibrates freely, it will be observed that the image comes to rest in any position to which it may be deflected, after vibrating with constant period and gradually diminishing range on each side of this position of rest. The occurrence of friction is shown by the needle coming to rest abruptly, or vibrating more quickly than proper. The reading obtained under these circumstances is, of course, of no value. The quicker vibrations obtained in using the induction plate must not be mistaken for vibrations indicating friction, from which they may be easily distinguished by their regularity.

If, as may possibly happen, the process of observing the deflections at different levels, and drawing up the fibre on that side

which is being raised while getting less sensibility, should only lead the operator to draw up one fibre till it bears the whole weight, while the other is seen to hang loosely, he should adjust them as nearly as he can by the eye to bear an equal share of the weight, and examine the position of the needle by looking through the glass of the jar just below the rim, the two quadrants in front of the mirror being drawn out, and the lantern taken off to let in plenty of light. He will probably find that the needle leans slightly downwards relatively to the quadrants on that side which he was drawing up while getting smaller deflections. To correct this is a delicate operation, which should only be attempted by a very careful operator. Though perfect symmetry of suspension is aimed at, it is not essential to the utility of the instrument. If it be desired to make the correction, first secure the needle as during transit; take off the cover, and while it is held by a careful assistant, or properly supported in a position in which it may be levelled, remove the lower guard tube (the wide brass tube hanging down the centre) after screwing out the small pin in its neck. It will be observed that the upper and narrower guard tube consists of two semi-cylindrical parts united. The part in front may now be removed by taking out the two screws which fasten it at the top, and the platinum wire which carries the needle may be examined. If it has got bent it must be straightened; if not, it may be bent carefully just above the needle, so as to raise that end of the needle which was observed to hang lowest. If the cover be supported so that it may be levelled, the needle may be set free, and the operator may observe whether he has succeeded in making it hang parallel to the surfaces above and below it. The needle must not, however, be allowed to hang by the fibres, while bending the platinum wire, or while removing or replacing the guard tubes.

The works being replaced, the process of observing the deflections at different levels and adjusting the tension of the fibres should be repeated, with the view of getting minimum sensibility in the level position.

The two unoccupied holes bored through the cover and flange of the jar are intended to receive the square-pointed keys, when not in use.

Grades of Sensitiveness.

439. There are several ways of making the connections to the terminals of the quadrants, frame, and induction plate, so as to get various degrees of sensitiveness for measuring potentials of various strengths.

1st Grade.

The following is the most sensitive arrangement, such as would be used for measuring a potential of about 1 volt:—

One pole of the battery would be connected, through the medium of a reversing key (Fig. 174, page 394), to one quadrant terminal, and the other to the frame of the instrument and to the second quadrant terminal. This, by reversing the key, would give about 50 divisions on either side of the 360, equal to 100 in all.

2nd Grade.

Leaving one pole of the battery to the frame, the next degree of sensitiveness is obtained by disconnecting the pair of quadrants that are connected to the frame, the electrode being raised for the purpose; the other connections must be the same as in the last case. By this arrangement the needle is acted upon by one pair of quadrants only.

440. By using the induction plate we may still further diminish the sensitiveness of the instrument. For instance, when we connect the pole of the battery to a pair of quadrants, those quadrants take the potential that it has; but if we connect it to the induction plate, then the charge in the quadrant below is only an induced one, and, since there is an interval between the plate and the quadrant, this induced charge will be small, and the effect on the needle proportionally small. Again, if we disconnect one pair of quadrants, and connect the wire from the battery to the induction plate and to the corresponding quadrants, then the charge will be partially *bound*. The effect on the needle will therefore be less still. The actual number of grades of sensitiveness with the induction plate are as follows:—

3rd Grade.

One pair of quadrants connected to one pole of battery. Induction plate and second pole of battery connected to frame. Second pair of quadrants disconnected by raising electrode.

4th Grade.

One pair of quadrants connected to one pole of battery, and also to induction plate. Second pole of battery connected to frame. Second pair of quadrants disconnected by raising electrode.

5th Grade.

Induction plate connected to a pole of battery. One pair of quadrants and second pole of battery connected to frame. Second pair of quadrants disconnected by raising electrode.

6th Grade.

Induction plate connected to pole of battery. Second pole of battery connected to frame. Both pairs of quadrants disconnected by raising the electrodes.

441. We can in each of the foregoing cases interchange the terminals of the quadrants, that is to say, we can use the left terminal where we used the right, and *vice versâ*.

442. There is one more point to mention in connection with the instrument, and that is, that it may be found, on raising one of the electrodes to disconnect it from the quadrants, that the act of doing so causes the image on the scale to deviate a few degrees from zero in consequence of a charge being induced thereby.

In the most recent form of instrument there is a small milled vulcanite head provided, by turning which the quadrants are connected to the frame, and the charge being thereby dissipated, the image returns to zero. When this is done the milled head must be turned back before commencing to test again.

The Use of the Electrometer.

443. The electrometer can be used in every test where a condenser is usually employed.

In using the condenser we have to charge it, and then note its discharge on the galvanometer, which gives the potential. With the electrometer we have simply to connect to its terminals the wires which would be connected to the condenser, and the permanent deflection on the scale gives us the potential, which can be observed at leisure.

Thus in measuring the resistance of the battery by the method given on page 331 (§ 357), we should first connect the battery wires to the electrometer (through the medium of the reversing key is best), note the deflection, then insert the shunt, again note the deflection, and calculate from the formula.

The great value of the electrometer, however, lies in the fact of its enabling us to notice the continuous fall of charge in a cable and not, like the condenser method, merely to determine what the

potential has fallen to after a certain time. We can see with unfailing accuracy when the charge has fallen to one-half, or any other proportion we please.

We see, in fact, exactly what is going on in the cable at any moment.

The connections for such a test could not well be simpler. We charge the cable, connect it to the electrometer, the frame being to earth, and then notice the deflection as it gradually falls down the scale. We do not even require a battery, as we can charge the cable with a few sparks from an electrophorus.

The degree of sensitiveness necessary for any particular cable we can, of course, only tell by experience.

Measurements from an Inferred Zero.

444. When very high resistances, such, for instance, as short lengths of highly insulated cable, are measured by the ordinary fall of charge method, the fall, even in a considerable time, would be so small that the test would be an unsatisfactory one, for the difference between the deflection at the beginning of the test, and that after the interval of time, could only be a small fraction of the whole length of the scale; and if the deflections are not accurately noted, still less can we be satisfied of the correctness of our result when worked out from a formula.

By means of a method suggested by the late Professor Fleeming Jenkin, however, such high resistances can be measured by the fall of charge test with considerable precision.

Professor Jenkin's improvement consists in virtually prolonging the scale and counting the divisions from an *inferred zero.*

An explanation of the method of making the test will best show what an inferred zero is.

One pole of the battery being to earth, the other pole is connected to one pair of quadrants.

The second pair of quadrants is connected to the cable.

By joining for an instant the two pairs of quadrants together, the cable and quadrants take the same potential; therefore, at the moment of disconnecting them, the needle will be at zero.

The potential, however, of the cable, and the quadrants connected to it, will fall, and the needle be deflected.

Suppose, now, one cell connected to the electrometer gave 100 divisions deflection, and suppose the battery which charged the cable was 100 cells, then if the cable lost 1 per cent. of its charge, the charge remaining would be 99, and as the other quadrant,

being permanently connected to the 100 cells, has the potential of 100, the difference between the two is $100 - 99 = 1$ cell, which, as we have said, gives 100 divisions. The 2 per cent. loss would give 200 divisions, and so on, whereas by the method mentioned in § 443, if we get 300 say, at first, then 1 per cent. loss would only move the image down to 297, and 2 per cent. would move it down to 294.

When all the charge is lost, the deflection would evidently be $100 \times 100 = 10{,}000$, which is the inferred zero. To obtain this zero for any particular battery, we should have to get the deflection from 1 cell and then determine, by the method given on pages 323 (§ 350) and 335 (§ 362), what the electromotive force of the testing battery is in terms of the 1 cell. Then by multiplying the 1 cell deflection by this value we get what we require.

The numbers representing the potentials we must evidently get by subtracting the deflections on the scale from the inferred zero.

To obtain the full range of the scale we should, at starting, get the image on the actual marked zero, which is, as we have before said, at the end, and not at the middle of the scale.

445. It is possible to use the electrometer without having the acid of the jar charged. For this purpose one pair of quadrants should be connected to the needle; by this arrangement the needle becomes charged by the same electricity that charges the quadrants to which the needle is connected. It will be seen, however, that under this condition the deflections will not be directly proportional to the potentials producing them, as the action is similar to that which takes place in the case of an electro-dynamometer (page 352); the deflections, in fact, will be proportional to the *squares* of the potentials.

The special advantage of the foregoing method of using the instrument is that it enables rapidly alternating potentials to be measured, as in the case with rapidly alternating currents through the electro-dynamometer.

CHAPTER XV.

MEASUREMENT OF HIGH RESISTANCES.

446. The highest resistance which it is possible to measure by means of the Wheatstone bridge described at the commencement of Chapter VIII. (page 210), is 1,000,000 ohms. It is true that some bridges have another set of resistances in the top row, which will enable the ratio 10 to 10,000 to be used, and consequently a resistance of

$$\frac{10{,}000 \times 10{,}000}{10} = 10{,}000{,}000 \text{ ohms}$$

to be measured; but this is not often the case, and the values of resistances much greater than this frequently require to be determined.

For this purpose a modification of the deflection method given in Chapter I., page 6 (§ 10), must be adopted.

447. Provide a single, and also about 100 constant cells. Find their respective electromotive forces by the discharge method given on pages 323 (§ 350) and 335 (§ 362). Thus, suppose the discharge taken from the 1 cell, which, as we have explained, should be taken first, gave a deflection of 300, the galvanometer shunt (S_2) being adjusted for this purpose to 560 ohms. Suppose also that the discharge from the 100 cells in the place of the 1 cell, gave a deflection of 302, with a shunt (S_1) of 6 ohms; then by multiplying the 302 by

$$\frac{G + S_1}{S_1}$$

we get the deflection we should have had if no shunt had been used; this will represent the electromotive force of the 100 cells. In like manner, by multiplying the 300 by

$$\frac{G + S_2}{S_2}$$

we get a number representing the electromotive force of the 1 cell. Taking the resistance of the galvanometer (G) to be 5000 ohms,

and giving the other numerical values to the quantities, the ratio of the electromotive force of the 1 cell to the electromotive force of the 100 cells would be

$$\frac{5000 + 560}{560} \times 300 : \frac{5000 + 6}{6} \times 302,$$

or as

$$2980 : 252{,}000.$$

If now we divide the greater number by the less, we get the value of the 100 cells in terms of the 1 cell. This value is 84·6, that is to say, the 100 cells are 84·6 times stronger than the 1 cell, and not 100 times. This might arise from some of the cells being defective, or imperfectly insulated. This does not matter, however, so long as we determine, as we have done, *how* much more powerful the 100 cells are than the 1 cell.

Calculation may be saved in the foregoing measurement if we adjust the galvanometer, by means of the directing magnet, so that a convenient discharge deflection is obtained with the 1 cell when there is *no* shunt between the terminals of the instrument. The exact value of this deflection being noted, the discharge deflection from the 100 cells is next taken with the $\frac{1}{99}$ shunt (page 63, § 63); then the latter deflection multiplied by 100 and divided by the first deflection, obviously at once gives the value of the 100 cells.

448. Having found the value of the 100 cells in terms of the single cell, we next proceed to join up the galvanometer, with a shunt, &c., between its terminals, in circuit with a resistance coil and the single cell, as shown by Fig. 175 (page 406).

Put a resistance of 10,000 ohms in A B (a resistance of 10,000 ohms in a separate box is often used for this measurement), and having first inserted all the plugs in S, press down the short-circuit key, and proceed to remove some of the plugs, until a deflection of, say, 300 is obtained, then raise the key and see if the spot of light comes back to zero properly; if it does not, then by disconnecting one of the wires, see that the cause is not from the short-circuit key not making proper contact. If this has not the required effect, the adjusting magnet of the galvanometer must be slightly shifted, and, if necessary, put a little lower down, so as to make the needle a little less sensitive. After a few trials this will be satisfactorily done, and the spot of light will always come back to the zero point when no current is passing through the galvanometer.

Let the deflection be 301½, the shunt being 7 ohms.

$$\text{Multiply } 301 \cdot 5 \text{ by } \frac{5000 + 7}{7}, \text{ which gives } 215{,}700.$$

This is the deflection we should get through 10,000 ohms, with no shunt to the galvanometer. There is really in the circuit, besides the 10,000 ohms, the resistance of the 1 cell, and also the

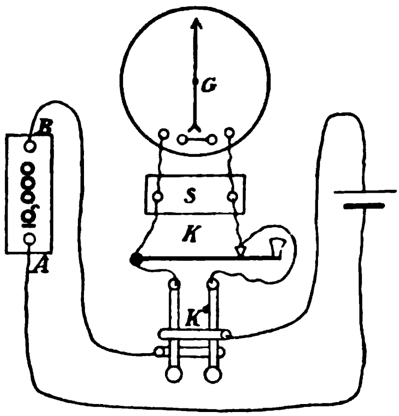


FIG. 175.

resistance of the galvanometer and shunt combined (which will be practically 7 ohms), but this will be so small as to be of no consequence; it may, however, be added on to the 10,000 when working out the results, if preferred.

Now, if we had used the 100 cells instead of the 1 cell, our deflection would have been 84·6 times as great as it was with the 1 cell. If, then, we multiply 215,700 by 84·6 we shall get the deflection obtainable with the 100 cells through a resistance of 10,000. This value will be found to be 18,248,000. Multiplying this number by 10,000 we get the *constant;* this constant is obviously the theoretical resistance which would give a deflection of 1 division with the 100 cells.

If it is required to use, say, 200 cells instead of 100 only, then in cases where galvanometer shunts of a fixed value ($\frac{1}{9}$th, $\frac{1}{99}$th, $\frac{1}{999}$th), only, are available, it would be advisable to employ 2 cells instead of the 1 cell, for making the test, so as to cause the deflections to be of an approximately equal value (page 101, § 92); this would not of course alter the foregoing process of calculation

in any way, it would only result in the numerical value of the "constant" being different. The actual *number* of cells used, it may be pointed out, has nothing to do with the calculations; in fact, it is usual to speak of the 100, or 200, cells as the "battery" simply. A *one*-cell battery is used for producing the permanent deflection through 10,000 ohms, because 100 cells would deflect the spot of light off the scale with the lowest shunt that could be used; one cell happens to be a convenient electromotive force to employ, but, as pointed out, it might be preferable to use two, or even more, in certain cases.

It may be pointed out that the constant deflection with 1 cell through 10,000 ohms may usually be taken with the $\frac{1}{999}$ shunt in the place of a shunt of a particular numerical value (as in the foregoing example); this simplifies calculation, as we have then simply to multiply the constant deflection by 1000 instead of by $\frac{G+S}{S}$.

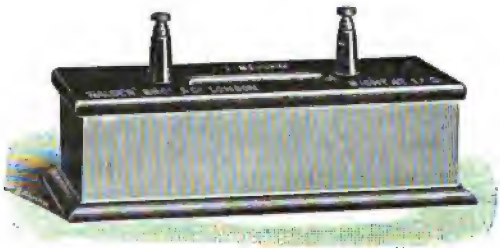


Fig. 176.

Fig. 177.

449. The foregoing process is simplified by using a resistance of 1,000,000 (1 megohm) in the place of 10,000. The constant can then be found with the "battery" at once. Fig. 176 shows one

form of "megohm" for this purpose. As it is useful sometimes to be able to use a less resistance than 1 megohm, the latter is sometimes divided into 10 resistances of 100,000 ohms each, as shown by Fig. 177.

450. Having measured and worked out the constant (which may be done by the help of logarithmic tables,* or by means of a slide rule †), we insert the resistance which is to be measured, in the place of A B, using the 100 cells in the place of the 1 cell. Having adjusted S till a deflection of 300, or near to 300, is obtained, note S and also the deflection. Let S be 2500, and deflection 298. Then the deflection without the shunt would be

$$298 \times \frac{5000 + 2500}{2500} = 894.$$

Dividing the "constant" by this number, we get

$$\frac{182,480,000,000}{894} = 204,100,000 \text{ ohms},$$

which is the value of the resistance.

451. Practically, we may say the value of the resistance is 204,000,000 ohms, or 204 megohms, for inasmuch as we can only be certain of the values of the observed deflections to 3 places of figures, so we can only be certain of the worked out values to 3 places of figures. A great deal of time is often wasted in working out results to 5 or 6 places of figures when, in the observations necessary to obtain those results, it is possible to be certain of their value to 3 places only.

Measurement of the Insulation Resistance of a Cable.

452. In measuring the insulation resistance of a cable, the constant having been taken in the foregoing manner, we should join up the galvanometer, shunt, short-circuit key, reversing key, battery switch, battery, and cable, as shown by Fig. 178 (page 409).‡

By having both a galvanometer reversing key and a battery switch the trouble of reversing the wires on the galvanometer, when the battery current is reversed, is avoided, as it can be done more readily by means of the key. The object of reversing the

* Chambers' 'Mathematical Tables' are those generally used.

† Fuller's slide rule is a particularly useful instrument for working out these calculations; besides being very easy of manipulation, it is quite accurate to places of figures, whilst the 5th figure can be closely approximated to.

‡ See also 'The Silvertown Compound Key for Cable Testing,' Chapter XXVII.

galvanometer connections when the battery is reversed is to obtain the deflection always on the same side of the scale.

453. Both ends of the core of the cable must be trimmed by means of a sharp and clean knife, care being taken that the outer surface of the gutta-percha, which has been exposed and oxidised by the air, is completely cut away; the clean surface thus exposed should not be touched by the fingers. It is a good plan to coat the trimmed ends with hot paraffin *wax* (not *oil*).

The ends being thus carefully insulated, and the further end left hanging free, so as not to touch anything, the nearer end of the cable must be connected, through the medium of the lead wire,

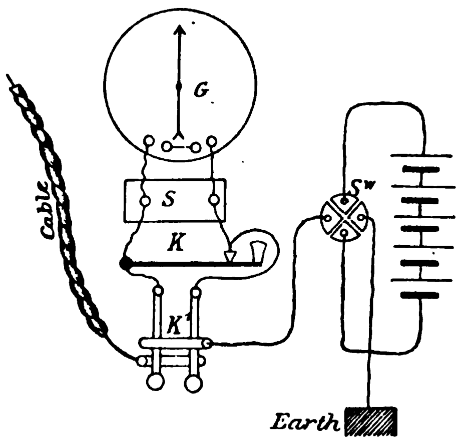


FIG. 178.

to the terminal screw of the reversing key, as shown in Fig. 178, care being taken not to touch the trimmed end in doing so. The switch plugs being inserted, the reversing key which puts the zinc pole to the cable must be clamped down, and (the short-circuit key being depressed) sufficient resistance inserted in the shunt to obtain a deflection of about 300.

At the end of a minute from the time the ***reversing*** key was clamped down, the exact deflection should be noted.

Electrification.

454. The deflection obtained, it will be found, is not a permanent one, but will gradually decrease as the current is kept on, falling rapidly at first, and then more slowly, until at length it

becomes practically stationary; the continued action of the current, in fact, apparently increases the resistance of the dielectric. This phenomenon is known as *Electrification*, and its cause is not well understood; it seems to be due to some kind of polarisation.*

The following shows the decrease in the deflection observed with a piece of cable core insulated with gutta-percha:—

Minutes' Electrification.	Deflection.
1	205
2	179
3	171
4	164
5	159
6	156
7	154
8	152
9	150
10	148·5
11	147
12	145·5
13	144
14	143
15	142

455. Electrification is much more marked at a low than at a high temperature; thus in an actual experiment it was found that with a piece of core (insulated with gutta-percha) at a temperature of 0° C. the deflection fell from 240 to 75 in 90 minutes; whereas with the same piece of core at a temperature of 24° C. the deflection fell from 240 to 173 only, in the same time.

456. The late Mr. Hockin verified the curious fact that it is not until some hours after the gutta-percha has taken its temperature that the resistance reaches its corresponding value.

457. The rate at which the deflection decreases also depends upon the nature of the insulating material; it is quicker in some kinds of gutta-percha than in others, being generally smallest in the best quality. In the case of gutta-percha, the rate of fall between the 1st and 2nd minute at a temperature of 75° would average about 2 to 5 per cent. In indiarubber the decrease is very rapid, being at 75° as much as 50 per cent. between the 1st and 5th minute.

* Although it is usually assumed that the decrease in the deflection is due to an increase in the resistance of the dielectric, it is very doubtful whether any such change in the resistance actually takes place; it is more probable that the diminution in the deflection is caused entirely by an opposing electromotive force of polarisation, which force increases (but at a decreasing *rate*) in strength so long as the battery is kept on.

458. If the cable or insulated wire under test is quite sound, the electrification should take place perfectly regularly, that is to say, the deflection on the galvanometer scale should decrease steadily. An unsteady electrification, as a rule, is a sign that the insulation is defective. It sometimes happens, however, that the unsteadiness is due to the testing battery being in a bad condition, or not properly insulated; if, therefore, the electrification is such as to raise a suspicion that the insulation of the cable or insulated wire under test is not perfect, the battery should be looked to, to see whether it is in proper order. An unsteady electrification may also be caused by the ends of the cable or of the lead wire not being properly trimmed, or from their becoming damp. Before concluding, therefore, that the cable is faulty, these points should be attended to.

A third cause of unsteady electrification occasionally exists in factories; this is due to induced currents set up by the movement of the machinery in the proximity of the tanks in which the cable is coiled. When a cable is being tested on board ship, the rolling of the latter induces comparatively strong currents in the cable, and causes the galvanometer deflections to be very erratic. The effects of these currents, in both cases, may be completely got rid of by the simple device suggested by the late Mr. J. May, of the Telegraph Construction Company, of making the insulation test with *both* ends of the cable connected to the testing apparatus, instead of with one end only.

459. Although the deflections after the 1st and 2nd minute with a zinc current are usually all that is necessary when testing each of the lengths of core (about 2 knots) of which a cable is composed, or when testing a cable during manufacture, yet, when the cable is complete a more elaborate test requires to be made.

460. Now it is found with a good cable, that if the battery be taken off after electrification has proceeded for some time, and the cable be put to earth through the galvanometer, a continually decreasing current will flow through the latter back from the cable.* Now if the deflection (called the "Earth Reading") be noted exactly 1 minute after the battery current is taken off, then the value of this deflection added to the reading taken just at the moment when the battery was taken off (that is, the last electrification reading), will equal the deflection observed after 1 minute's electrification of the cable. And, again, the earth reading at the

* Compare with note on page 410.

end of 2 minutes if added to the last electrification reading, will equal the deflection obtained after 2 minutes' electrification, and so on. Thus the "earth readings" obtained from the cable referred to on page 410 were as follows:—

	Earth Readings.
After 1 minute	59
„ 2 minutes	38
„ 3 „	30
„ 4 „	25
„ 5 „	22

The last electrification reading (at the 15th minute), it will be seen, was 142; if we add to this the 1st minute earth reading, viz. 59, we get $142 + 59 = 201$, which is approximately the same as the 1st minute electrification reading, viz. 205. Again the last electrification reading added to the 2nd minute earth reading, viz. 38, gives $142 + 38 = 180$, which is approximately the same as the 2nd minute electrification reading, viz. 179. If great care is taken to read the deflections at the exact termination of the minute intervals, the calculated and observed values will agree much more closely than in the actual examples just given. Considerable skill, however, is required in making the observations, as the fall in the deflection being very rapid at first, it may happen that the observed deflections are three or four divisions too much or too little, in consequence of the observations being made a second too soon or too late. The relation between the electrification and earth readings, as has been before stated, will only hold good if the cable is sound, and the accordance between the two may therefore be taken as an index of the good condition of the cable. It is not always the case, however, that the earth readings are noted.

The process of manipulation for taking the earth readings is as follows:—A few seconds before the completion of the last minute for electrification (usually the 15th minute)* the last electrification reading is noted and the galvanometer short-circuit key is raised, then exactly at the termination of the minute, one of the battery switch plugs is removed and placed in the adjacent hole, so that the battery becomes disconnected, and the galvanometer terminal connected to earth. The galvanometer short-circuit key being then depressed the current flows through the galvanometer, and the readings are taken at the exact termination

* The fall in the deflection is so slow after about the 10th minute that the actual deflection at the exact termination of the 15th minute would be practically the same as it was a few seconds before that time.

of each successive minute, the 1st minute being counted from the time the battery was taken off. It is not usual to take more than 5 earth readings.

Electrification readings are next taken with the copper pole of the battery connected to the cable. For this purpose the second reversing key of the galvanometer should be clamped down, and the first one released, so as to reverse the instrument; the plugs of the battery switch should then be inserted so that the battery sends its current to the cable in the reverse direction to that it did at first; this being done, the deflections on the galvanometer should be noted at intervals of a minute, as before, until the same number of readings are obtained. The readings in this case should be the same as those observed when the zinc pole was joined up, that is, provided the cable is sound, and also provided it is free from any absorbed charge when the current is put on. The current which causes the earth deflections, however, continues for a considerable period, and therefore to render a cable neutral after it has been tested with any particular current it requires to be put to earth for a certain time, which varies according to the length of the cable. If the latter is not more than 10 or 15 miles long, half an hour will usually be sufficient to render it neutral, but greater lengths require a proportionately longer time. It can easily be seen when the absorbed charge is got rid of, for if the cable is neutral no deflection will be observed on depressing the short-circuit key, but if a charge is still retained a slight constant deflection will be produced.

461. When the cable is put to earth great care must be taken that the short circuit key K (Fig. 175, page 406) is first raised, otherwise the whole static discharge (which is quite distinct from the current which causes the earth deflections) will pass through the galvanometer coils and the needles may be demagnetised or, at least, their magnetic power be altered.

462. Although it is advisable if possible to take a set of readings with a zinc and with a copper current, the cable being neutral in both cases, yet if time is an object the test with the copper current (which is usually made after the test with the zinc current) can be taken before the earth current due to the zinc test has ceased. In this case, however, the average readings will be higher than would be the case if the cable were neutral, in fact if we take the last of the earth readings observed in the case of the zinc test, and we deduct it from the first minute electrification reading of the copper test, then the result should *approximately* equal the first minute electrification reading of the zinc test. Thus in the case of

the cable the zinc readings on which were given on page 410, the electrification readings obtained with the copper pole of the battery connected to the cable, were as follows:—

Minutes' Electrification.	Deflection.
1	227
2	200
3	189
4	182
5	178
6	174
7	171
8	168
9	165
10	163
11	161
12	159
13	157
14	156
15	155

Now the last earth reading taken in the case of the zinc test (page 412) was 22, and this deducted from 227 (the first copper electrification reading) gives 205, which is the first zinc electrification reading (page 410).

In making the test in practice, as soon as the last earth reading of the zinc test is observed, the galvanometer short-circuit key should be raised and the battery reversed, then one minute after this moment the first electrification reading should be noted.

When a copper current test is made in the foregoing manner, that is to say with the cable not neutral, we cannot compare all the copper with all the zinc readings, as it would be necessary to make a deduction from each of the former; but inasmuch as these deductions would have to be less and less from each successive reading (for the earth current which causes the copper reading to average lower than the zinc reading is a continually decreasing quantity) and as we do not know at what rate the diminution takes place we cannot make the comparison; the uniformity of the electrification, however, and the approximate agreement between the first minute zinc reading, and the first minute copper reading minus the last zinc earth reading, is sufficient to indicate the condition of the cable under test.

When all the electrification readings with the copper current are observed, a set of earth readings should be taken as in the case of the zinc current electrification test. The first earth reading added to the last electrification reading should, in this case, approximately equal the first zinc electrification reading. In

the cable in question the actual earth readings observed were as follows:—

					Earth Readings.
After 1 minute	..	..	..		50
„ 2 minutes	..	..	..		28
„ 3 „	..	..	..		21
„ 4 „	..	..	..		17
„ 5 „	..	..	..		14

It will be seen that in this case the first earth reading, viz. 50, added to the last electrification reading, viz. 155, is 205, which is the same as the first zinc electrification reading.

463. If there is not time to take readings both with the zinc and copper currents the zinc should be the one employed, as in the case of a fault it renders the latter more apparent, the copper current having the effect, to a certain extent, of sealing up a defect.

The measurements being made, the resistance at the end of the first minute with the zinc current, and the percentage of electrification between the first and second minute and (in the case of a completed cable) also between the 1st and last (usually the 15th minute) should be worked out. It is not usual or necessary to carry the calculations beyond this.

Price's Method of Correcting for Surface Leakage.

464. This methed, devised by Mr. W. A. Price, is shown by Fig. 179. A A are the two ends of a coil of cable brought out of

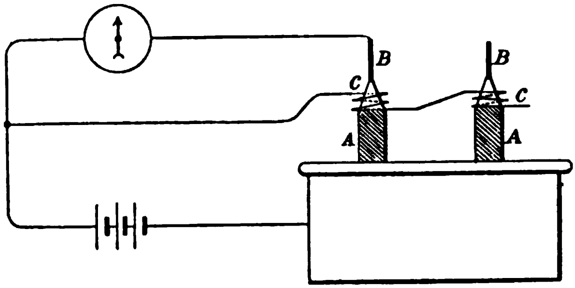


FIG. 179.

the tank of water in which the coil is immersed. The ends are shown trimmed into long tapering cones in the usual way, so as to expose a large clean surface of the dielectric, as well as the ends, B B, of the conductor. C C is a thin copper wire wound closely two or three times round the middle of the taper ends, and con-

nected as shown. Then if the resistances of the surface films on B, C, B, C, are large compared with that of the galvanometer, B, C, B, C, are all at the same potential, and no leakage will take place from B, B, over the trimmed surfaces.

If the conductivity a of the film between B and C is comparable with that of the galvanometer, g, the galvanometer deflection will be reduced in the proportion of $g : a + \gamma$, and must be multiplied by $\frac{a + \gamma}{\gamma}$ to obtain the correct result.

A practical test of this method, applied to a piece of sound gutta-percha core immersed in water, showed the following result:—

The ends were carefully cleaned and trimmed, and projected from the water by several feet. The connections were made first without any guard wire, and 20 divisions of deflection were obtained on the galvanometer scale, indicating a dielectric resistance of 13,000 megohms. The ends were lowered into the water till only a few inches remained exposed, and these were rubbed with black lead. The galvanometer deflection increased to 124,000 divisions (124 with $\frac{1}{1000}$ shunt). The guard wire was then connected without any other alteration, and the deflection obtained was 20 divisions as before.

Equally clear results were obtained by connecting an artificial leak of known resistance (one megohm) between the conductor and the earth, and joining the guard lead to different points of it.

465. Price's method, it has been pointed out by Mr. H. Savage (Electrician at Messrs. Henley's Works, North Woolwich), may be applied with great advantage for preventing instrument leakage when testing in damp weather. This leakage causes considerable trouble, rendering accurate testing impossible. It will generally be found that the greatest trouble is with the galvanometer, as while the keys and shunt box may be insulated by placing them on blocks of paraffin wax, it is difficult and undesirable to move the galvanometer to take this special precaution, and despite any amount of drying the galvanometer persistently leaks. This trouble may be avoided in the following simple way:—Connect the levelling screws of the galvanometer together and join them to the insulated terminal of the battery key. Of course, the galvanometer will require to be insulated to avoid short-circuiting the battery, but this is already usually done by standing the galvanometer on ebonite buttons, or suspending it by rubber tubes.

This guard wire method succeeds where the ordinary methods fail by reason of moisture on the surface. Of course, the same system might be extended to the shunt box, &c., due care being

taken to avoid accidental contact. During damp weather a leakage of 200 scale divisions with 750 volts was reduced to one division by this method.

466. When the cable is connected to the testing instruments by a long leading wire, then at the commencement of the test the end of the lead should be disconnected from the cable, and insulated; if any deflection is observable on the galvanometer when the battery current is put on, this deflection must be subtracted from the deflection obtained when the cable is attached to the lead. In making this correction care must be taken that the same shunt (if any) is connected to the galvanometer as will be employed when the cable is connected to the lead, or if no shunt is used with the lead then the necessary allowance for this must not be forgotten to be made.

The ends of the lead must be trimmed in the same manner as the ends of the cable.

The practical way of noting down and working out these tests will be found in Chapter XXVI.

467. At the works of Messrs. Siemens and Co., Charlton, the method which has been described of testing the completed cable is not generally adopted, the following test being preferred:—The testing battery is applied through a galvanometer to the cable in the usual way, and readings for five consecutive minutes with the zinc pole of the battery to the cable are observed, the battery is then immediately reversed and five more minute readings taken; the battery is then again reversed, and so on until six sets of five minute readings have been noted, viz. three with a zinc and three with a copper current, taken alternately. If the cable is in good condition, then the last two sets of readings should be identical in value.

Shunt Tables.

468. When a large number of cables have to be tested daily at a factory, any contrivances or methods for shortening calculations are of great value. Now the use of shunts of different values for obtaining readable deflections on the galvanometer scale with different cables is continual, and the working out of the multiplying powers of these shunts is a somewhat tedious operation when a large number have to be calculated. If the resistance of the galvanometer used for making the tests were constant, a small table could easily be calculated which would show the multiplying power of any particular shunt at a glance; but the resistance of a

galvanometer varies considerably with change of temperature, and therefore under ordinary conditions a table of the kind cannot be employed.

469. A very simple method of getting over the foregoing difficulty, devised by Mr. Herbert A. Taylor, is in use in the testing rooms of the Telegraph Construction and Maintenance Company. The method is to have a small adjustable set of resistance coils directly in circuit with the galvanometer, so that the resistance of the latter can practically be always preserved the same.

The resistance of the ordinary reflecting galvanometer usually averages between 5000 and 6000 ohms; by having the galvanometer wound, therefore, so that in the hottest weather the latter value is never exceeded, and by having a set of resistance coils adjustable from 1 up to about 1000 ohms, the resistance in the circuit can always be kept up to 6000 under all conditions, and, therefore, a table giving the multiplying power of shunts for a galvanometer of 6000 ohms resistance can always be made use of. Tables of this description will be found at the end of the book. The tables also give the combined resistance of the galvanometer and shunt, which may sometimes be required to be taken into account.

470. In the insulation testing of submerged cables the effects of earth currents are often to render the readings somewhat unsteady, so that considerable discrimination is required to determine whether the observed unsteadiness is due to this cause or to the existence of a fault. In the case of single cored cables there is no method of eliminating these effects of earth currents, but if the cable is multiple-cored then Mr. F. Jacob points out that by a simple device the earth current difficulty can be entirely eliminated and excellent results obtained. This device consists in testing two of the cores at the same time, the second core being connected to the pole of the battery which in the ordinary insulation test is put to earth, and the battery being well insulated. In this case the total measured insulation of the two cores is double that of a single core, hence the mean insulation per mile of the total length of two cores will be the total measured insulation multiplied by one-half the length of one core.

471. Mr. Jacob further points out that this method may be applied in other tests, those for capacity, for example, it being only requisite to replace all the connections which are usually put to earth, by connections to the other core, the distant ends of the two cores of course being left separated and insulated. In this

case, since the two cores are in series, the joint capacity of the two will be one-half that of a single core, so that the capacity of one core will be double that of the total measured capacity.

If a condenser of large capacity, i.e. of a capacity not greatly disproportionate to the total capacity under test, is available, the earth current effect could probably be greatly reduced by interposing this condenser between the testing apparatus and the cable under test. In this case if F_1 be the capacity of the cable, F_2 the capacity of the condenser, and f the joint capacity (in cascade) of the two, then (page 310, § 330),

$$f = \frac{F_1 F_2}{F_1 + F_2},$$

hence

$$F_1 = \frac{F_2 f}{F_2 - f}.$$

472. As multiple core cables usually have not less than three cores, by making a series of tests in the manner indicated for conductor resistance tests, page 256, the individual insulation resistance of each wire can be obtained in a precisely similar way. If two separate cables which lie between the same termini are tested on Mr. Jacob's plan, the readings obtained will be much steadier than when each cable is tested separately in the ordinary manner, but they will seldom be absolutely steady, showing how local and variable the earth current changes are. In order to ascertain the individual insulation of each cable by a test of this kind, the approximate relative values of the insulation of each cable can be ascertained by balancing one cable against the other in a Wheatstone bridge, and then dividing the total observed insulation of the two in the proportion of these relative values.

473. It may be mentioned that it has been proved experimentally * that the insulation resistance of a cable core or insulated wire in perfect condition is the same whether it is measured with a low or with a high battery power. It has been stated, however, that this is not the case when the power is of several thousand volts potential.

* The 'Electrical Review,' April 11th, 1890, page 393; 'Journal of the Institution of Electrical Engineers,' No. 95, page 620.

CHAPTER XVI.

MEASUREMENT OF RESISTANCES BY POTENTIALS.

474. There are two distinct ways of measuring resistances by potentials :—

1st. By noting the fall of potential along a known resistance with which the unknown resistance is in connection.

2nd. By noting the rate at which a condenser, of a known capacity, loses its potential when it discharges itself through the unknown resistance.

Fall of Potential Method.

475. If we connect a battery to a resistance R + *x*, as shown by Fig. 180, the potential of the battery may be regarded as* falling regularly along the resistance, being full at *a* and zero

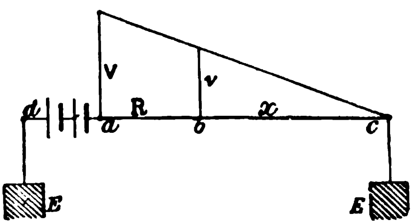


Fig. 180.

at *c*. The same would be the case if *c* and *d* were connected together instead of being put to earth. By similar triangles we have

$$V : v :: R + x : x,$$

therefore

$$V x = v R + v x,$$

or

$$x (V - v) = R v,$$

* See Chapter XI., page 320, § 344.

from which

$$x = R\frac{v}{V - v};\qquad [A]$$

V being the potential at a, and v the potential at b. So that, if R is a known resistance, we can—by observing the values of V and v—determine the value of x.

For example.

If $R = 1000$ ohms, $V = 300$, and $v = 200$, then

$$x = 1000\frac{200}{300 - 200} = 2000 \text{ ohms.}$$

476. The relative values of the potentials can be measured by means of a condenser. To do this we should join up our condenser and galvanometer, as shown by Fig. 142, page 313, the only difference being that the terminals which are there represented as being in connection with a battery would, in the present case, be connected to the points a and d (or c) for determining V, and to b and d (or c) for determining v. The condenser discharges in the two cases give V and v.

Another, and for most cases a preferable, method of measuring the potentials, is to insert a galvanometer between the point at which the potential is to be measured and the earth, there being in the circuit a resistance several thousand times greater than the resistance of the conductor of the cable. The permanent deflections in this case indicate the potentials (§ 348, page 322).

477. Instead of measuring the potential V, we can, if we please, at once determine the value of $V - v$ by connecting the wires from the condenser, &c. (or from the galvanometer and high resistance), to the points a and b; the deflection in this case at once gives us $V - v$. So that if we call v' this difference of potential, we get

$$x = R\frac{v}{v'}.\qquad [B]$$

478. The conditions for making the test by formula [A] in the best possible manner are precisely similar to those in the case of the "Divided Charge Method" of measuring the electrostatic capacity of a cable or condenser (page 379); for equation [A] (page 379) in this latter test is similar to (though not identical with) equation [A] (given above) of the test under consideration. We must, in fact, adjust R until we make v approximately equal to $\frac{V}{3}$, that is to say, we must make R about twice as large as x.

In the case of equation [B] (page 421) the conditions are slightly different, for here the quantity v' replaces $(V - v)$, and although v' and $(V - v)$ are equal, yet inasmuch as v' is the result of a single observation only, there can be but one error in it; consequently, to determine the best conditions for making the test, we must take equation [A] (page 421), and assume an error δ to exist in v only.

Let λ be the error in x caused by an error δ in v, then

$$x + \lambda = R \frac{v + \delta}{V - (v + \delta)};$$

but since

$$x = R \frac{v}{V - v}, \quad \text{or,} \quad R = x \frac{V - v}{v},$$

therefore

$$\lambda = x \left\{ \frac{V - v}{v} \times \frac{v + \delta}{V - (v + \delta)} - 1 \right\} = x \frac{V \delta}{v \{ V - (v + \delta) \}};$$

or, since δ is a very small quantity, we may say

$$\lambda = x \frac{V \delta}{v (V - v)}.$$

Now we have to make λ as *small* as possible; this we shall do, since x, V, and δ are constant quantities, by making $v (V - v)$ as *large* as possible.

But

$$v (V - v) = \frac{V_2}{4} - \left(\frac{V}{2} - v \right)^2;$$

and to make this expression as large as possible we must make $\frac{V}{2} - v$ as small as possible; that is, since v must be positive, we must make it equal to 0, or

$$\frac{V}{2} - v = 0,$$

therefore

$$V = 2 v;$$

but

$$v' = V - v,$$

therefore

$$v' = 2 v - v = v;$$

in which case we get

$$x = R;$$

that is to say, in order to make the test as accurately as possible, we must make R approximately equal to x.

479. If, instead of introducing the unknown resistance x, and the known resistance R, between the points a and c, we join the pole a of the battery direct on to b, we can determine the value of x by simply noting V, and then inserting an adjustable resistance in the place of x, and altering it until we make the potential at b to be V, as at first, when of course $x = R$.

Best Conditions for making the Test.

480. In the case of formula

$$x = R\frac{v}{V - v}, \qquad [A]$$

make R approximately equal to $2x$.
In the case of formula

$$x = R\frac{v}{v'}, \qquad [B]$$

make R approximately equal to x.

Possible Degree of Accuracy attainable.

In the case of formula [A]

$$\text{Percentage of accuracy} = \frac{\delta\,(V + v)\,100}{v\,(V - v)}.$$

In the case of formula [B]

$$\text{Percentage of accuracy} = \frac{\delta\,(v + v')\,100}{v\,v'}; \qquad [C]$$

where δ is the fraction of a division to which each of the deflections V, v, and v' can be read.

LOSS OF POTENTIAL METHOD.

481. In Chapter XIII., page 363, an equation

$$F = \frac{T}{2\cdot 303\ R \log \frac{V}{v}}$$

was obtained, where F was the electrostatic capacity, in microfarads, of a condenser, or cable, the potential of whose charge fell from V to v when it was discharged during T seconds through a resistance of R megohms.

Now if F is the known and R the unknown quantity, then

$$R = \frac{T}{2\cdot 303\ F \log \frac{V}{v}};$$

so that we can determine the value of a resistance by a capacity and loss of charge measurement.

482. The connections for making such a test would be precisely similar to those given for determining electrostatic capacities by loss of charge (§ 405, page 368).

If we were measuring the resistance of a short cable by this method, the discharge deflection V, compared with the discharge deflection obtained with the same battery from a standard condenser, would give us the value of F. For long cables, however, as we have before explained, this does not give correct results, so the capacity must be determined by other methods, Thomson's for example (page 369).

483. From (§ 398, page 364) it is obvious that we must have

Best Conditions for making the Test.

Make v as nearly as possible equal to $\frac{V}{3 \cdot 5}$.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = R \frac{200\,\delta}{2 \cdot 303\, v \log \frac{V}{v}}$$

where δ is the fraction of a division to which each of the deflections V and v can be read.

Thomson's Method.*

484. In Fig. 181, which shows the connections for the test, G is a galvanometer; C, a condenser; K, a Rymer-Jones key (page 316); k, a short-circuit key; S, a Varley slide resistance (page 231). The test is then made in the following manner.

The slides are set so that a convenient potential, V, can be obtained at the slider d. Key K is then moved over to the left so as to charge the cable for, say, five seconds, and then moved to the "insulate" position so that the battery is cut off. t seconds afterwards the key is again turned to the left (key k being depressed), and the deflection produced by the inrush which replaces the charge which has leaked out during the "insulate" period, is noted. This series of operations is repeated several times, and the mean of the observed deflections is taken as being the correct result.

Now if during the "insulate" period the potential of the

* This method was devised by Sir William Thomson (Lord Kelvin) in 1875, but appears to have been forgotten. Dr. A. Tobler (Professor in the Federal Polytechnic School at Zurich) called attention to it in the 'Electrical Review' for October 8th, 1897.

charge has fallen from V to, say, v_1, then the potential, v, lost will be

$$v = V - v_1,$$

and this will be the potential which is added to the charge when the inrush deflection is noted.

To determine the value of v, turn key K to the right so as to discharge the cable, and shift the slides so that the potential at d is altered in value, depress key k and then turn key K over to the left. If the new adjustment of the slides happens to be such

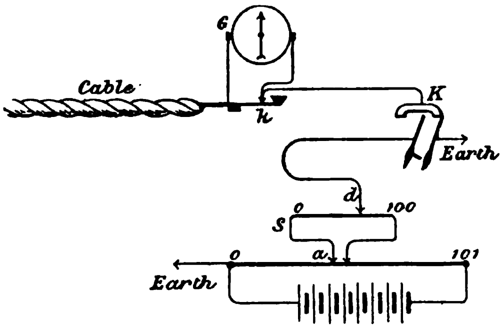


FIG. 181.

that the potential at d is equal to v, then the movement of key K will have caused a similar deflection to that obtained in the first instance. If, however, the deflection is not the same, then the position of the slides must be altered until the movement of the key K causes the original deflection to be reproduced, and consequently the correct value of v to be obtained.

Since the resistance, R, of the cable is given by the formula

$$R = \frac{T}{2 \cdot 303 \text{ F} \log \frac{V}{v}},$$

F being the capacity of the cable (page 423), and since

$$v = V - v_1, \quad \text{or,} \quad v_1 = V - v,$$

we have

$$R = \frac{T}{2 \cdot 303 \text{ F} \log \frac{V}{V - v}}.$$

For example.

The following values of V and v were found on the slides:—

$$V = 10{,}000 \qquad v = 8500,$$

the capacity, F, of the cable was 5 microfarads, and the time, T, one minute (60 seconds); what was the resistance of the cable?

We have

$$V - v = 10{,}000 - 8500 = 1500,$$

therefore

$$R = \frac{60}{2 \cdot 303 \times 5 \log \frac{10{,}000}{1500}} = \frac{60}{2 \cdot 303 \times 5 \times \cdot 824} = 6 \cdot 32 \text{ megohms.}$$

The great advantage of the above-described test over the ordinary loss of potential method (page 423) lies in the fact that the deflection which represents the loss of charge in the cable, can be exactly reproduced by the operation, whilst in the ordinary method, two sometimes widely different deflections (at beginning and end of time T) have to be compared.

Gott's Proof Condenser Method.

485. An excellent method of determining the relative values of V and v in the "loss of potential" test (page 423) has been suggested by Mr. J. Gott. This method avoids the necessity of discharging the cable, and consists in applying what may be termed a "proof" condenser to the latter, and then measuring the discharge from the same. This condenser should be of small capacity, so as not to remove an appreciable portion of the charge from the cable; if this is the case it is obvious that the discharge obtained from the condenser, after it has been connected for a few seconds to the cable at any particular time, will represent the potential which the cable has at that time.

486. When the insulation resistance of a cable is measured by the foregoing methods, the result obtained is a mean of the resistances which the cable has at the commencement and at the end of the test, as *electrification* (page 409) goes on the whole time the charge is falling.

487. Experimental results show that in the case of a cable whose core is insulated with gutta-percha, if the cable be charged 10 seconds before taking the discharge V, and again 10 seconds before insulating it preparatory to observing the discharge v, then

the value of R after 1 minute, obtained from the formula, agrees with that obtained by the constant deflection method given in the last chapter (§ 452, page 408).

488. If we know the potential which the cable has when fully charged, and also its potential after a certain time, we can determine the potential it will have after any other time in the following manner :—

A charged cable loses equal percentages of its charge in equal times, that is to say—if, for example, 5 per cent. of its charge were lost during the first second, then five per cent. of *what remained* would be lost in the second second.

Let V be the potential at first;
v „ „ after 1 sec.;
v_1 „ „ „ t_1 secs.;
v_2 „ „ „ t_2 „

and let us suppose the charge loses $\frac{1}{n}$th of its potential during the first second; then the potential at the end of first second will be

$$V - \frac{V}{n} = v = V\frac{v}{V}; \qquad [1]$$

and the potential at end of the second second will be

$$v - \frac{v}{n}; \qquad [2]$$

but from equation [1] we get

$$n = \frac{V}{V - v};$$

therefore, substituting this value in [2] the latter quantity becomes

$$\frac{v^2}{V}, \quad \text{which equals} \quad V\left(\frac{v}{V}\right)^2,$$

and consequently the potential at the end of t_1 seconds will be

$$V\left(\frac{v}{V}\right)^{t_1} = v_1.$$

Also we must have

$$V\left(\frac{v}{V}\right)^{t_2} = v_2;$$

therefore

$$t_1 = \frac{\log \frac{v_1}{V}}{\log \frac{v}{V}},$$

and

$$t_2 = \frac{\log \frac{v_2}{V}}{\log \frac{v}{V}};$$

that is,

$$t_2 = \frac{\log \frac{v_2}{V}}{\log \frac{v_1}{V}} \cdot t_1 = \frac{\log \frac{V}{v_2}}{\log \frac{V}{v_1}} \cdot t_1.$$

For example.

The potential at first was 300 (V), and after 20 seconds (t_1) it fell to 200 (v_1). After what time (t_2) would it fall to 100 (v_2)?

$$t_2 = \frac{\log \frac{300}{100}}{\log \frac{300}{200}} \times 20 = \frac{\overline{2 \cdot 4771213} \atop \overline{2 \cdot 0000000}}{\overline{2 \cdot 4771213} \atop \overline{2 \cdot 3010300}} \times 20 = 54 \text{ secs.}$$

489. It being usually required to know the time the charge in a cable will take to fall to half charge, the formula becomes

$$t_2 = \frac{\cdot 30103}{\log \frac{V}{v_1}} \cdot t_1.$$

490. The formulæ we have given are capable of various modifications, which, however, are more of a fanciful than of an actual and practical value.

Thus the formulæ

$$R = \frac{T}{F \log_e \frac{V}{v}}, \text{ and, } F = \frac{T}{R \log_e \frac{V}{v}}$$

may be simplified if we make $v = \frac{V}{2}$, for in this case

$$\log_e \frac{V}{v} = \log_e 2 = \cdot 693;$$

therefore

$$R = \frac{T}{\cdot 693\, F} = 1\cdot 443\, \frac{T}{F}.$$

To obtain experimentally the time occupied in falling to half charge, repeated trials would be necessary, and the time taken in doing this would hardly compensate for the advantage of using a simpler formula.

The object of obtaining the time of fall to half charge is to get a convenient unit for comparison with other cables, and this time of fall is easily calculated from the formula before given, in which the potential after any time may be used, this being obtained by one observation only.

491. A useful formula is that suggested by Sir W. H. Preece, which is obtained in the following manner :—

In the equation

$$t_2 = \frac{\cdot 30103}{\log \frac{V}{v_1}} \cdot t_1$$

let n = percentage of loss of time t_1, then

$$n = \frac{(V - v_1)\, 100}{V};$$

therefore

$$v_1 = V\, \frac{100 - n}{100};$$

substituting this value of v_1 in the above equation, we get

$$t_1 = \frac{\cdot 30103}{\log \frac{100}{100 - n}} \cdot t_1 = \frac{\cdot 30103}{2\cdot 000 - \log (100 - n)} \cdot t_1.$$

For example.

If a cable lost 20 per cent. of its charge in five minutes, in how many minutes would it fall to half charge ?

$$t_2 = \frac{\cdot 30103}{2\cdot 000 - \log (100 - 20)} \times 5 = 15'\, 32''.$$

492. From the equations

$$V \left(\frac{v}{V}\right)^{t_1} = v_1,$$

$$V \left(\frac{v}{V}\right)^{t_2} = v_2 \text{ (page 427).}$$

we can find what would be the potential, v_2, after a certain interval of time, t_2, the potential at first, and the potential v_1, after a time, t_1, being given.

Thus we have from the foregoing equations

$$\left(\frac{v_1}{V}\right)^{t_2} = \left(\frac{v_2}{V}\right)^{t_1};$$

therefore

$$v_2 = V\left(\frac{v_1}{V}\right)^{\frac{t_2}{t_1}}.$$

This formula we should have to work out by the aid of logarithmic tables.

For example.

The potential of the charge in a cable when full was 300 (V). After 20 minutes (t_1) the potential fell to 200 (v_1). What would be the potential v_2 at the end of 30 minutes (t_2)?

$$v_2 = 300\left(\frac{200}{300}\right)^{\frac{30}{20}} = 300\left(\frac{2}{3}\right)^{\frac{3}{2}}$$

$$\begin{array}{rl}
\log 2 = & \cdot 3010300 \\
\log 3 = & \cdot 4771213 \\
\hline
& \bar{1}\cdot 8239087 \\
& 3 \\
\hline
2) & \bar{1}\cdot 4717261 \\
\hline
& \bar{1}\cdot 7358631 \\
\log 300 = & 2\cdot 4771213 \\
& 2\cdot 2129844 = \log \text{ of } \underline{163\cdot 3}.
\end{array}$$

493. In connection with the foregoing tests it may be mentioned that, in testing cables, it is very usual to make fall of charge measurements, but not to work out the results by any of the foregoing formulæ. The general practice is to simply calculate and record the percentage of fall.

CHAPTER XVII.

LOCALISATION OF FAULTS BY FALL OF POTENTIALS.

Clark's Method.

494. In Fig. 180 (page 420) in the last chapter, if $b\ c$ were a portion of a cable making *full* earth at c, then by determining $b\ c$ by the method described we should find the position of the break.

Supposing, however, a cable had a fault which did not make full earth, then the potential would not fall to zero at that point, but would have a value depending upon the resistance of the fault. The potential, however, would be the same as the potential at the further end of the cable, provided that end were insulated.

If we can determine the value of this potential, we can readily localise the position of the fault.

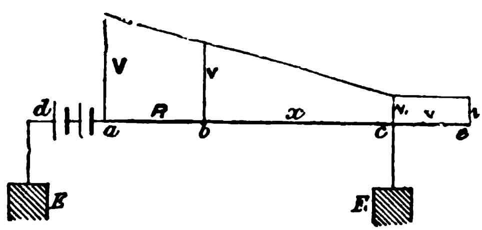


Fig. 182.

In Fig. 182 let $b\ e$ be the cable which has a fault at c, the end of the cable at e being insulated; and let R be the resistance between the battery at the end of the cable b, then

$$V - v_1 : v - v_1 :: R + x : x;$$

therefore

$$x(V - v_1) = (v - v_1)R + (v - v_1)x,$$

or

$$x[(V - v_1) - (v - v_1)] = R(v - v_1);$$

therefore

$$x = R \frac{v - v_1}{(V - v_1) - (v - v_1)},$$

that is,

$$x = R \frac{v - v_1}{V - v}. \qquad [A]$$

495. If, as explained in the last test (§ 476, page 421), we at once determine the value of $V - v$, by connecting the wires from the condenser, &c. (or from the galvanometer and high resistance), to the points a and b, then if we call v' this difference on potential, we get

$$x = R \frac{v - v_1}{v'}. \qquad [B]$$

496. In order to determine the relative values of the potentials at the two ends of the cable, their values with reference to some standard of potential or electromotive force must be obtained. For this purpose any of the standard cells mentioned in Chapter VII. (page 157) may be used.

The way in which such standard cells would be employed for making the test we have been considering, would be as follows :—

The electrician at a charges a condenser from one of the standard cells, and notes the discharge deflection on his galvanometer. This deflection, then, represents the potential of the cell.

The wires from the standard cell are now disconnected, one wire is connected to earth, and the other to a, and again a discharge reading is taken; then this reading, divided by the reading obtained with the standard cell, and multiplied by the electromotive force of the cell, gives the value of V in terms of the potential or electromotive force of the standard cell. The wire at a is then disconnected and joined to b, and another discharge measured, which result divided by the standard discharge, and multiplied by the force of the cell, gives the value of v.

The electrician at the other end, e, of the cable makes a similar test, and thus determines the value of v_1.

The capacities of the condensers at the two stations, it may be observed, need not be alike.

For example.

The discharge deflection obtained from a condenser at station a, with a standard cell of $1 \cdot 453$ volts electromotive force, was 150 divisions. The potentials V and v, measured with the same condenser, gave deflections equivalent to 2550 and 1050 divisions respectively; therefore

$$V = \frac{2550}{150} \times 1 \cdot 453 = 24 \cdot 701.$$

$$v = \frac{1050}{150} \times 1 \cdot 453 = 10 \cdot 171.$$

The discharge deflection obtained from a condenser at station e, with a standard cell of $1 \cdot 462$ volts electromotive force, was 180 divisions. The potential v_1, measured with the same condenser, gave a discharge deflection of 360 divisions; therefore

$$v_1 = \frac{360}{180} \times 1 \cdot 462 = 2 \cdot 924.$$

R was equal to 1000 ohms. What was the value of x?

$$x = 1000 \frac{10 \cdot 171 - 2 \cdot 924}{24 \cdot 701 - 10 \cdot 171} = 498 \cdot 8 \text{ ohms},$$

showing that the fault was $498 \cdot 8$ ohms distant from the end b of the cable. If the length of the cable were, say, 80 knots, and its total conductivity resistance 800 ohms, or 10 ohms per knot, then the distance of the fault from b would be $\frac{498 \cdot 8}{10}$, or $49 \cdot 88$, knots.

The value of v_1 when obtained at e would be telegraphed to b; this could be done, since the cable would not be entirely broken down.

If the potentials are measured by observing the permanent deflections obtained through a high resistance (§ 348, page 322), the observations with the standard cells must be made in the same manner.

497. In making the test we are liable to make errors in V, v, and v_1, and these errors will produce the greatest total error in x when the errors in V and v_1 are minus, and the error in v is plus; let each of the errors be δ, and let λ be the total error produced in x, we then have

$$x + \lambda = R \frac{(v + \delta) - (v_1 - \delta)}{(V - \delta) - (v + \delta)} = R \frac{v - v_1 + 2\delta}{V - v - 2\delta},$$

or

$$\lambda = R \frac{v - v_1 + 2\delta}{V - v - 2\delta} - x;$$

but

$$x = R \frac{v - v_1}{V - v}, \quad \text{or,} \quad R = x \frac{V - v}{v - v_1},$$

therefore

$$\lambda = x \left[\frac{V - v}{v - v_1} \times \frac{v - v_1 + 2\delta}{V - v - 2\delta} - 1 \right] = x \frac{2\delta (V - v_1)}{(v - v_1)(V - v - 2\delta)};$$

but, since δ is small, we may say

$$\lambda = x \frac{2\delta (V - v_1)}{(v - v_1)(V - v)},$$

or

$$\lambda = x \frac{2\delta (V - v_1)}{(v - v_1)[(V - v_1) - (v - v_1)]}.$$

Now if we regard $(V - v_1)$ as a constant quantity, then in order to make λ as small as possible we must make the denominator of the fraction as small as possible; from § 477, page 421, we can see that in order that this may be the case we must make

$$(v - v_1) = \frac{(V - v_1)}{2},$$

that is to say, we must make R approximately equal to x.

In the case of formula [B] (page 432) the conditions for making the test in the most satisfactory manner, are slightly different from the foregoing; for since $(V - v)$ in this case is obtained by a single measurement, v', there can be but one error, δ, in it. We have, in fact,

$$\lambda = x \left[\frac{V - v}{v - v_1} \times \frac{v - v_1 + 2\delta}{V - v - \delta} - 1 \right] = x \frac{\delta (2V - v - v_1)}{(v - v_1)(V - v - \delta)};$$

but, since δ is very small, we may say

$$\lambda = x \frac{\delta (2V - v - v_1)}{(v - v_1)(V - v)},$$

or

$$\lambda = x \frac{\delta [2(V - v_1) - (v - v_1)]}{(v - v_1)[(V - v_1) - (v - v_1)]}.$$

Now this equation is of the same form as equation [F] (page 125), consequently the investigation there given may be applied to the present case. In the latter, the coefficients of $(V - v_1)$ and $(v - v_1)$ are 2 and -1 respectively; if therefore in equation [G] (page 125) we substitute $-\frac{1}{2}$ for k, and also if we substitute x and R, for C_2 and C_1, respectively, we shall obtain the conditions we require; we have then

$$v - v_1 = \frac{V - v_1}{\sqrt{-\frac{1}{2} + 1} + 1}, \quad \text{or,} \quad V - v_1 = (v - v_1)\left(\frac{1}{\sqrt{2}} + 1\right)$$
$$= (v - v_1)\, 1{\cdot}7071;$$

that is to say, we must have

$$R = 1 \cdot 7071\, x.$$

Practically we may say, make

$$R = 2\,x$$

approximately.

We have therefore

Best Conditions for making the Test.

498. In the case of formula

$$x = R\frac{v - v_1}{V - v}, \qquad [A]$$

make R approximately equal to x.

In the case of formula

$$x = R\frac{v - v_1}{v'}, \qquad [B]$$

make R approximately equal to $2\,x$.

Possible Degree of Accuracy attainable.

In the case of formula [A]

$$\text{Percentage of accuracy} = \frac{\delta\,(V - v_1)\,200}{(v - v_1)\,(V - v)}.$$

In the case of formula [B]

$$\text{Percentage of accuracy} = \frac{\delta\,(2\,v' + v - v_1)\,100}{(v - v_1)\,v'}.$$

Where δ is the fraction of a division to which each of the deflections can be read.

SIEMENS' EQUAL POTENTIAL METHOD.

499. In Fig. 183 (page 436) let B E be the cable which has a fault at c, x and y being the distances on either side of the fault, and z the equivalent length of the latter. Suppose that one pole of a battery is connected at B, the other pole being to earth, then if the end of the cable at E is insulated we shall have, as in the last test, the potential at E to be the same as the potential at the fault. Next suppose that the battery at B is removed, and that that end of the cable is insulated; then, if a battery is connected to E, of such a strength that the potential at the fault, and therefore at B, is the same as was the potential at E in the first case, then V_2 will be the new potential at E.

Now,

$$V_1 - v_1 : V_2 - v_1 :: x : y,$$

therefore

$$\frac{x}{y} = \frac{V_1 - v_1}{V_2 - v_1}.$$

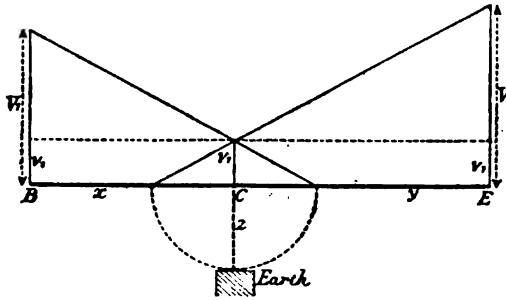


FIG. 183.

If l be the length of the cable, then

$$l = x + y, \quad \text{or,} \quad y = l - x;$$

therefore

$$\frac{x}{l - x} = \frac{V_1 - v_1}{V_2 - v_1};$$

that is,

$$x(V_2 - v_1) = l(V_1 - v_1) - x(V_1 - v_1),$$

or

$$x = l\frac{V_1 - v_1}{(V_2 - v_1) + (V_1 - v_1)}.$$

For example.

In a faulty cable 500 knots (l) long, after adjusting the potentials according to the foregoing method, the values of the same were found to be

$$V_1 = 200,$$
$$V_2 = 300,$$
$$v_1 = 40.$$

What was the distance (x) of the fault from B?

$$x = 500\frac{200 - 40}{(300 - 40) + (200 - 40)} = 190 \cdot 5 \text{ knots.}$$

500. In making the test practically, the following course would be pursued:—

Station B first connects one pole of a battery direct on to the

cable, the other pole being to earth, whilst E insulates his end of the cable. This being done, B notes the potential V_1, and E the potential v_1. When B thinks that sufficient time has elapsed for E to have taken his observation, he removes the battery and insulates his end of the cable. E noting that his potential has fallen to zero connects up his speaking apparatus, and B having done the same, E communicates to B the result he has obtained.

Station E now connects up his battery to the cable, taking care that the pole connected to the latter is similar to that employed by B in the first instance. The latter observes the potential at his end of the cable, and if it is not the same as that previously obtained at E, he informs the latter, by means of signals agreed upon, that such is the case, whereupon E increases or decreases his battery power, and regulates it by varying a resistance in its circuit, until the potential at B is made the same as it was at E on the first occasion. The potential V_2 is then noted by E, and the result being reduced to terms of a standard cell,* is communicated to B. The latter station, having also reduced his results to terms of a standard cell, then works out the formula, and thus determines the position of the fault.

501. For localising faults in long cables this method is more accurate than the previous one, as it is not so much influenced by the *resultant* fault † produced by the conductive power of the insulating sheathing, more especially if the fault is near the middle of the cable.

It must be understood that both tests are only accurate in cases where the total insulation resistance of the cable is very high compared with the resistance of the fault, for in such cases the fall of potential is practically represented by a straight line, and the formulæ are constructed on this assumption.

When, however, the cable is very long and the total insulation resistance consequently comparatively low, then the potential cannot be regarded as falling regularly from end to end, but must be graphically represented by a curve, and the potential at the fault is less than that indicated in the straight line diagram, and the potential at the extreme end is lower than this still. The exact formulæ for these tests are considered in Chapter XXII.

502. From the nature of the test it must be evident that there are no particular conditions which enable a maximum degree of accuracy to be obtained, except in so far that the battery power employed should be sufficient to enable high deflections to be produced.

* See § 495, page 432.

† See § 320, page 300.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{200\,\delta}{V - v_1},$$

where δ is the fraction of a division to which each of the deflections can be read.

SIEMENS' EQUILIBRIUM METHOD.*

503. If two batteries have their opposite poles connected to the ends of a perfect cable, their other poles being to earth, then the fall of potential along the cable is continuous and cuts the latter at a certain point. The position of this point can be varied by altering the relative electromotive forces of the batteries, or by adding in resistances between the batteries and the ends of the cable. In the case of a faulty cable, if the fault is at this point, then no current passes from the fault to earth, consequently any alteration in the resistance of the fault does not affect the values of the potentials at the different points along the line of fall.

By observing what arrangement of resistances and electromotive forces is necessary to bring the zero point to the fault, the position of the latter can be accurately determined.

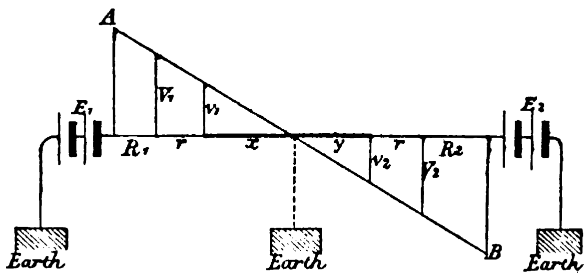


FIG. 184.

In Fig. 184 let x and y be the portions of the cable on either side of the fault, and let r, r be equal resistances connected to either end of the cable, also let R_1 and R_2 be resistances whose values can be varied at pleasure.

Now, in making the test we have to adjust R_1 and R_2 so that the potential at the fault shall be at zero, and consequently that

* 'Journal of the Society of Telegraph-Engineers,' vol. v. page 61. This method, it should be remarked, is substantially the same as that devised by M. Emile Lacoine, and described in vol. iv. page 97, of the same journal.

A B shall be a straight line. To obtain this result we must have

$$v_1 : v_2 :: x : y,$$

or

$$\frac{v_1}{x} = \frac{v_2}{y};$$

and also

$$V_1 : v_1 :: r + x : x,$$

or

$$V_1 - v_1 = \frac{v_1 r}{x};$$

and again

$$V_2 : v_2 :: r + y : y,$$

or

$$V_2 - v_2 = \frac{v_2 r}{y} = \frac{v_1 r}{x};$$

from which we get

$$V_1 - v_1 = V_2 - v_2;$$

that is to say—in order that A B may be a straight line the differences of the potentials on either side of r, at both ends of the cable, must be the same.

504. To obtain this result in practice only one of the resistances R_1 and R_2 need be adjusted. The best way of making the test would then be as follows:—

The two stations should first adjust their galvanometers by means of the movable magnets so that they both give precisely the same deflections when a current from a standard cell through a standard resistance is sent through them. This being done, batteries E_1 and E_2 are connected by the two stations on to the ends of the cable, and then the adjusted galvanometers are severally connected on each side of the respective resistances r and r at the two stations, there being in the circuit of each galvanometer very high, but equal, resistances. Station A, say, now adjusts R_1 and watches the effect on his galvanometer; B also watches the effect on his own galvanometer, and from time to time signals to A the deflection he obtains: this signalling is easily done by having the front contact of a well-insulated key connected to the end of the cable, and the back contact connected to earth, whilst the lever of the key is connected to one terminal of a small condenser whose second terminal is to earth. By pressing down this key a small quantity of the charge in the cable will rush into the condenser, and a momentary movement of the galvanometer needle at station

A will be produced; by arranging then that so many movements shall represent a particlar deflection, B can easily communicate his results to A.

When exact adjustment is obtained, that is to say, when $(V_1 - v_1)$ and $(V_2 - v_2)$ are equal, the galvanometers are disconnected from either side of r and r, and the potential v_1 is measured; x is then obtained from the formula

$$x = r \frac{v_1}{v'}$$

where v' equals $(V_1 - v_1)$, as in the "Fall of Potential Method" of measuring a resistance, page 420.

505. To make the foregoing test as accurately as possible it is advisable, for the reason explained in § 477, page 421 (after the value of x has been obtained by a rough test), to adjust r and r so that they shall each be approximately equal to x.

With regard to the "Possible degree of accuracy attainable," we are liable to make an error in obtaining the value of $V_1 - v_1$, but inasmuch as $V_1 - v_1$ may itself contain an error due to $V_2 - v_2$ being incorrectly measured, the actual total error which may exist in x must be twice that given by formula [C] (page 423); consequently we have

Best Conditions for making the Test.

Make r, r, each approximately equal to x.

Possible Degree of Accuracy attainable.

$$\text{Percentage of accuracy} = \frac{\delta\,(v_1 + v')\,200}{v_1\,v'},$$

where δ is the fraction of a division to which each of the deflections can be read.

Fall of Potential Test for an Earth Fault when the Conductor is not broken. Rymer Jones' Null Method between Ship and Shore.

506. Condenser discharges through a Thomson marine galvanometer (page 66) cannot be accurately compared on board a ship when the latter is rolling or pitching; the deflections are also too abrupt to be easily read. For measuring discharges, a well-balanced Sullivan galvanometer (page 70) is a great improvement

on the old ironclad marine instrument, yet large throws are very rapid and difficult to note with accuracy, consequently three or four discharges must be noted to make sure of the exact reading, even when this remains constant. It is preferable, therefore, in such a case to employ a null method in order to measure potentials.

Shore Measurements (Fig. 185).—At some full minute, prearranged between the ship and shore, the latter, we will suppose, applies the *negative* pole of his battery to the cable through a

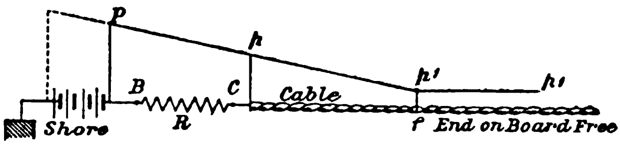


FIG. 185.

resistance R, which should approximate to the conductor resistance of the cable up to the fault *plus* the resistance of the fault, as found by a preliminary test when the distant end on board is insulated.

A condenser is then charged in turn from the cable and battery ends of R, while the cable end on board is *free*, and a series of discharges from both these points is recorded during (say) 5 or 10 minutes. The number of readings required will depend on their regularity, since in the case of a high resistance fault the throws will often vary greatly. Widely different readings—indicative of great momentary changes at the fault—should be discarded. The true determination of the position of an unsteady fault will often entirely depend on the judgment of the operator in selecting the most reliable discharges.

As the potential at B is very much less affected by a change in the fault resistance than the potential at C, the greater number of readings should be recorded from C; and only when these vary considerably will it be necessary to do more than occasionally check the discharge from B.

The mean values for d and D, obtained from the two series of discharge readings at C and B, represent respectively the potentials p and P.

To compare these values with p' measured on board, it is necessary to know the exact values of P and p in volts, or in terms of a standard cell similar to that used on board for determining p', as explained in § 496, page 432.

Accuracy can only be secured by a large number of throws—

hence the advantage of using with an astatic reflecting galvanometer, a magnetised knife blade, or better still, a very small electro-magnet placed near to the galvanometer, and energised by a single cell and a suitable reversing key close to the operator's hand, to bring the spot of light quickly back to zero after each discharge (page 89).

For this reason a dead-beat Sullivan galvanometer—as constructed for shore use—is well suited for this part of the test; but when using a galvanometer on the d'Arsonval principle (or indeed any form of astatic galvanometer) for measuring throws, a *universal shunt* (page 96) should be employed to insure discharges differing in strength being strictly comparable.

Ship Measurements (Fig. 186).—Concurrently with tests being made on shore, the ship is measuring the potential p', caused by

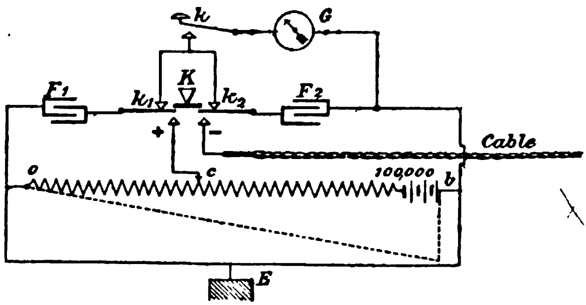


FIG. 186.

the current applied to the cable from the shore station, by a null method based on Thomson's (Lord Kelvin) capacity test (page 369), which differs from it only in the fact that *potentials* are compared instead of *capacities*.

Suppose F_1 and F_2 are condensers of *equal* capacity. Should the fault f have a high resistance, a capacity of one-third, or one microfarad, will suffice for each; but if the fault resistance be not great, a larger capacity (say 5 or 10 microfarads) will be more suitable. Battery b is joined up to a high resistance—a slide resistance of 100,000 ohms if available—and through this the potential falls as shown by the dotted line. The electromotive force of b need be of no particular value provided that at some point along the slide resistance a potential exists equal (but *opposite* in sign) to p' at the end of the cable. This will generally be much below the potential of the battery on shore. We will, however,

suppose that the ship uses about the same number of cells as the shore station so as to allow a wide margin.

Depressing key K charges F_2 *negatively* from the cable, and F_1 *positively* from a point of the slide resistance (to be found by repeated trials) of equal potential to that of the cable. Since these two condensers have equal capacity and are oppositely charged, it follows that if, on allowing key K to rise, and keys k_1 and k_2 to rise with it against their upper contacts, the two charges neutralise each other, the potential at the end of the cable will be equal to that at contact c. In other words, if, after allowing the charges to mix for, say 5 seconds, no discharge passes through the galvanometer when k is depressed, then the potential at the slide contact c will be equal (though *opposite* in sign) to the potential at the end of the cable, and will be given by the index slide in terms of the total electromotive force of the battery power circulating through its coils, which is represented by 100,000. Thus, if a balance be produced with the slide contact at 4000 from the zero end, the potential p' (in volts) at the end of the cable will be

$$\frac{4000}{100000} \times \text{E.M.F. of } b \text{ used on board.} \qquad [1]$$

The two condensers on board need not be of the same capacity. Thus, supposing F_1 to be one-fifth of F_2, a balance may be obtained if F_1 be charged by a potential five times as high as p', at the end of the cable—i. e. c has to be 20,000 instead of 4000. In this case, to obtain the true potential p', the scale readings indicated by the slide contact c must be divided by the known ratio (5) between the two condensers, i.e. $\frac{20,000}{5} = 4000$ (as above).

To make p' comparable with P and p, and all three applicable to Clark's fall of potential formula (page 431), viz.:—

$$\text{Distance of fault} = \text{R}\,\frac{p - p'}{\text{P} - p} \text{ ohms from the shore station,}$$

it is necessary to know the exact electromotive force of battery b on board, either in volts, or in terms of a standard cell similar to the one used on shore.

On board this is found by a modification of Poggendorf's method (page 185), the slide resistance being used as a Clark's potentiometer (page 200). The slide contact c is moved up the scale till, on tapping the key momentarily, no current influences the galvanometer G, thus showing equilibrium between the potential of the standard

cell S, and that on the slide at this point c, at a ohms from the zero end of the coils; then

$$\text{E.M.F. of } b = \frac{100000}{a} \text{ standard cells,}$$

which, inserted in equation [1] (page 443), gives

$$p' = \frac{4000}{100000} \times \frac{100000}{a} \times \text{E.M.F. of the standard cell}$$

$$= \frac{4000}{a} \text{ standard cells.}$$

If the ship and shore standard cells are similar (as might be the case), it is unnecessary to know their electromotive force; but should the two differ, either their respective voltages or else the precise *ratio* which they bear to each other (as shown by a discharge test), must be known, in order to correct the ship's value p' by this ratio and make it comparable with P and p.

CHAPTER XVIII.

TESTS DURING THE LAYING OF A CABLE.

507. The immediate detection of a fault which may occur in a cable during its submersion is a point of great importance. To enable this to be done, a good system of testing is requisite.

Whatever the system be, it should be a continuous one, that is to say, the cable should be continuously and visibly under test, so that the moment a fault occurs it may be detected by the ship and traced.

System for Compound Cables.

508. For laying cables which are not more than 200 miles or so in length, and which have several wires, the method shown by Fig. 187 may be employed.

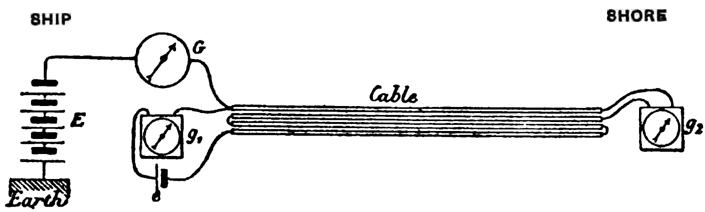


Fig. 187.

In this system the wires are all connected up in one continuous length as shown. Should there be an odd number of wires, the odd one would have to be coupled on to one of the others in "multiple arc."

In Fig. 187, g_1 and g_2 are two ordinary "detector" galvanometers *well* insulated. The battery *e*, of one or two cells (also well insulated), keeps a continuous current circulating through these galvanometers and the conducting wires of the cable; this serves as a "continuity" test, for if any of the wires should break within their insulating sheathing, the circuit becomes interrupted, and consequently the needles of both galvanometers will fall back

to zero. In the case of a cable with an odd number of wires, should the conductor of either of the two which are coupled together become broken, then the needles will only fall back a little way and not back to zero; this, however, will be quite sufficient to indicate that the conductor is fractured.

The galvanometer G is of the marine description, shown on page 67, and is connected to one of the wires. The battery E, of about 200 cells, keeps a continuous current flowing through the galvanometer and through the insulation covering of the wires. If a fault occurs in the insulation, the current by escaping direct to earth causes an immediate and very large increase in the deflection of the needle of G.

In order to keep up communication with the shore, the current from battery e is reversed after certain equal intervals of time. If the shore perceives that the reversal has not taken place, or that the needle of g_2 is not steadily deflected, he knows that something has gone wrong, or that the ship wishes to communicate with him, and he joins up his speaking apparatus and tries to communicate with the ship. The galvanometers g_1 and g_2 could be used for this purpose by having *well* insulated keys inserted in their circuit at the ship and shore, these keys being so arranged that their depression breaks the circuit; the movements of the needles could then be worked according to the ordinary Morse code, and communication be kept up without interrupting the insulation test.

System for Single Wire Cables.

509. The method just described is only applicable to a cable which has more than one wire, for although with the latter the insulation test would be kept up, there would be no means of communicating with the shore. In such cases the following plan may be adopted:—

The end of the cable on board the ship is well insulated, and connected to a battery and Thomson galvanometer as in the previous test and as shown by Fig. 188 (page 447). On shore (Fig. 189) a condenser is provided, one terminal of which is connected to a brass lever which plays between two insulated contacts; one of these contacts is connected to the second terminal of the condenser, which latter terminal is also connected, through a Thomson galvanometer, to earth; the other contact is connected to the conductor of the cable. The battery connected to the cable on board the ship charges the former to a certain potential, and the value of this potential will be the same throughout the whole

length, provided no fault exists. If the lever on shore be moved against the contact connected to the cable, a portion of the charge in the latter will rush into the condenser, and will charge up the set of plates, to which it is connected, to the same potential as the cable; the second set of plates will become charged to the opposite potential by a charge rushing in from earth through the galvanometer; this in-rush will produce a *throw*, or momentary deflection

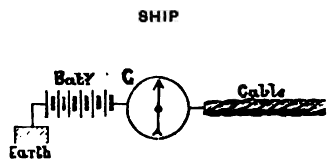


FIG. 188.

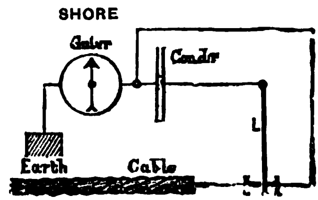


FIG. 189.

of the needle, the amount of which will represent the potential of the charge in the condenser, that is, the potential at the end of the cable. If now the lever be moved from the cable contact to the contact connected to the condenser, the latter will be short-circuited and discharged. The rush of the charge into the condenser when the latter is connected to the cable contact, produces a simultaneous rush into the cable from the battery on the ship, and as this takes place through the galvanometer on board the ship a sudden throw is produced on the needle. Now if a fault occurs during the laying, the *steady* deflection on the ship's galvanometer, which is due to a flow of current through the dielectric of the cable, and which is distinct from the *throw* which takes place when the condenser becomes connected to the cable at the shore end, becomes greatly increased and renders the presence of the fault evident immediately. On the shore the effect of the fault is to reduce the potential at that end of the cable, and consequently the charge which the condenser takes becomes correspondingly reduced; when, then, the condenser becomes charged through the galvanometer, a reduced throw is produced, which thus shows the shore the existence of the fault.

The lever on shore which charges and discharges the condenser is moved by clockwork which causes it to act every five minutes, so that every hour twelve throws are observed on each galvanometer. At the end of every hour the ship reverses the battery so that the direction of the throws is changed.

In order to enable the ship to communicate with the shore,

instructions are given that if at the end of the hour the throws do not become reversed, or if they become reversed before the expiration of the hour, it is a sign that the ship wishes to communicate with the shore; in this case, then, the shore disconnects the cable from the clock lever and connects it with the speaking apparatus, and as the ship does the same, the necessary communications can be carried on. If, on the other hand, the shore wishes to call the attention of the ship, he can do so by moving a lever, corresponding to the clock lever, two or three times quickly by hand; the ship then observing that the throws on her galvanometer take place quickly, instead of at intervals of five minutes, immediately joins up her speaking apparatus, and thus communicates with the shore.

The movement of the lever L in the foregoing system of testing is effected, as has been pointed out, by means of a clock, but L may be a hand-worked key, and this is sometimes preferred, as although a clock ensures the discharges being obtained after regular intervals of time, yet the hand method ensures the necessary watchfulness of the electrician on shore, which is a point of importance.

Willoughby Smith's System.

510. For long single-wire cables a refinement of the foregoing method, devised by the late Mr. Willoughby Smith, has been adopted. This system is shown by Figs. 190 and 191 (page 449).

On shore, the cable is connected to a key K, galvanometer G_2, and condenser C_1 as in the last method of testing. To the cable there is also connected a resistance in circuit with a galvanometer G. This resistance is very much greater than the total insulation resistance of the cable, and consequently it does not appreciably affect the potential measured by the key K, whilst it allows sufficient current to pass through the galvanometer G to produce a sensible deflection of its needle.

The high resistance is made of selenium, and it must be carefully excluded from light, and kept at as uniform a temperature as possible, otherwise it will vary considerably.

On the ship the cable is connected to a slide resistance Wheatstone bridge similar to that shown by Fig. 102, page 234.

The working of the apparatus is then as follows:—

On the ship, plugs are inserted at p_1 and p_2, and balance is kept on the galvanometer G_4 by adjusting the slides of the slide resistances, the resistance R being preserved constant. This gives the insulation of the cable.

Galvanometer G_5 is kept short-circuited under ordinary conditions, it being only used occasionally for the purpose of ascertaining whether the batteries are in good condition.

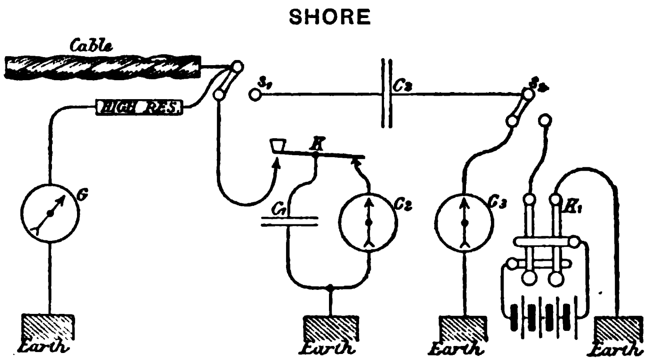


FIG. 190.

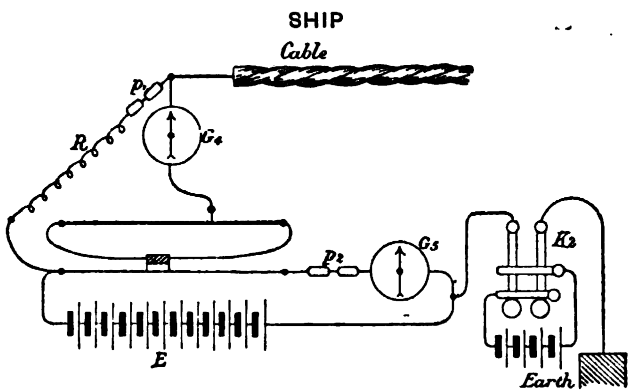


FIG. 191.

Should it be thought advisable, as a check, to take an ordinary deflection insulation test,* this can be done by removing the plugs p_1 and p_2; the current then passes direct from the battery through the galvanometer G_4 into the cable.

On shore the potential at the end of the cable is observed on G_2 by depressing the key K every five minutes. The deflections obtained are noted and recorded.

* Page 408.

The battery E is reversed every fifteen minutes by the ship, and this is observed on the galvanometer G and shows that the conductor of the cable is entire. If the ship requires to communicate with the shore, it reverses the battery several times after short intervals; this is acknowledged by the shore by means of the key K; when this is done, the shore moves over the switch S_1 and receives signals from the ship on galvanometer G_3 through the medium of the condenser C_2. The insulation test is not interrupted by this signalling, as the cable remains insulated the whole time. The effect of working the signalling key K_2 is only to add or subtract a little from the charge in the cable through the medium of the condenser, and thereby to produce momentary deflections on the galvanometer G_3. The same in the case when the shore signals to the ship, the switch S_2 being moved over to key K_1 for that purpose.

Various slight modifications have been, and are, employed in practically using this method, but the general arrangement is that which has been indicated.

See also "Rymer Jones' Null Method," page 440.

CHAPTER XIX.

JOINT-TESTING.

511. Joints are the weak points in a cable, and it is therefore essential that they should be not only carefully made but carefully tested.

A joint, being a very short length of the core, offers, or should offer, a very high resistance; it would consequently be impossible to test it by a direct deflection method, that is, a method similar to that by which the insulation resistance of a cable is taken (page 408). Even with a very powerful battery, the galvanometer deflection, provided the joints were good, would be quite inappreciable. One or other of the following methods must therefore be adopted.

A condenser can be charged through the medium of the joint, and after a noted time the discharge taken, which gives the amount which has leaked through the joint. This is known as *Clark's accumulation method.*

Or a charged condenser may be allowed to discharge itself through the joint, and the amount lost after a certain time noted.

In both these methods the discharge deflections are compared with the results obtained with a few feet of perfect core.

Clark's Accumulation Method.

512. A gutta-percha or ebonite trough is provided, which is suspended by long ebonite rods from any convenient hook.

The good insulation of the trough is a point of great importance, and consequently the suspending rods should be quite dry and clean. The most effectual way of obtaining this result is to well scour the surface of the ebonite with a glass or emery paper; this is a much better method than covering the surface with hot paraffin wax as is sometimes done.

513. We may here remark that *surface* leakage is almost the only medium of loss to be feared in electrical apparatus, and this should always be seen to by keeping all surfaces over which

leakage is likely to occur, in proper condition. The peculiar formation of ebonite causes minute quantities of sulphuric acid to form on its surface, so that the latter should be often rubbed over with a dry cloth. Hot paraffin wax painted over the dry surfaces is very advantageous, but, where appearance is immaterial, nothing is so effectual as a surface well scoured with glass or emery paper.

514. The trough is filled with water, and the joint to be tested is immersed and held down in it by two hooks placed at the bottom.

The portion of the core on either side of the joint should be carefully dried (not paraffined), for the same reason that the suspending rods were so treated.

The connections for the test, shown by Fig. 192, are very

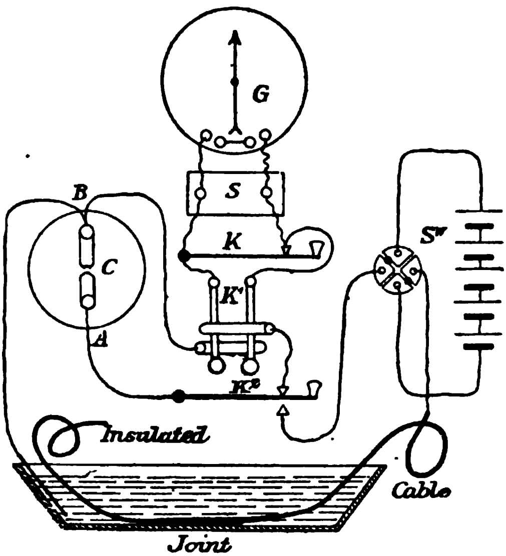


FIG. 192.

similar to those shown by Fig. 142, page 313; the only difference being that the pole of the battery, which in that figure was connected directly to the condenser, is, in the joint test, connected to it through the medium of the joint. The battery used should be as large as possible; 200 Daniell cells is the power very commonly employed.

515. After the joint is placed in the trough for testing, it is necessary to see that the latter is sufficiently well insulated. To do this the pole of the battery, which for the regular test would be connected to the core, must be connected to the wire attached to the plate in the trough, and the discharge key pressed down; this charges the condenser; the battery being then disconnected from the plate, an interval of time (usually one minute) equal to that which would be occupied by the test of the joint, is allowed to elapse, and then the "Discharge" trigger is pressed and the discharge noted; this should be equal, or very nearly so, to the instantaneous discharge.

516. The good insulation of the trough being satisfactorily obtained, and the connections being made as shown by Fig. 192 (page 452), the short-circuit plug of the condenser must be inserted in its place, the discharge key pressed down, and then the short-circuit plug removed; the battery then charges the condenser through the joint.

After a certain time, usually one minute, the discharge deflection must be noted. A similar measurement must also be made, using a length of perfect core in the place of the joint. If, in the latter case, the discharge deflection after the same interval of time is much less than that obtained from the joint, the latter is defective and must be remade.

517. It is a very important point in making the test to insert the short-circuit plug in the condenser previous to depressing the discharge key; if this is not done, an induced charge is thrown into the condenser by the sudden rush of the battery current into the core when the discharge key is depressed. This induced charge will give a considerable deflection when the condenser is discharged, which deflection is in no way due to *leakage* through the joint, though it might be mistaken for such. By keeping the condenser short-circuited this induced charge is dissipated.

518. If the joint is good, the discharge deflection seldom exceeds two or three divisions. Indeed, the fact that it does not do so is usually a quite sufficient proof of the soundness of the joint, and it is not often the case that a comparison with a piece of perfect core is necessary.

Discharge Method.

519. This is a reversal of the foregoing, and consists in charging the condenser full and letting it discharge itself through the joint.

The connections for making this test would be similar to those employed in measuring high resistances by the loss of potential method given in Chapter XVI., page 423 (§ 481), the one end of the core taking the place of one end of the resistance, and the plate in the trough the place of the other end.

520. When a joint is made in a cable core at sea, neither end can be got at. The joint, however, could be tested by making the connections as for the discharge method of testing, only instead of joining the core to the condenser terminal, the latter, and also the cable end, would be put to earth. To carry out the test in this manner, arrangements would have to be made with the shore, previous to the manufacture of the joint, that at a certain time the end of the cable shall be put to earth.

As a matter of fact, joints made at sea are never tested, though there seems no reason why they should not be so.

521. We may if we please, in both the foregoing tests, place the galvanometer between the back terminal of the key and the condenser, and join the two terminals from which it was removed, by a piece of wire. We should then get a charge as well as a discharge deflection, and there is this advantage, that if the joint is very bad or the trough not well insulated, we should get a permanent deflection after the charge deflection has taken place.

522. The connections should always be so made that the zinc pole of the battery is connected to the core and the copper pole to the plate.

523. It is very advisable to employ a special condenser for making these tests, for if one is used which has been charged at any time with a high battery power, it will often be found that a portion of this charge will have become absorbed, and when the condenser is left to itself, this portion will become free and give a discharge which may be mistaken for an accumulation through the joint.

Electrometer Method.

524. Although the preceding methods of testing are often the only ones which can be adopted, yet when possible it is best to make joint tests by means of an electrometer, as the results are always more trustworthy than those obtained by the condenser method, since they are free from the source of error mentioned at the end of the last paragraph.

Fig. 193 shows the connections for making this test, which is executed in the following manner:—

After the insertion of the joint in the trough, the insulation of the latter must be tested; this is done by pressing down key K_1 and moving the switch S over to its *well* insulated contact

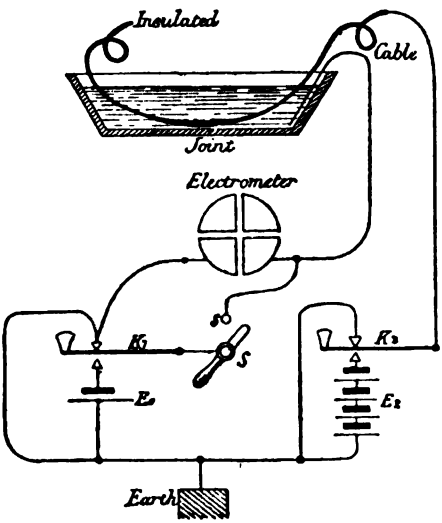


Fig. 193.

stop *s*; this puts the ten-cell battery E_1 in connection with the quadrants of the electrometer, and thereby charges them and causes a steady deflection of the needle. Key K_1 being kept depressed, switch S is now opened and the deflection watched for two minutes to see whether there is any sensible fall due to the charge on the quadrants leaking to earth through the medium of the trough; if this loss is only equal to two or three divisions, the insulation of the trough may be considered to be good.

Key K_1 is now released and switch S closed so as to discharge the electrometer. Switch S is now again opened and key K_2 depressed; this puts the 200-cell battery E_2 in connection with the core of the cable, and the momentary rush of current into the latter causes an induced charge to rush out of the trough and produce a sudden deflection of the electrometer needle; it is usual to record this deflection, although it is of no value, except to show

that the various connections have been properly made, and that the joint has been placed in the trough.

Key K_2 being kept depressed, switch S is now moved over to *s* (so as to discharge the electrometer), and then again opened. The scale of the electrometer is then watched, as the current leaking through the joint into the trough accumulates and causes a gradually increasing deflection of the needle; the amount of this deflection should be noted at the end of one and two minutes after the opening of the switch.

After the observations with the joint have been made, a piece of perfect core must be inserted in the trough and a similar test executed, the results of which should not differ much from those obtained with the joint. It always happens that a joint gives a greater accumulation than an equal length of perfect core, unless indeed the joint has been made several days before being tested, which is seldom, if ever, the case.

CHAPTER XX.

SPECIFIC MEASUREMENTS.

525. In order to compare the relative "Conductivity," "Insulation," and "Inductive Capacity" of the materials used in the construction of the cores of submarine and other cables, it is necessary that each of them should be expressed in terms of some unit with which the comparison can be made.

The standard conductivity, resistance, and inductive capacity units are respectively the conductivity, resistance, and inductive capacity between the opposite faces of a centimetre cube of an imaginary material having a conductivity, resistance and inductive capacity represented by 1; and the conductivity, resistance, or inductive capacity of any material expressed in terms of a centimetre cube of the latter, are respectively the *specific conductivity, specific resistance, and specific inductive capacity* of the same.

Relative Conductivity.

526. For practical purposes, "specific conductivity" expressed in terms of a centimetre cube is not a convenient unit for comparison, the generally adopted standard is that of pure (according to Dr. Mathiessen) * copper taken as 100.†

Now, according to Dr. Matthiessen, 1 metre of pure soft (that is *annealed*) copper wire weighing 1 gramme, has a resistance of

* Improvements which have been effected in the manufacture of copper, have enabled better results to be obtained than that given by Matthiessen's standard. Wire giving 101 per cent. conductivity is frequently met with, and 102 per cent. is not uncommon. It may be mentioned that the specific gravity of the copper with which Mathiessen's experiments were made appears to have been such that 1 cubic foot weighed 555 lbs.

† The actual resistance of a centimetre cube of pure soft copper (Matthiessen's), at 60° F. is ·0000016964, consequently the specific conductivity at that temperature is $\frac{1}{\cdot 0000016964} = 589490$. But since the standard for pure copper is taken at 100, if we wish to obtain the actual specific conductivity of any wire we must multiply its percentage of conductivity by 589490 and divide by 100, i.e. we must multiply the percentage by 5895.

·1440 B.A. (British Association) ohm at a temperature of 0° Cent. (32° F.); but it appears that Dr. Matthiessen actually made his exact determination of the resistance of copper, at a temperature of about 60° F., and that the value ·1440 was obtained by a correction coefficient obtained from the formula

$$\left.\begin{array}{r}\text{Resistance at}\\ 32^\circ\text{ F.}\end{array}\right\} = \frac{\text{Resistance at temperature } t^\circ \text{ F.}}{1 - \cdot 00215006\,(t^\circ - 32^\circ) + \cdot 00000278\,(t^\circ - 32^\circ)^2}.$$

In order to put the question of the standard of pure copper upon a definite (if not strictly correct) basis, it has been decided (May 1899) by a committee of manufacturers and experts, that ·1440 corrected up to 60° F. by the above formula shall in future be taken as the standard, and that this value, viz.:

$$\cdot 1440 \times \cdot 9866\,^* \times 1 \cdot 061596 = \cdot 150821 \text{ standard ohms,}$$

shall, for accurate determinations, be corrected for temperature, by "Clark, Forde & Taylor's" coefficients (page 467), whilst for commercial purposes, the formula

$$R = r\,(1 + \cdot 00238\, t^\circ),$$

shall be used, the constant ·00238 being a mean value of Clark, Forde & Taylor's coefficients.

Now, since 1 metre-gramme of pure soft copper wire has at 60° F. a resistance of ·150821 S. ohms (·152870 B.A. ohms), therefore l metres of a similar quality of wire will at the same temperature have a resistance l times as great. But l metres of the wire will weigh, not 1 but l grammes, consequently l metres weighing 1 gramme must have a resistance of $l \times l$, or l^2, $\times$ ·150821 S. ohms; and, further, if the l metres weighed w grammes, then the resistance, R, would be

$$R = \frac{l^2 \times \cdot 150821}{w} \text{ S. ohms at } 60^\circ \text{ F.}$$

If l is expressed in feet and w in grains, then

$$R = \frac{l^2 \times \cdot 216226}{w} \text{ S. ohms at } 60^\circ \text{ F.} \qquad [A]$$

This formula enables the conductivity of any copper wire to be determined; for having measured off a definite length of the latter, and ascertained its weight, temperature, and resistance, then the

* 1 standard (S.) ohm = ·9866 B.A. ohm.

latter, compared with the resistance of a pure copper wire of the same length, temperature, and weight, gives us by direct proportion, what we require.

For example.

Suppose the length of our sample wire were 20 feet, its weight 500 grains, its resistance ·1740 S. ohm, and its temperature 50°; then by the foregoing formula the resistance at 60° of a pure copper wire of a similar length and weight will be

$$R = \frac{20 \times 20 \times \cdot 216226}{500} = \cdot 17298 \text{ S. ohm.}$$

But by the table, page 467, the relative resistances of a pure copper wire at 50° and at 60° are as 1·04279 to 1·06665, therefore the resistance of the pure copper wire at 50° will be

$$\cdot 17298 \times \frac{1 \cdot 04279}{1 \cdot 06665} = \cdot 16911 \text{ S. ohm.}$$

To get then the conductivity (x) of the wire sample, we have the inverse proportion

$$\cdot 1740 : \cdot 16911 :: 100 : x$$

or

$$x = \frac{\cdot 16911 \times 100}{\cdot 1740} = 97 \cdot 2;$$

that is to say, the conductivity of the wire sample is 97·2 per cent. of that of pure copper.

527. In the case of a cable where the weight per knot of the conductor is always known, the calculations are much simpler, as they can be made by reference to Table II., which gives the resistances corresponding to various percentages of conductivity of a conductor 1 knot long weighing 1 lb., and at a temperature of 75° F. This table is calculated in the following manner:—

Resistance of 1 metre-gramme at 60° F. = ·150821 S. ohm $= \text{(by table, page 467)} \cdot 150821 \times \dfrac{1 \cdot 10258}{1 \cdot 06665} = \cdot 155901$ S. ohm at 75° F.

Resistance of 1 knot, that is 2029 yards, or $\dfrac{2029}{1 \cdot 093633}$ metres, weighing $\dfrac{2029}{1 \cdot 093633}$ grammes $= \cdot 155901 \times \dfrac{2029}{1 \cdot 093633}$ S. ohms at 75° F.

Resistance of 1 knot weighing 1 lb. (453·59 grammes)

$$= \cdot 155901 \times \frac{2029}{1 \cdot 093633} \times \frac{2029}{1 \cdot 093633} \times \frac{1}{453 \cdot 59} = 1183 \cdot 06 \text{ S. ohms.}$$

This latter value then is the resistance of a knot-pound of copper of 100 per cent. conductivity, and is entered in the table as such, the other values being obtained by direct proportion.

528. The way in which the table would be used is as follows:—

Supposing we had a cable whose conductor weighed 107 lbs. per knot (this is a very usual weight for the conductor of a cable), and whose resistance per knot at 75° F. was found by experiment to be 11·202 S. ohms, then by multiplying 11·202 by 107 we get the resistance of a knot-pound of copper of a corresponding conductivity. $11 \cdot 202 \times 107 = 1198 \cdot 61$ and this resistance in the table corresponds to a conductivity of 98·7, which is therefore the percentage of conductivity of the conductor of the cable.

529. It is sometimes required to determine the conductivity of a wire whose length and diameter (d) are known; for this purpose the weight of 1 cubic foot of soft copper may be taken to be 555 lbs. We have then,

$$\left.\begin{array}{r}\text{Weight of wire 1 foot long} \\ \text{and 1 mil in diameter}\end{array}\right\} = 1 \times \left(\frac{1}{1000 \times 12}\right)^2 \times \cdot 785398^*$$

$$\times\ 555 \times 7000 \text{ grains} = \cdot 0211894 \text{ grains.}$$

$$\left.\begin{array}{r}\text{Resistance of wire 1 foot long} \\ \text{weighing } \cdot 0211894 \text{ grains}\end{array}\right\} = \frac{\cdot 216226}{\cdot 0211894}\dagger$$

$$= 10 \cdot 2045 \text{ S. ohms, at } 60^\circ \text{ F.}$$

Since the resistance of a wire varies inversely as its sectional area, that is, inversely as the square of its diameter (d), we must have:—

$$\left.\begin{array}{r}\text{Resistance of } l \text{ feet of pure} \\ \text{copper wire } d \text{ mils in diameter}\end{array}\right\} = \frac{l \times 10 \cdot 2045}{d^2} \text{ S. ohms, at } 60^\circ \text{ F.}$$

If the temperature of the wire is not 60° F., then a correction, by means of coefficients, has to be made.

For example.

The resistance of 50 feet (l) of copper wire, 14 mils (d) in diameter, is found to be 2·7460 S. ohms at a temperature of 65° F.; what is the conductivity of the wire?

* $\frac{\pi}{4} = \cdot 785398$.

† See equation [A], page 458.

$$\left.\begin{array}{l}\text{Resistance of 50 feet of pure}\\ \text{copper wire 14 mils in diameter}\end{array}\right\} = \frac{50 \times 10\cdot2045}{14 \times 14} \text{ S. ohms, at } 60^\circ \text{ F.}$$

Applying the coefficients given in the table, page 467, we get,

$$\text{Resistance at } 65^\circ \text{ F.} = \frac{50 \times 10\cdot2045}{14 \times 14} \times \frac{1\cdot07861}{1\cdot06665} = 2\cdot6324 \text{ S. ohms;}$$

then by inverse proportion we have

$$2\cdot7460 : 2\cdot6324 :: 100 : x,$$

or

$$x = \frac{2\cdot6324 \times 100}{2\cdot7460} = 95\cdot86\,;$$

that is to say, the conductivity of the wire sample is 95·86 per cent. of that of pure soft copper.

530. The diameter of the wire in mils may readily be measured by means of the gauge shown by Fig. 194. The complete turn of the finger screw represents $\frac{1}{50}$th of an inch, and as the screw has a scale on it divided into 20 parts, each part being $\frac{1}{20}$th of a complete revolution of the screw, therefore each part or division represents $\frac{1}{50 + 20} = \frac{1}{1000}$th of an inch, i.e. 1 mil.

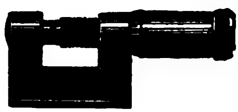


FIG. 194.

In order to prevent undue force or pressure being exerted by the screw when the latter is screwed up so as to grip the wire being measured, these gauges are sometimes fitted (as suggested by Mr. Herbert Taylor) with a cap which runs spring-tight over the part of the screw which is manipulated by the fingers, so that as soon as the screw is turned sufficiently to fairly grip the wire, this cap (which is being held by the fingers) slips round, and a limit is thus placed to the pressure that can be exerted.

531. In the case of small wires where it is difficult to measure the diameter with great accuracy, it is preferable to test for specific conductivity by weight rather than by gauge, for by taking a sufficient length of wire we can determine the value of the weight as accurately as we please.*

532. Table III. shows the resistances, &c., of various gauges of pure copper wire at 60° F.

* Diameter in mils = length in feet $\times \sqrt{\begin{array}{c}\text{weight}\\ \text{in grains}\end{array}} \times \cdot68697$.

Specific Insulation.

533. By ***Specific Insulation*** is meant the specific resistance (§ 525, page 457) of any insulating material. In this case, as we have no pure standard material with which to make a comparison, we cannot, as in the case of wire, get a piece of a certain length and compare it by measurement with another piece whose value has been fixed experimentally, and which we can call 100; the resistance therefore of a centimetre (or multiple of a centimetre) cube of an imaginary material whose value is taken as 1 megohm is adopted as a standard; the resistance of any material expressed in terms of such a cube, i.e. the specific resistance of the material, gives the relative value required.

Now, the form in which gutta-percha or other insulating material is used for submarine or other cables is that of a cylinder, in which the conducting wire is concentrically placed; and we have to determine from the observed resistance of a known length of the cable, and from the dimensions of the external and internal diameters of the cylinder, what the resistance of a centimetre cube of the insulating material would be. As this is an interesting problem, we will give it at length.

Looking at a transverse section, let us suppose the sheathing to be divided into a number of concentric circles, such that the resistance of the piece between any two circles equals ρ. For this to be the case, it is evident that the circles nearer the circumference must be of a greater thickness than those near the centre, since their circumferences are greater.

Let there be n of these circles, so that $n\ \rho = \mathrm{R}$ (ρ here corresponds to the little interval of time t in the loss of charge problem, page 361, § 397).

Now, if $\left.\begin{matrix} d_d \\ d_{d+1} \end{matrix}\right\}$ be the internal and external radii or diameters of any one cylinder, and if the difference $d_{d+1} - d_d$ is very small, the resistance of the cylinder will be

$$\frac{(d_{d+1} - d_d)\,\sigma}{2\,\pi\,l\,r_d},$$

where l is the length of the cable, and σ the specific resistance of the insulating material.

Now, the smaller we make $d_{d+1} - d_d$, the nearer will this be true. But in order to do this, we must make ρ small and n large.

Now

$$\frac{(d_{d+1} - d_d)\,\sigma}{2\,\pi\,l\,r_d} = \rho,$$

since ρ equals the resistance of each cylinder; therefore

$$d_{d+1} = d_d\left(1 + \frac{2\,\pi\,l\,\rho}{\sigma}\right);$$

then, as in the problem we have referred to,

$$d_n = d_d\left(1 + \frac{2\,\pi\,l\,\rho}{\sigma}\right)^n,$$

where d_n, or D, is the external, and d_d, or d, the internal radius or diameter of the sheathing; that is,

$$D = d\left(1 + \frac{2\,\pi\,l\,\rho}{\sigma}\right)^n = d\left(1 + \frac{2\,\pi\,l\,R}{\sigma\,n}\right)^n,$$

and the larger n is, the nearer is this true; therefore make $\rho = 0$, and $n = \infty$, so that $n\,\rho$ still equals R; we then get a perfectly accurate result. Let

$$\frac{2\,\pi\,l\,R}{\sigma\,n} = \frac{1}{x},$$

so that $x = \infty$ when $n = \infty$. Then

$$D = d\left[\left(1 + \frac{1}{x}\right)^x\right]^{\frac{2\,\pi\,l\,R}{\sigma}}$$

when $x = \infty$, but when this is the case, the expression within the square brackets is known to be equal to e,* thus

$$\frac{D}{d} = e^{\frac{2\,\pi\,l\,R}{\sigma}};$$

therefore

$$R = \frac{\sigma \log_e \frac{D}{d}}{2\,\pi\,l}, \quad \text{or,} \quad \sigma = \frac{R\,l\,2\,\pi}{\log_e \frac{D}{d}}, \qquad [A]$$

* Todhunter's Algebra, Chapter XXXIX.

or putting the numerical values of the constants, and substituting common for natural logs,

$$\sigma = \frac{\mathrm{R}\,l\,2\cdot728}{\log\frac{\mathrm{D}}{d}}.$$

Since D and d are in the form of a proportion, it is immaterial in what units they are expressed, but l must be in centimetres and R in megohms, R being the total resistance of the length l.

534. In practice the numerical value worked out from the foregoing formula is, for all insulating materials, very excessive, hence it is preferred to make the practical dimension of σ to be $\frac{1}{1,000,000,000}$th of its actual dimension, so that the formula for practical use is

$$\sigma = \frac{\mathrm{R}\,l\,2\cdot728}{\log\frac{\mathrm{D}}{d}\times 1,000,000,000}.$$

For example.

The core of a cable 1 statute mile long had an insulation of 5000 megohms. Its external diameter was ·5 inch, and internal diameter ·314 inch. What was its specific insulation resistance?

$$\sigma = \frac{5000\times 160930\times 2\cdot728}{\log\frac{\cdot5}{\cdot314}\times 1,000,000,000} = 10\cdot865.$$

If we arrange that R is the resistance of a *statute mile* of the wire, then since

$$\frac{160930\times 2\cdot728}{1,000,000,000} = \cdot000439,$$

therefore the formula becomes

$$\sigma = \frac{\mathrm{R}\times\cdot000439}{\log\frac{\mathrm{D}}{d}}. \qquad [\mathrm{B}]$$

If R is the resistance per *knot*, then we have

$$\sigma = \frac{\mathrm{R}\times\cdot000506}{\log\frac{\mathrm{D}}{d}}. \qquad [\mathrm{C}]$$

535. Referring to equation [A], page 463, since*

$$\log.\frac{\mathrm{D}}{d} = 2\left\{\frac{\mathrm{D}-d}{\mathrm{D}+d} + \frac{1}{3}\left(\frac{\mathrm{D}-d}{\mathrm{D}+d}\right)^3 + \frac{1}{5}\left(\frac{\mathrm{D}-d}{\mathrm{D}+d}\right)^5 + \ldots;\right.$$

* Todhunter's Algebra, Chapter XXXIX.

then if D and d do not differ largely, we may neglect all the terms after the first without considerable error; so that equation [A] becomes

$$\sigma = \frac{R\, l\, \pi}{\frac{D - d}{D + d}};$$

or if we substitute thickness, t, of material, we get

$$\sigma = R\, l\, \pi \frac{d + t}{t}.$$

Further, if we substitute the value of π, &c., then equation [B] becomes simplified to

$$\sigma = R \times \cdot 0005056 \frac{d + t}{d},$$

which, since the simplified formula actually makes σ rather too large, may be written

$$\sigma = R \times \cdot 0005 \frac{d + t}{t} = R \frac{d + t}{t} \div 2000.$$

Similarly, formula [C] becomes

$$\sigma = R \times \cdot 0005829 \frac{d + t}{d},$$

which for practical purposes may be written

$$\sigma = R \frac{d + t}{t} \div 1700.$$

536. It should be understood that these simplified formulæ are only approximate, but they can be relied upon to within about 1 per cent. if the thickness of the insulation is not more than about $\frac{1}{8}$th of the diameter of the conductor; examples like this are continually met with in the case of electric light leads, though not in the case of submarine cables, for which the simplified formulæ can be only regarded as roughly approximate, and convenient only for check purposes.

Specific Inductive Capacity.

537. From what has been said on page 423, § 481, it will be evident that the formula for giving the specific inductive capacity (k) of a cable-core will be

$$k = F \frac{\log \frac{R}{r}}{\cdot 000506},$$

where F is the capacity per knot of the core in microfarads.

CHAPTER XXI.

CORRECTIONS FOR TEMPERATURE.

538. In order to make tests for Conductivity, Insulation Resistance, or Electrostatic Capacity, strictly comparative, it is either necessary that they be made at the same temperature, or, when this cannot be done, the temperatures at which they are taken should be noted, and a correction made.

Corrections for Conductor Resistance.

539. For a considerable period it was regarded as proved, that when the temperatures were not very widely different, then for every degree of increase in temperature, an equal percentage of increase in resistance took place; that is to say, if the resistance increased at a certain rate per cent. by a rise of one degree of temperature, it would be increased by the next degree of rise, at the same rate per cent. calculated on the new resistance.

If this were so, it will be evident, on consideration, that the percentage of increase for a certain number of degrees will be the same at whatever part of the scale these degrees are taken. Thus, if a resistance increased 25 per cent. between 30° and 40°, it would increase 25 per cent. between 65° and 75°.

Results obtained from time to time, however, appeared to indicate that it was doubtful if this "compound interest" law could be regarded as being correct, and recent and very careful experiments made by Messrs. Clark, Forde & Taylor have practically confirmed the supposition that the law is not a correct one. Messrs. Clark, Forde & Taylor have found that as regards pure copper, the formula

$$R_t = R_{32}\,(1 + \cdot 0023708\,(t - 32°) + \cdot 00000034548\,(t - 32°)),$$

in which R_t is the resistance at a temperature t, and R_{32} is the resistance at freezing point, is accurate within $\frac{3}{10}$ths of a degree

Fahrenheit for all temperatures between 85° and 32° F. The following table is calculated from this formula, R_{32} being taken as unity.

Degrees. t	Resistance. R_t	Degrees. t	Resistance. R_t	Degrees. t	Resistance. R_t
85	1·12662	67	1·08340	49	1·04040
84	1·12422	66	1·08101	48	1·03802
83	1·12181	65	1·07861	47	1·03564
82	1·11940	64	1·07622	46	1·03326
81	1·11700	63	1·07383	45	1·03088
80	1·11459	62	1·07143	44	1·02850
79	1·11219	61	1·06904	43	1·02612
78	1·10979	60	1·06665	42	1·02374
77	1·10739	59	1·06426	41	1·02137
76	1·10498	58	1·06187	40	1·01899
75	1·10258	57	1·05949	39	1·01661
74	1·10018	56	1·05710	38	1·01424
73	1·09778	55	1·05471	37	1·01186
72	1·09538	54	1·05232	36	1·00949
71	1·09299	53	1·04994	35	1·00712
70	1·09059	52	1·04755	34	1·00474
69	1·08819	51	1·04517	33	1·00237
68	1·08580	50	1·04279	32	1·00000

540. As the principal use of the coefficients is to enable a resistance at any temperature to be corrected to 75°, for this purpose a modification of the foregoing table is necessary, as follows:—

This modification consists in making the coefficient for 75° equal to 1, and reducing all the other coefficients in the ratio of 1·10258 (the coefficient for 75° in the table) to 1; thus the coefficient for 60°, for example, becomes

$$\frac{1{\cdot}06665}{1{\cdot}10258} = {\cdot}96741,$$

that is to say ·96741 is the coefficient for correcting to 75°, a resistance which is at a temperature of 60°; and we should have to *divide* the resistance at 60°, by ·96741, in order to so correct it.

541. In most cases it is preferable to have *multiplying* coefficients for the purpose, and this we obtain by using the reciprocals

of the coefficients obtained in the foregoing manner; thus the 60° coefficient becomes

$$\frac{1}{\cdot 96741} = 1\cdot 0337.$$

Table IV., given at the end of the book, is compiled in this manner.

542. This table can of course be used for the purpose of correcting from any one temperature to any other temperature other than 75°, thus if we required to correct from, say, 40° to 60°, then we *multiply* the resistance at 40° by the 40° coefficient, viz. 1·0821, and divide the result by the 60° coefficient, viz. 1·0337.

Influence of Conducting Power upon Variation of Resistance by Change of Temperature.

543. According to Matthiessen,* the influence of temperature upon the resistance of metals varies according to the conducting power of the metal, the law being, that "the percentage of decrement in the conducting power of an impure metal, between 0° C. and 100° C., is to that of the pure one, between 0° C. and 100° C., as the conducting power of the impure metal at 100° C. is to that of the pure one at 100° C." A numerical example will best explain this law:—

Supposing we have two wires of the same metal, one of which is pure and the other impure, and we take such a length of each that they both have a resistance of 300 ohms at 0° C.; and suppose that the relative specific conductivities of the two kinds of metal are as 100 to 90. Now if we found that the pure sample increased its resistance from 300 to 420 ohms, or 40 per cent., when the temperature was increased to 100° C., then we should find that the impure sample when raised to 100° C. would have increased its resistance to 408 ohms, or 36 per cent., for

$$100 : 90 :: 40 : 36.$$

From this it appears that the correction coefficients require to be varied according to the purity of the metal, but if we know what the coefficients are for the pure metal, and also the specific conductivity of the metal, we can correct the coefficients accordingly. Let R be the resistance of both the pure and impure metals at a temperature t, and R_1 the resistance of the *pure* metal at a temperature t_1, and let κ be the coefficient required to correct R to the latter temperature, that is, let

$$R_1 = R\,\kappa. \qquad [1]$$

Let R_2 be the resistance of the impure metal at the temperature t_1, and let κ_1 be the coefficient required to correct R to this temperature, that is, let

$$R_2 = R\,\kappa_1. \qquad [2]$$

Also let C and C_1 be the specific conductivities of the pure and impure metals.

Lastly, let p and p_1 be the percentages of increase in resistance of the two samples respectively, between the temperatures t and t_1.

* Phil. Trans., 1864, p. 167.

We then have the following equations:—

$$p = \frac{R - R_1}{R} 100 \qquad [3]$$

$$p_1 = \frac{R - R_2}{R} 100 \qquad [4]$$

and the proportion

$$p : p_1 :: C : C_1$$

or

$$\frac{p}{p_1} = \frac{C}{C_1};$$

but from equations [3] and [4] we get

$$\frac{p}{p_1} = \frac{\frac{R - R_1}{R} 100}{\frac{R - R_2}{R} 100} = \frac{R - R_1}{R - R_2};$$

therefore

$$\frac{C}{C_1} = \frac{R - R_1}{R - R_2},$$

or, substituting the values of R_1 and R_2, obtained from the equations [1] and [2], we get

$$\frac{C}{C_1} = \frac{R - R\kappa}{R - R\kappa_1} = \frac{1 - \kappa}{1 - \kappa_1};$$

therefore

$$1 - \kappa_1 = \frac{C_1}{C}(1 - \kappa),$$

that is

$$\kappa_1 = 1 - \frac{C_1}{C}(1 - \kappa) = 1 + \frac{C_1}{C}(\kappa - 1).$$

As the specific conductivity of the pure metal is always taken as 100, the formula becomes

$$\kappa_1 = 1 + \frac{C_1}{100}(\kappa - 1). \qquad [A]$$

For example.

From Table IV., the correction coefficient for correcting from 45° to 75° is 1·0695, for pure copper. What is the coefficient for copper whose conductivity is 96 per cent. of that of the pure metal?

$$\kappa_1 = 1 + \frac{96}{100}(1{\cdot}0695 - 1) = 1{\cdot}0665.$$

In practice the foregoing correction is seldom, if ever, used.

Determination of the Temperature of a Wire by Change of Resistance.

544. By a reverse process to the foregoing we can tell what the temperature of a wire is, if we know what is its resistance at one temperature, and also its resistance at the unknown tempera-

ture. For all we have to do is to divide one resistance by the other, and note with what number of degrees of temperature the coefficient so obtained corresponds; then this result shows the number of degrees the wire has above or below the temperature at which the wire was measured.

Thus, for example, if it were known that the conductor resistance of a cable measured at a temperature of 75° F. was 2000 ohms, and at an unknown temperature it was 1960 ohms, then

$$\frac{2000}{1960} = 1 \cdot 0204;$$

and this coefficient from Table IV. corresponds to a temperature of 66° F., which is consequently the temperature of the cable.

Corrections for Insulation Resistance.

545. The law of change of resistance by change of temperature for *Insulators* is the reverse of that for *Conductors*, that is to say, *increase* of temperature *diminishes* their resistance, and *vice versâ*. The "compound interest" law (page 466) although not applicable for conductor resistance corrections, is generally applied in the case of "insulation resistances." This law is similar to the law for the fall of potential in an insulated cable (page 428). We have simply, in fact, to substitute resistances for potentials, and degrees of temperature for intervals of time, in any of the formulæ relating to the fall of potential, and we get our formulæ for change of resistance by change of temperature.

At the end of Chapter XVI. (page 430) we obtained a formula

$$v_2 = \mathrm{V}\left(\frac{v_1}{\mathrm{V}}\right)^{\frac{t_2}{t_1}};$$

if, then, we suppose a resistance to have *de*creased from R to r, by an *in*crease of temperature of $n°$, and to r_1, by an increase of $n_1°$, by substitution in the foregoing formula, we get the equation

$$r_1 = \mathrm{R}\left(\frac{r}{\mathrm{R}}\right)^{\frac{n_1°}{n°}}$$

or

$$\frac{r_1}{\mathrm{R}} = \left(\frac{r}{\mathrm{R}}\right)^{\frac{n_1°}{n°}}, \text{ that is, } \frac{\mathrm{R}}{r_1} = \left(\frac{\mathrm{R}}{r}\right)^{\frac{n_1°}{n°}},$$

as representing the relation between these quantities.

Now $\frac{R}{r_1}$ is the ratio of increase of resistance for $n_1°$ of difference of temperature, hence if we have a resistance R_t at a temperature $t°$, of an insulating material similar to that of R (in the foregoing case), and we wish to know its resistance r_{t_1} when increased in temperature to $t_1°$, then if $n_1° = t° - t_1°$, we have

$$\frac{R_t}{r_{t_1}} = \left(\frac{R}{r}\right)^{\frac{t° - t_1°}{n°}}.$$

Suppose then we had a wire insulated with, say, gutta-percha, the insulation resistance, R, of which at 40° was found to be 22,000 megohms, and whose insulation resistance r at 85° was 700 megohms, then the insulation resistance had decreased its value from 22,000 to 700, by an increase of temperature of 85° − 40° = 45° ($n°$). We therefore know that any other wire insulated with a similar material will decrease its insulation resistance to the same relative amount by a decrease of 45° of temperature, that is, we have

$$\frac{R_t}{r_{t_1}} = \left(\frac{15{,}000}{700}\right)^{\frac{t° - t_1°}{45°}} = (1{\cdot}0796)^{t° - t_1°},$$

that is,

$$r_{t_1} = \frac{R_t}{(1{\cdot}0796)^{t° - t_1°}}.$$

The quantity

$$(1{\cdot}0796)^{t° - t_1°}$$

then, is the coefficient by which R_t must be divided in order to determine its value when its temperature is increased from $t_1°$ to $t°$.

546. If, as is usually the case, the resistances have to be corrected to 75° F., then $t° = 75°$. In Tables V., VI., and VII., coefficients for various differences of temperature are given, calculated in the foregoing manner.

547. The influence of temperature is very much greater on insulators than on conductors; thus, whereas a copper wire would only increase its resistance from 1000 ohms to 1036 ohms by an increase of 15° F. of temperature; a gutta-percha core would increase its resistance from 1000 megohms to about 9000 megohms by the same amount of decrease of temperature. The amount of the change of resistance by change of temperature, which takes place in the case of insulating materials, is dependent

upon the quality of the latter, and, therefore, the correction coefficients for the same can only be regarded as approximately correct.

Effect of Temperature on Inductive Capacity.

548. Mr. W. J. Murphy has proved experimentally that neither pressure nor temperature affect the true capacity of a cable core insulated with gutta-percha. Careful tests were made for several days on 150 knots of cable, prior to and after the laying of the same into a depth of some 1000 to 1100 fathoms of water, which meant changing the temperature of the cable from 83° F. to about 37° F., and increasing the pressure from 1 to nearly 20 atmospheres. The tests were made while the cable was in the ship's tanks and at the higher temperature, and repeated while the cable was being laid out, and again after it had been down for some time, and had therefore settled down to the lower temperature and greatly increased in dielectric resistance in consequence. No change in its true capacity could be detected.

549. In the case of wires insulated with indiarubber, however, there is marked effect at a high temperature; thus the results of experiments made by Mr. W. Maver * showed that a change of temperature from 100° F. to 212° F. increased the capacity nearly 140 per cent. The dimensions of the insulated wire on which the experiments were made, were as follows:—Conductor, 97 mils; thickness of insulation, 126 mils. No data, it is believed, exist as to the effect of change of temperature below 100° F.

Corrections when Two Sections of a Cable are at Different Temperatures.

550. It is very often the case in cable factories that two sections of the cable are in different tanks at different temperatures. As the whole length must be tested in one section, it may be necessary to know what correction must be applied to the measured resistance of the whole length of cable to correct it to the value it would have at one uniform temperature.

Corrections for "Conductor Resistance" when Two Sections of a Cable are at Different Temperatures.

551. Let l_1 and l_2 be the two lengths of the cable in the different tanks, also let r_1 and r_2 be the respective conductor

* 'The Electrical Engineer,' (New York), August 12th, 1891.

resistances of the two sections at the temperatures of the tanks, and let P_c be the resistance of the two together. Also let k_1 and k_2 be the coefficients by which r_1 and r_2 must be *multiplied* respectively in order to reduce them to the values they would have at one uniform temperature, and let R_c be the total resistance of the cable at this uniform temperature; we then have the following equations:

$$\begin{aligned} P_c &= r_1 + r_2, \\ R_c &= r_1 k_1 + r_2 k_2, \\ \frac{l_1}{l_2} &= \frac{r_1 k_1}{r_2 k_2}; \end{aligned} \qquad [1]$$

therefore

$$\frac{R_c}{P_c} = \frac{r_1 k_1 + r_2 k_2}{r_1 + r_2}, \quad \text{or,} \quad R_c = P_c \frac{r_1 k_1 + r_2 k_2}{r_1 + r_2};$$

and also we may say

$$\kappa l_1 = r_1 k_1, \quad \text{or,} \quad r_1 = \frac{\kappa l_1}{k_1},$$

$$\kappa l_2 = r_2 k_2, \quad \text{or,} \quad r_2 = \frac{\kappa l_2}{k_2},$$

where κ is a constant; therefore

$$R_c = P_c \frac{\kappa (l_1 + l_2)}{\kappa \left(\frac{l_1}{k_1} + \frac{l_2}{k_2}\right)} = P_c \frac{l_1 + l_2}{\frac{l_1}{k_1} + \frac{l_2}{k_2}}.$$

If l_1 and l_2 are the lengths of the portions of the cable in knots, then the corrected resistance per knot (r_c) will be

$$r_c = P_c \frac{l_1 + l_2}{\frac{l_1}{k_1} + \frac{l_2}{k_2}} \div (l_1 + l_2) = \frac{P_c}{\frac{l_1}{k_1} + \frac{l_2}{k_2}}. \qquad [2]$$

For example.

At a cable factory there were 15 knots (l_1) of manufactured cable lying in a tank whose temperature was 50° F. Connected to this cable were 5 knots (l_2) of core in a tank whose temperature was 55° F. The total observed conductor resistance of the 20 knots was 215 ohms (P_c). What would be the conductor resistance per knot (r_c) of the cable and core at 75° F.?

From Table IV. we have

$$k_1 = 1{\cdot}0573, \quad k_2 = 1{\cdot}0454;$$

therefore

$$r_t = \frac{215}{\dfrac{15}{1 \cdot 0573} + \dfrac{5}{1 \cdot 0454}} = 11 \cdot 33 \text{ ohms.}$$

Corrections for "Insulation Resistance" when Two Sections of a Cable are at Different Temperatures.

552 Let l_1 and l_2 be the lengths of the two sections, r_1 and r_2 their respective insulation resistances at the temperature of the tanks, P_t the combined resistance of the two sectious, k_1 and k_2 the coefficients by which r_1 and r_2 must be *divided* in order to reduce them to the values they would have at one uniform temperature, also let R_t be the combined resistance of the two sections at this uniform temperature; then we have the following equations:

$$P_t = \frac{r_1 \, r_2}{r_1 + r_2},$$

$$R_t = \frac{\dfrac{r_1}{k_1} \cdot \dfrac{r_2}{k_2}}{\dfrac{r_1}{k_1} + \dfrac{r_2}{k_2}} = \frac{r_1 \, r_2}{r_1 \, k_2 + r_2 \, k_1}, \qquad [3]$$

$$\frac{l_1}{l_2} = \frac{\dfrac{r_2}{k_2}}{\dfrac{r_1}{k_1}} = \frac{r_2 \, k_1}{r_1 \, k_2};$$

therefore

$$\frac{R_t}{P_t} = \frac{r_1 + r_2}{r_1 \, k_2 + r_2 \, k_1}, \quad \text{or,} \quad R_t = P_t \frac{r_1 + r_2}{r_1 \, k_2 + r_2 \, k_1};$$

and also we may say

$$\kappa \, l_1 = r_2 \, k_1, \quad \text{or,} \quad r_2 = \frac{\kappa \, l_1}{k_1},$$

$$\kappa \, l_2 = r_1 \, k_2, \quad \text{or,} \quad r_1 = \frac{\kappa \, l_2}{k_1},$$

where κ is a constant; therefore

$$R_t = P_t \frac{\kappa \left(\dfrac{l_1}{k_1} + \dfrac{l_2}{k_2} \right)}{\kappa \, (l_1 + l_2)} = P_t \frac{\dfrac{l_1}{k_1} + \dfrac{l_2}{k_2}}{l_1 + l_2}.$$

If l_1 and l_2 are the lengths of the sections in knots, then the corrected resistance per knot (r_t) will be

$$r_t = R_t(l_1 + l_2) = P_t\left(\frac{l_1}{k_1} + \frac{l_2}{k_2}\right).$$

For example.

Taking the same lengths and temperatures as in the previous example, let us suppose that the total observed insulation resistance of the 20 knots was 160 megohms (P_t); what would be the insulation resistance per knot (r_t) at 75° F., the insulator being Willoughby Smith's gutta-percha?

From Table VI. we find

$$k_1 = 6 \cdot 928, \quad k_2 = 4 \cdot 704,$$

therefore

$$r_t = 160\left(\frac{15}{6 \cdot 928} + \frac{5}{4 \cdot 704}\right) = 516 \cdot 5 \text{ megohms.}$$

Approximate Corrections for Conductor and Insulation Resistance when Two Sections of a Cable are at Different Temperatures.

553. Instead of correcting each section from its actual to the required temperature, in the way shown, we could assume that the whole length had a temperature which was a mean between the two actual temperatures, and correct the resistances by the coefficients (both for conductor and insulation resistance) corresponding to that mean temperature. This mean temperature may be calculated approximately, on the evident assumption that its value is closer to the temperature of the longer length, in proportion to the proportion which the longer length bears to the shorter length. Let us therefore have

$$\begin{aligned} t_1^\circ &= \text{temperature of length } l_1 \\ t_2^\circ &= \quad ,, \qquad\quad ,, \qquad l_2 \\ t_m^\circ &= \text{mean temperature;} \end{aligned}$$

then, assuming t_1 to be the higher temperature, we have

$$t_1^\circ - t_m^\circ : t_m^\circ - t_2^\circ :: l_2 : l_1;$$

therefore

$$t_1^\circ l_1 - t_m^\circ l_1 = t_m^\circ l_2 - t_2^\circ l_2,$$

or

$$t_m^\circ l_2 + t_m^\circ l_1 = t_1^\circ l_1 + t_2^\circ l_2,$$

that is,

$$t_m^\circ = \frac{t_1^\circ l_1 + t_2^\circ l_2}{l_1 + l_2}.$$

For example.

At a cable factory there were 15 knots (l_1) of manufactured cable lying in a tank whose temperature was 50° (t_1°) F. Connected to this cable were 5 knots (l_2) of core in a tank, whose temperature was 55° (t_1°) F. What was the mean temperature (t_m°) of the whole length?

$$t_m^\circ = \frac{50 \times 15 + 55 \times 5}{15 + 5} = \frac{1025}{20} = 51 \cdot 25^\circ.$$

As a rule the temperature coefficients are only given for degrees and half degrees, as in Tables IV., V., VI. and VII.; consequently, in cases where the mean temperature works out to a fraction of a degree other than ·5, we should take the coefficient of the degrees which are nearest in value to the calculated temperature; this is usually sufficient for all practical purposes. In the above example it is obvious that the coefficient is a mean value between the coefficients for 51° and 51·5°, consequently, in such a case, we can by a very simple calculation see what the exact (practically) coefficient would be. Thus for example, the coefficients in Table IV. for 51° and 51·5° are 1·0549 and 1·0537 respectively, consequently the coefficient for 51·25° is obviously $\frac{1 \cdot 0549 + 1 \cdot 0537}{2}$, or 1·0543.

Practical Application of Corrections for Temperature.

554. When a cable is in course of manufacture, the insulated conductor (or "core" as it is called), before being covered with the protecting sheathing, is placed in water heated to a temperature of 75° F., and is kept in the same for a period of not less than 24 hours. By this lengthened immersion the core acquires the temperature of the water throughout its mass. Careful tests are then made. After the core has been sheathed, it is coiled into a tank and kept covered with water at a normal temperature, and tests are made regularly every day to ascertain its condition. As regards the testing of the conductor, it was usually the practice to take a resistance measurement and then to correct the same to 75° by means of the coefficient corresponding to the temperature of the water in the tank in which the cable is coiled; this corrected

result, if the conductor were in proper condition, would obviously correspond with the results obtained from the core at the 75° temperature.

Now, owing to the slowness with which the gutta-percha covering conducts heat, unless a considerable time has elapsed after the immersion of the sheathed cable in the tank, the temperature of the water in the latter would not necessarily indicate the precise temperature of the conductor, consequently the actual tests at 75° and the corrected observed results might not correspond. The object of a conductor test, made in the foregoing manner, would of course be to ascertain whether the conductor had deteriorated in any way during the course of manufacture; experience has, however, shown that no such deterioration ever does take place, and that consequently a corrected conductor test, even if it were accurate, is practically of no value. At the present time, therefore, it is the invariable practice to make use of the conductor test for the purpose of ascertaining the internal temperature of the core, so that a correct reduction coefficient may be applied to the insulation test of the insulating covering. The method of obtaining the temperature has been indicated in § 544, page 469. In cases where the calculated internal, and the observed external, temperatures do not correspond, the reduction coefficient corresponding to the mean of the two is sometimes taken for correcting the insulation resistance.

When the manufacture of a cable is quite completed, and the latter has been allowed to remain in the tanks for 24 hours or more, the external observed, and the internal calculated, temperatures will generally correspond very closely.

555. In certain cases it happens that a length of cable may be coiled in two different tanks which are at different temperatures; in such cases the temperature calculated from the conductor resistance is, of course, a mean of the two.

556. It should be remarked that the corrections indicated in paragraphs 551, 552, and 553, although not generally used, are still employed in some cases.

557. As has been pointed out in § 456, page 410, the late Mr. Hockin verified the curious fact that it is not until some hours after the gutta-percha has taken its temperature that the resistance reaches its corresponding value.

CHAPTER XXII.

LOCALISATION OF FAULTS AT HIGH RESISTANCE.

Faults in Cables.

558. In all the tests for localising faults hitherto described, with the exception of the loop test (§ 310, page 294), it has been assumed that the insulation resistances of the portions of cable on either side of the fault are infinitely great compared with the resistances of the conductor. Such an assumption practically holds good in cases where the cable under test is short, and also if the resistance of the fault is small, but when we come to deal with long cables having faults of high resistance, the formulæ we have obtained are no longer correct. The following investigation* is made for the purpose of obtaining a test which shall be correct for cables of any length and having faults of any resistance:—

Let A B (Fig. 195) be a cable of any length connected to a

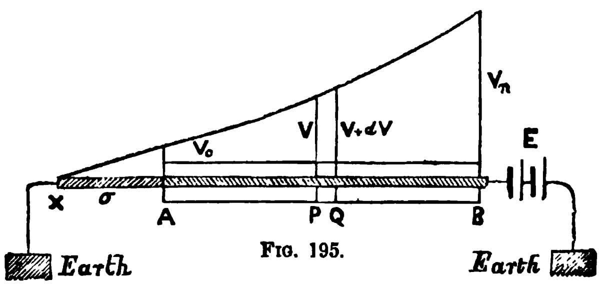


Fig. 195.

battery as shown, and having its further end to earth through a resistance σ. By putting $\sigma = 0$ the end of the cable will be direct to earth, and by putting $\sigma = \infty$, it will be insulated.

$$\text{Let the length } A B = n$$
$$\text{,, \quad ,, \quad } A P = x$$
$$\text{,, \quad ,, \quad } P Q = dx.$$

* See 'On the Leakage of Submarine Cables,' by A. B. Kempe, B.A., 'Journal of the Society of Telegraph-Engineers,' vol. iv. page 90.

Let the potential at $A = V_0$
" " " $B = V_n$
" " " $P = V$
" " " $Q = V + dV.$

Let the current strength at $A = C_0$
" " " $B = C_n$
" " " $P = C$
" " " $Q = C + dC.$

Let the resistance of $XAP = R$
" " " $XAQ = R + dR$
" " " $XAB = R_n$
" " " $XA = R_0 = \sigma.$

Also let resistance of unit length of conductor $= r$
And " " " " sheathing $= i.$

Calling E the electromotive force of the battery, then since the flow of electricity from any point to any other point close to it is from the point of higher to that of lower potential, and is equal to the difference of potential divided by the resistance separating the two points, therefore the current along A B at P is

$$\frac{(V + dV) - V}{r\,dx} = \frac{dV}{r\,dx} = C.$$

The resistance of the wire P Q is evidently $r\,dx$, because it varies *directly* as the length of the wire, but the resistance of the insulating sheath P Q is $\frac{i}{dx}$, because it varies *inversely* as the length. Hence the "leakage" or the current from the surface of the conductor between the points P and Q to the earth where the potential is zero, is

$$\frac{V - 0}{\frac{i}{dx}} = \frac{V\,dx}{i} = dC.$$

Hence

$$\frac{dC}{dx} = \frac{V}{i};$$

but

$$C = \frac{dV}{r\,dx},$$

therefore

$$\frac{dC}{dx} = \frac{1}{r} \cdot \frac{d^2 V}{dx^2};$$

therefore

$$\frac{d^2 V}{dx^2} = \frac{r V}{i} = m^2 V,$$

where

$$m^2 = \frac{r}{i}, \text{ i.e., } m = \sqrt{\frac{r}{i}}.$$

The solution of this differential equation, obtained by the well-known method,* is

$$V = A e^{mx} + B e^{-mx}, \qquad [1]$$

and

$$C = \frac{1}{r} \cdot \frac{dV}{dx} = \frac{m}{r}\left[A e^{mx} - B e^{-mx}\right]. \qquad [2]$$

Now when $x = n$

$$V = V^n = E, \text{ and } C = C_n,$$

therefore, since resistance $= \dfrac{\text{potential}}{\text{current strength}}$,

$$R_n = \frac{V_n}{C_n} = \frac{E}{C_n};$$

and similarly when $x = 0$

$$V = V_0, \text{ and } C = C_0,$$

and

$$R_0 = \sigma = \frac{V_0}{C_0}.$$

Taking, therefore, $x = n$, we get

$$E = V_n = A e^{mn} + B e^{-mn}, \text{ by [1]}$$

$$C_n = \frac{m}{r}\left[A e^{mn} - B e^{-mn}\right], \text{ by [2]},$$

therefore

$$R_n = \frac{E}{C_n} = \frac{r}{m}\left[\frac{A e^{mn} + B e^{-mn}}{A e^{mn} - B e^{-mn}}\right]. \qquad [3]$$

* See Boole's 'Differential Equations,' Second Edition, Chapter IX., p. 194.

Again, taking $x = 0$, we have

$$\sigma = \frac{V_0}{C_0} = \frac{A + B}{\frac{m}{r}(A - B)};$$

therefore

$$\frac{A}{B} = \left(\frac{\sigma\frac{m}{r} + 1}{\sigma\frac{m}{r} - 1}\right), \qquad [4]$$

and

$$R_n = \frac{r}{m}\left[\frac{e^{mn}\left(\sigma\frac{m}{r} + 1\right) + e^{-mn}\left(\sigma\frac{m}{r} - 1\right)}{e^{mn}\left(\sigma\frac{m}{r} + 1\right) - e^{-mn}\left(\sigma\frac{m}{r} - 1\right)}\right]. \qquad [5]$$

559. Let us now see how we can apply this investigation so as to obtain a test which shall be strictly accurate for a cable of any length and resistance. The following, which accomplishes this, is in reality the fall of potential test given on page 431, with the formula corrected.

Let $b\,c$ (Fig. 196) be the cable having a fault z at c, x and y being the *lengths* on either side of the fault, and let R be a resistance.

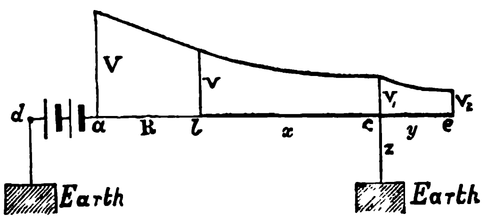


FIG. 196.

Now from equation [1] (page 480) we have

$$v = A\,e^{mx} + B\,e^{-mx} \qquad [6]$$

and at c, where $x = 0$,

$$v_1 = A + B; \text{ therefore, } A = v_1 - B;$$

therefore

$$v = v_1\,e^{mx} - B\,e^{mx} + B\,e^{-mx},$$

or

$$B = \frac{v_1 e^{mx} - v}{e^{mx} - e^{-mx}}.$$

Now from [6]

$$v - 2 B e^{-mx} = A e^{mx} - B e^{-mx},$$

therefore

$$v - 2 e^{-mx} \frac{v_1 e^{mx} - v}{e^{mx} - e^{-mx}} = \frac{v (e^{mx} + e^{-mx}) - 2 v_1}{e^{mx} - e^{-mx}} = A e^{mx} - B e^{-mx}. \quad [7]$$

Calling R_n the resistance beyond b, we have

$$\frac{V}{v} = \frac{R + R_n}{R_n};$$

therefore

$$\frac{R}{V - v} = \frac{R_n}{v} = \frac{R_n}{A e^{mx} + B e^{-mx}} \qquad \text{from [6];}$$

but from [3] (page 480)

$$R_n = \frac{r}{m} \left[\frac{A e^{mx} + B e^{-mx}}{A e^{mx} - B e^{-mx}} \right],$$

therefore

$$\frac{R}{V - v} = \frac{r}{m} \left[\frac{1}{A e^{mx} - B e^{-mx}} \right],$$

therefore from [7]

$$\frac{R}{V - v} = \frac{r}{m} \left[\frac{e^{mx} - e^{-mx}}{v (e^{mx} + e^{-mx}) - 2 v_1} \right]. \quad [8]$$

Again, considering the portion of the cable y, we have

$$\frac{v_1}{v_2} = \frac{A_1 e^{my} + B_1 e^{-my}}{A_1 + B_1},$$

and from [4]

$$\frac{A_1}{B_1} = \frac{\sigma \frac{m}{r} + 1}{\sigma \frac{m}{r} - 1},$$

in which, since the end of the cable is insulated, $\sigma = \infty$; therefore

$$\frac{A_1}{B_1} = 1,$$

from which

$$\frac{v_1}{v_2} = \frac{e^{my} + e^{-my}}{2};$$

therefore

$$v_1 = v_2 \frac{e^{my} + e^{-my}}{2};$$

inserting this value in [8], we get

$$\frac{R}{V - v} = \frac{r}{m}\left[\frac{e^{mx} - e^{-mx}}{v(e^{mx} + e^{-mx}) - v_2(e^{my} + e^{-my})}\right].$$

Now if l be the length of the cable, then $y = l - x$, therefore

$$\frac{R\frac{m}{r}}{V - v} = \frac{e^{mx} - e^{-mx}}{v(e^{mx} + e^{-mx}) - v_2(e^{m(l-x)} + e^{-m(l-x)})}.$$

Multiplying the top and bottom of the equation by e^{mx} we get

$$\frac{R\frac{m}{r}}{V - v} = \frac{e^{2mx} - 1}{v(e^{2mx} + 1) - v_2(e^{ml} + e^{-ml}e^{2mx})}$$

$$= \frac{e^{2mx} - 1}{e^{2mx}(v - v_2 e^{-ml}) + v - v_2 e^{ml}};$$

therefore

$$e^{2mx} = \frac{(V - v) + (v - v_2 e^{ml}) R\frac{m}{r}}{(V - v) - (v - v_2 e^{-ml}) R\frac{m}{r}},$$

from which

$$x = \frac{1}{2m}\log_e \frac{(V - v) + (v - v_2 e^{ml}) R\frac{m}{r}}{(V - v) - (v - v_2 e^{-ml}) R\frac{m}{r}}. \qquad [9]$$

In this form the formula cannot be practically used, as we require to know r and m, that is $\sqrt{\frac{r}{i}}$, r being the resistance per unit length of the conductor, and i the resistance per unit length of the insulating sheathing. These we cannot determine individually, for the measurement made when the end of the cable is to earth is not that of the conductor alone but that of the conductor diminished by the insulation resistance; and similarly, when the

end is insulated the measurement made is not that of the insulating sheathing alone but of the latter combined with the resistance of the conductor. If, however, we know what these measured values are, we can substitute them in the formula in the place of m and r.

Let the measured resistance of the cable when to earth at the further end be R_e and when insulated R_i; then

$$R_e = \frac{r}{m}\left[\frac{e^{ml} - e^{-ml}}{e^{ml} + e^{-ml}}\right],$$

$$R_i = \frac{r}{m}\left[\frac{e^{ml} + e^{-ml}}{e^{ml} - e^{-ml}}\right].$$

This value of R_e we obtain from equation [5] (page 481) by putting $\sigma = 0$, and of R_i by putting $\sigma = \infty$.

By multiplying one equation by the other, we get

$$R_e\, R_i = \frac{r^2}{m^2};$$

therefore

$$\frac{m}{r} = \frac{1}{\sqrt{R_e\, R_i}}. \qquad [10]$$

Also, we have

$$\frac{R_e}{R_i} = \left(\frac{e^{ml} - e^{-ml}}{e^{ml} + e^{-ml}}\right)^2;$$

therefore

$$e^{ml}(\sqrt{R_e} - \sqrt{R_i}) = -\,e^{-ml}(\sqrt{R_e} + \sqrt{R_i});$$

therefore

$$e^{2ml} = \frac{\sqrt{R_i} + \sqrt{R_e}}{\sqrt{R_i} + \sqrt{R_e}}; \qquad [11]$$

that is

$$\frac{1}{2\,m} = \frac{l}{\log_e \dfrac{\sqrt{R_i} + \sqrt{R_e}}{\sqrt{R_i} + \sqrt{R_e}}}; \qquad [12]$$

and also from [11] we have

$$e^{ml} = \sqrt{\frac{\sqrt{R_i} + \sqrt{R_e}}{\sqrt{R_i} - \sqrt{R_e}}}, \text{ and } e^{-ml} = \sqrt{\frac{\sqrt{R_i} + \sqrt{R_e}}{\sqrt{R_i} - \sqrt{R_e}}}, \qquad [13]$$

We have thus determined $\frac{1}{2\,m}$, $\frac{m}{r}$, e^{ml}, and e^{-ml}, and can consequently insert their values in any equations we may require.

560. Instead of employing the resistance R (Fig. 196, page 481),

we may make the test by connecting the battery direct on to b through a galvanometer, so that the resistance R_s of the cable can be measured by the ordinary deflection method (§ 10, page 6). Then, since

$$V : v :: R + R_s : R_s,$$

therefore

$$V - v = v \frac{R}{R_s}.$$

If we substitute this value of $V - v$ in equation [9] (page 483), we get

$$x = \frac{1}{2m} \log_e \frac{\frac{v}{R_s} + (v - v_2 e^{ml}) \frac{m}{r}}{\frac{v}{R_s} - (v - v_2 e^{-ml}) \frac{m}{r}}.$$

$$= \frac{1}{2m} \log_e \frac{v + (v - v_2 e^{ml}) R_s \frac{m}{r}}{v - (v - v_2 e^{-ml}) R_s \frac{m}{r}}.$$

For example.

A cable 1000 knots (l) long had a very small fault in it which was required to be localised. When the cable was good its resistance with the further end insulated, after five minutes' electrification, was 700,000 ohms (R_i), and its resistance with the further end put to earth, 5000 ohms (R_e). When the cable was faulty its resistance with the end insulated, after five minutes' electrification, was 100,000 ohms (R_s). The potentials at the ends of the cable, after five minutes' electrification, were 300 (v) and 292 (v_2) respectively. What was the distance (x) of the fault from the nearer end of the cable?

$$\frac{m}{r} = \frac{1}{\sqrt{5000 \times 700,000}} = \frac{1}{59,161},$$

$$\frac{1}{2m} = \frac{1000}{\log \frac{\sqrt{700,000} + \sqrt{5000}}{\sqrt{700,000} - \sqrt{5000}} \times 2 \cdot 3026} = 5902 \cdot 1,$$

$$e^{ml} = \sqrt{\frac{\sqrt{700,000} + \sqrt{5000}}{\sqrt{700,000} - \sqrt{5000}}} = 1 \cdot 0884,$$

$$e^{-ml} = \sqrt{\frac{\sqrt{700,000} - \sqrt{5000}}{\sqrt{700,000} + \sqrt{5000}}} = \cdot 9188.$$

Inserting these values in the equation, we get

$$x = 5902 \cdot 1 \times \log \frac{300 + [300 - (292 \times 1 \cdot 0884)]\, 100{,}000 \times \dfrac{1}{59{,}161}}{300 - [300 - (292 + \cdot 9188)]\, 100{,}000 \times \dfrac{1}{59{,}161}}$$

$\times\ 2 \cdot 3026 = 538$ knots.

561. Since in the case of a small fault the difference between the potentials at the two ends is comparatively small it is essential that they should be measured with great accuracy, otherwise a small error made in determining their value will make a considerable error in the value of x. The readings on the scale of the galvanometer or electrometer must therefore be made as high as possible; it is even advisable to extend the length of the scale so that this may be done more effectually.

562. The relative values of the potentials at the two ends of the cable must be determined in the manner described in Chapter XVII. (§ 496, page 432).

Localisation of Faults in Insulated Wires.

563. The usual method of localising faults in lengths of insulated wire, or cable core which has not been covered with the iron sheathing which forms the complete cable, is to pass the wire through water in an insulated trough, one pole of a battery of several hundred cells being connected to the conductor, the other pole to one terminal of a galvanometer, and the other terminal of the latter to the water in the trough. With this arrangement, immediately the fault in the wire passes into the trough the galvanometer needle is deflected. This method, however, is not adapted for detecting extremely minute faults, such as are of not unfrequent occurrence.

Warren's Method.*

564. This method is adapted for localising very minute faults, i.e. faults of high resistance.

The length of wire to be operated on is immaterial, provided that the whole or a portion of it can be coiled on an insulated drum, and that between the parts coiled the surface of the core for a length of 6 or 8 inches can be cleaned and dried so as to prevent conduction.

* 'Philosophical Magazine,' No. 314, June 1879.

In the first case (when the whole can be coiled on a drum), one-half is coiled off on a second drum, the two drums being carefully insulated. The surface of the core between the drums is well cleaned and dried. The conductor is attached to one set of quadrants of an electrometer, the other set being to earth, and the two drums are connected to earth by an attendant at each drum; one pole of the battery (whose second pole is to earth) is then connected to the conductor, so that the whole becomes charged, the battery is then disconnected from the electrometer, and the earth-wires simultaneously taken off the drums. It is best to leave the battery on until the earth-wires are removed from the drums.

The insulation of the drums and electrometer should be such that no loss can be perceived after a few minutes, when, if the earth-wire be applied first to one drum and then to the other, the fault will be found on that drum which causes the greatest fall in the electrometer. The wire is coiled from the faulty side to the other, and the test repeated as often as is required. A mile of core with a small fault in it can by a little practice be put right in an hour or two, involving no more waste than a portion of the insulator which can be held between the fingers, and without even cutting the conductor. The position of the fault, when it is obtained between the two drums, can be found more closely by cleaning and drying the surfaces on each side of it, and then touching the place where the fault appears to be, with the earth-wire, and seeing whether there is a fall in the electrometer.

In the second case, where the bulk would prevent the whole from being insulated, we should continue to coil the core upon an insulated drum until the fault disappeared—that is, until it was coiled on the drum. This is a useful method when dealing with "served core," that is core with its hemp covering only, at a cable factory.

565. By the foregoing method a *joint* may be tested with great ease by immersing it in an insulated trough of water, and putting the latter to earth, or even by simply touching the moist joint with the earth-wire.

566. The tests can be made with a galvanometer instead of an electrometer, although it is not such a sensitive arrangement. In this case the battery would be connected through the galvanometer to the conductor, as in an ordinary insulation test, and then the drums be connected to earth alternately, when the deflection of the needle shows on which drum the fault exists; but as the lengths on each drum may be very unequal, and consequently one

drum may show a greater deflection simply in virtue of its having a greater length of core on it, the rush of current alone is not sufficient to enable the drum on which the fault is, to be found; but by carefully watching the electrification, and seeing whether the fall is regular or not, no difficulty will be found in fixing upon the drum containing the fault. The battery-power required will vary with the magnitude of the fault and the sensitiveness of the instrument, and can only be determined by experience and experiment.

Jacob's Method.

567. This method, which is a very satisfactory one, consists in winding the faulty core, no matter what the resistance of the fault happens to be, on to a drum or platform which requires to be only partially insulated, so that a wooden stand or even the floor is often quite sufficient; the battery with one pole to earth is then applied direct to the conductor of the core the other end of which is insulated, the galvanometer is inserted between the drum and earth; thus so long as the resistance between the drum and earth along the surface of the core or otherwise is not too small, so as to shunt the galvanometer too much, the current through the fault, if it be on the drum, will flow through the galvanometer to earth, but if the fault is not on the drum the current will pass direct to earth since the tank in which the rest of the core is coiled will be to earth; the core will be then wound off or on accordingly until the fault is found. Often this fault will not be visible to the naked eye and the exact place can be electrically determined by passing the end of the wire leading from the galvanometer, over the surface of the core, while the battery is connected as above described. If two drums are used, the core on one will have its surface connected to earth, and that on the other connected to the galvanometer, or *vice versâ*, so that it can be seen that the fault has not disappeared in coiling over. It should be added that in this test there is no necessity for cleaning the surface of the portion of core between the two drums, and that any description of core except that covered with a metallic sheathing,* can be so tested.

* When a cable is to be laid in seas where the teredo worm abounds, it is now usual to cover the insulated core with a close fitting lapping of thin brass tape.

CHAPTER XXIII.

LOCALISATION OF A DISCONNECTION FAULT IN A CABLE.

Localisation of a Total Disconnection.

568. The localisation of a total disconnection in a cable is a very easy matter. The conductor being broken inside the insulating sheathing, a battery connected to the end of the cable will charge the latter up as far as the fault only, consequently if we measure the discharge, and compare it with the discharge from a condenser of a known capacity charged from the same battery, we shall obtain the capacity of the portion up to the fault. Also since the capacity per knot of the cable is always known, the observed capacity of the length in question, divided by the capacity per knot, will give at once the distance of the fault.

Localisation of a Partial Disconnection.

569. Partial disconnection faults, although they are seldom met with in cables with gutta-percha cores, frequently occur in those whose insulating material is indiarubber. This arises from the elasticity of the substance; for when any undue strain is put on the core the conductor breaks, but the indiarubber only stretches, and an earth fault is not made. When the strain is taken off, the two ends of the conductor come together and make contact more or less perfectly. If the break is noticed at the moment the cable is being laid from the ship, its position is of course known. But in some cases a fault of this nature does not develop itself until some time after the submersion; its locality can then only be found by testing.

Such faults are difficult to localise, as none of the ordinary tests are applicable to them. The following method, however, devised by the author, is susceptible of considerable accuracy if carefully made.

In Fig. 197, B K represents the cable with its further end to earth; R and r are the resistances of the portions of the cable on either side of the disconnection, and y is the resistance of the latter; a, b, and c are the three sides of a Wheatstone Bridge, of which the cable forms the fourth side; g and g_1 are two galvanometers, the former being of the ordinary Thomson form and the latter also a Thomson, but provided with heavy needles, so that its movements are very sluggish.

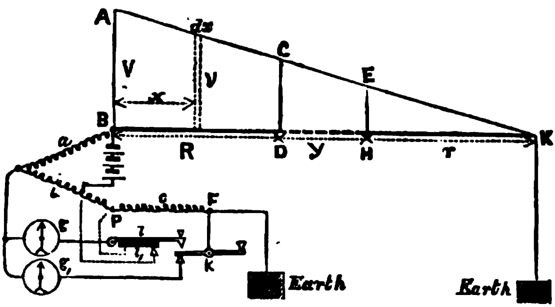


FIG. 197.

Connected to the battery, and also to the galvanometers, is a key formed in two parts. The ordinary lever k of the key has its back-stop connected through the galvanometer g_1 to the junction of the resistances a, b; thus when the key is in its normal position the galvanometer g_1 is connected to earth. The second portion of the key consists of a lever l, to the underneath part of which is fixed the metal piece l_1, which is insulated from l. Normally, as shown in the figure, l_1 rests on a stop connected to one pole of the battery, the other pole of the latter being connected to B. The point P is connected permanently with l_1, whilst the lever l is itself permanently connected to the galvanometer g.

Now, the result of this arrangement is, that normally the battery is connected between the points B and P, and the galvanometer g_1 is connected between the junctions of a and b and with earth, that is with the end B of the cable; the whole arrangement, in fact, forms an ordinary Wheatstone Bridge.

Now, if a, b, and c are adjusted so that balance is produced, then the needle of the galvanometer g_1 will stand at zero; if, when this is the case, the key k be depressed, g_1 will be disconnected, and when the lever of k touches the end of l, g will be put in circuit in the place of g_1; but immediately this takes place l_1 will be lifted off its contact, and the battery will be cut off;

exactly at this moment then the charges in the cable will discharge and divide themselves, portions flowing out at the further end and the other portions flowing out through g, a, and b, and thence through c to earth. A *throw* of the needle of the galvanometer will thus be produced.

Supposing the key k to be in its normal position, so that the battery causes a current to flow through the cable, whilst the resistance a, b, and c, are so adjusted that the galvanometer g_1 is unaffected, then let V be the potential at the beginning, and v be the potential at any other point of the portion B D.

If now the key be depressed, the charges in the cable represented by the areas A B C D and E H K, will flow out at the two ends of the cable in proportions dependent upon the values of the resistances R, y, and r, and the combined resistances of a, b, g, and c.

Let $v\,dx$ be a differential part of the charge A B D C, then this portion will split, and the portions flowing out at the two ends of the cable will be inversely proportional to the resistances on either side of $v\,dx$; thus the portion flowing out at B will be

$$d\,Q' = v\frac{R + y + r - x}{R_1 + R + y + r}\,dx,$$

where R_1 is the combined resistance of a, b, g, and c.

Now

$$V : v :: R + y + r : R + y + r - x,$$

therefore

$$v = V\frac{R + y + r - x}{R + y + r},$$

that is

$$d\,Q' = V\frac{(R + y + r - x)^2}{(R_1 + R + y + r)\,(R + y + r)}\,dx,$$

and the integral of this between the limits

$$x = R \text{ and } x = 0$$

will give the quantity Q' flowing out at B, that is

$$Q' = \int_0^R V\frac{(R + y + r - x)^2}{(R_1 + R + y + r)\,(R + y + r)}\,dx$$

$$= \frac{V}{(R_1 + R + y + r)\,(R + y - r)}\int_0^R (R + y + r - x)^2\,dx$$

$$= \frac{V}{(R_1 + R + y + r)\,(R + y + r)}\left[\frac{(R + y + r)^3 - (y + r)^3}{3}\right]$$

$$= \frac{V}{3}\cdot\frac{(R + y + r)^3 - (y + r)^3}{(R_1 + R + y + r)\,(R + y + r)}.$$

Similarly we should find that the quantity Q″ flowing out from the portion r of the cable would be

$$Q'' = \frac{V}{3} \cdot \frac{r^3}{(R_1 + R + y + r)(R + y + r)},$$

and therefore the total quantity Q flowing through the galvanometer will be

$$Q' + Q'' = \frac{V}{3} \cdot \frac{(R + y + r)^3 - (y + r)^3 + r^3}{(R_1 + R + y + r)(R + y + r)} = Q. \quad [1]$$

Now the total quantity Q_1 which the cable would take if its further end were insulated and the end B maintained at the potential V, would be

$$Q_1 = V(R + r).$$

Again, if f be the capacity in microfarads of such a length of the cable as would have a conductor resistance of 1 ohm, then $(R + r) f$ will be the actual total capacity of the cable, and if Q_2 be the charge held by a condenser of F microfarads capacity, also charged to the potential V, then

$$Q_1 : Q_2 :: (R + r) f : F;$$

therefore

$$Q_1 = \frac{Q_2 (R + r) f}{F} = V(R + r),$$

or

$$V = \frac{Q_2 f}{F}.$$

Substituting then this value of V in equation [1] we get

$$Q = \frac{Q_2 f}{3 F} \cdot \frac{(R + y + r)^3 - (y + r)^3 + r^3}{(R_1 + R + y + r)(R + y + r)}.$$

Let

$$R + y + r = L, \quad \text{therefore,} \quad y + r = L - R;$$
$$R + r = L_1, \quad \text{therefore,} \quad r = L_1 - R.$$

Substituting these values in the last equation we get

$$Q = \frac{Q_2 f}{3 F} \cdot \frac{L^3 - (L - R)^3 + (L_1 - R)^3}{(R_1 + L) L}$$

$$= \frac{Q_2 f}{3 F} \cdot \frac{L^3 - L^3 - 3 L R^2 + 3 L^2 R + R^3 + L_1^3 + 3 L_1 R^2 - 3 L_1^2 R - R^3}{(R_1 + L) L}$$

$$= \frac{Q_2 f}{3 F} \cdot \frac{L_1^3 + 3 R (L^2 - L_1^2) - 3 R^2 (L - L_1)}{(R_1 + L) L};$$

therefore

$$\frac{3\,Q\,F\,(R_1 + L)\,L}{Q_2 f} - L_1^3 = 3\,R\,(L + L_1)(L - L_1) - 3\,R^2(L - L_1);$$

therefore

$$R^2 - R\,(L + L_1) = -\frac{3\,Q\,F\,(R_1 + L)\,L - Q_2 f L_1^3}{3\,Q_2 f (L - L_1)};$$

therefore

$$R^2 - R\,(L + L_1) + \left(\frac{L + L_1}{2}\right)^2 = \left(\frac{L + L_1}{2}\right)^2 -$$

$$\frac{3\,Q\,F\,(R_1 + L)\,L - Q_2 f L_1^3}{3\,Q_2 f\,(L - L_1)},$$

that is

$$R = \frac{L + L_1}{2} - \sqrt{\frac{(L + L_1)^2}{4} - \frac{3\,Q\,F\,(R_1 + L)\,L - Q_2 f L_1^3}{3\,Q_2 f (L - L_1)}}. \quad [2]$$

Now the quantity Q discharged at B will split between the resistances g, and $a + b$, the quantity Q_3 passing through the galvanometer being

$$Q_3 = Q\,\frac{b + c}{b + c + g},$$

from which

$$Q = Q_3\,\frac{b + c + g}{b + c}.$$

The value of R_1, the combined resistance of a, b, g, and c, will be

$$R_1 = a + \frac{(b + c)\,g}{b + c + g},$$

and since balance is produced in the bridge

$$L = \frac{a\,c}{b},$$

therefore

$$R_1 + L = a + \frac{(b + c)\,g}{b + c + g} + \frac{a\,c}{b}$$

$$= (b + c)\frac{g}{b + c + g} + (b + c)\frac{a}{b} = \frac{(b + c)}{(b + c + g)\,b}[b\,g +$$

$$a\,(b + c + g)] = \frac{b + c}{(b + c + g)\,b}[g\,(a + b) + a\,(b + c)];$$

and therefore

$$Q(R_1 + L) = \frac{Q_3}{b}[g(a+b) + a(b+c)].$$

Substituting this value in equation [2] we get

$$R = \frac{L + L_1}{2} - \sqrt{\frac{(L+L_1)^2}{4} - \frac{\frac{3\,Q_3 F}{b}[g(a+b)+a(b+c)]\,L - Q_2 f L_1^3}{3\,Q_2 f(L - L_1)}}$$

in which, as we have before stated,

$$L = \frac{a\,c}{b}.$$

Should it be necessary to employ a shunt for the galvanometer g, of the $\frac{1}{n}$th value say, then the observed deflection will have to be multiplied by n in order to give the value of Q_3, and also the value of g in the formula will be $\frac{1}{n}$th of the actual resistance of the galvanometer.

For example.

In localising a partial disconnection in a cable by the foregoing test, the branches a and b of the bridge were made 100 ohms each, and balance was obtained on g when c was adjusted to 5000 ohms; consequently since a and b are equal

$$L = C = 5000.$$

The conductor resistance L_1 when the cable was perfect was 2000 ohms.

The resistance of the galvanometer was 5000 ohms, but when the discharge was noted it was necessary to employ the $\frac{1}{10}$th shunt, so that in the formula we must put

$$g = \frac{5000}{10} = 500.$$

The discharge deflection observed on depressing the key was 248 divisions, therefore

$$Q_3 = 248 \times 10 = 2480.$$

The discharge deflection Q_2 observed from a condenser of 1 microfarad capacity (F) when charged to the potential V was 202 divisions with no shunt, therefore

$$Q_2 = 202.$$

The cable having a conductor resistance of 10 ohms per mile, and an inductive capacity of ·3 microfarad per knot, the capacity in microfarads of such a length of the cable as would have a conductor resistance of 1 ohm, would be $\frac{\cdot 3}{10} = \cdot 03$ microfarad, that is

$$f = \cdot 03;$$

then

$$R = \frac{5000 + 2000}{2} - \sqrt{\frac{(5000 + 2000)^2}{4} - \frac{\frac{3 \times 2480 \times 1}{100} \left[500\,(100 + 100) + 100\,(100 + 5000)\right] 5000 - 202 \times \cdot 03 \times 2000^3}{3 \times 202 \times \cdot 03\,(5000 - 2000)}}$$

$$= 3500 - 2996 = 504 \text{ ohms.}$$

570. In making this test practically, after c and Q_3 have been obtained, the cable must be disconnected from the bridge, and a resistance equal to L be connected between B and F, the potential at the point B will then still be V, and further the galvanometer can be removed without altering this potential; the condenser and galvanometer must then be joined up in the manner shown by Fig. 142, page 313, the wires, however, which in that figure are shown as connected to the battery, being connected in the present case to the points B and F, Fig. 197 (page 490); then the discharge obtained, multiplied by the shunt (if one is employed), gives Q_2.

571. It will sometimes be found that the cable is traversed by an earth current. The effects of this may best be neutralised in the manner indicated on page 293, § 309, Chapter IX., the compensating battery being connected between the cable and the point B, and adjustment effected with the lever $l\,l_1$, raised so as to cut the testing battery off; when the galvanometer g_1 is unaffected the adjustment is correct, the lever $l\,l_1$ is then let down, and the test made as if no earth current existed.

572. As it would be a matter of considerable difficulty, if not of impossibility, to adjust the bridge balance with an ordinary Thomson galvanometer (g_1) in consequence of the latter being greatly affected by slight changes in the earth current, a galvanometer with a heavy needle whose movements are very sluggish,

and which is consequently unaffected by slight and sudden changes of current, is necessary for the purpose. For measuring the discharge, however, a more sensitive instrument (g) is necessary, which must be brought into use only at the exact moment required, since it is necessary that its needle be steady at zero at that time. By the arrangement of key shown in Fig. 197 (page 490), this object is completely effected, as the galvanometer g is only brought into use at the moment when the battery is cut off, and the cable discharged.

CHAPTER XXIV.

A METHOD OF LOCALISING EARTH FAULTS IN CABLES.

Localisation of Fault when Cable is not broken.

573. This test is of the same nature as the foregoing, and possesses the advantage of having all the necessary observations taken simultaneously, and from one end of the cable only.

In Fig. 198, R and p represent the resistances of the portions of the conductor of the cable on either side of the fault, and r represents the resistance of the fault itself. As in the previous

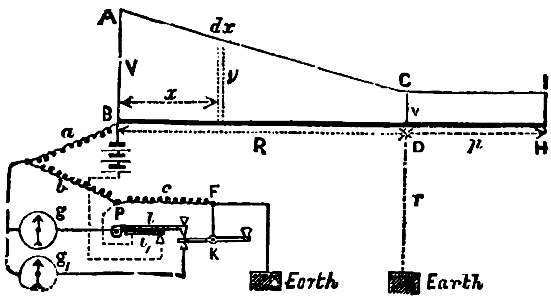


Fig. 198.

test, a, b, and c are the three sides of a Wheatstone bridge, of which the cable forms the fourth side, and g and g_1 are two galvanometers. $k\,l\,l_1$ is a key, the construction and working of which were fully described in the previous test (page 490).

Supposing the key to be in its normal position, then let V represent the potential at the beginning of the cable, V the potential at the fault and also at the further end H, and v the potential at any point between B and D.

If now the key k is depressed, the charge in the cable, which

is represented by the area A B C D, will flow out at B and at D in proportions dependent upon the values of the resistances r, R, and the combined resistances a, b, g, and c.

Let $v\,dx$ be a differential part of the charge. Then the portion of this which will flow out at B will be

$$d\,q_1 = v\,\frac{\mathrm{R} + r - x}{\mathrm{R}_1 + \mathrm{R} + r}\,dx,$$

where R_1 is the combined resistance of a, b, g, and c.

Now

$$\mathrm{V} : v :: \mathrm{R} + r : \mathrm{R} + r - x;$$

therefore

$$v = \mathrm{V}\,\frac{\mathrm{R} + r - x}{\mathrm{R} + r};$$

therefore

$$d\,q_1 = \mathrm{V}\,\frac{(\mathrm{R} + r - x)^2}{(\mathrm{R}_1 + \mathrm{R} + r)(\mathrm{R} + r)}\,dx; \qquad [1]$$

and the integral of this between the limits $x = \mathrm{R}$ and $x = 0$ will give the total quantity q_1, due to the charge A B D C, flowing out at B, that is

$$\begin{aligned} q_1 &= \int_0^{\mathrm{R}} \mathrm{V}\,\frac{(\mathrm{R} + r - x)^2}{(\mathrm{R}_1 + \mathrm{R} + r)(\mathrm{R} + r)}\,dx \\ &= \frac{\mathrm{V}}{(\mathrm{R}_1 + \mathrm{R} + r)(\mathrm{R} + r)}\int_0^{\mathrm{R}} (\mathrm{R} + r - x)^2\,dx \\ &= \frac{\mathrm{V}}{(\mathrm{R}_1 + \mathrm{R} + r)(\mathrm{R} + r)}\left[\frac{(\mathrm{R} + r)^3 - r^3}{3}\right] \\ &= \frac{\mathrm{V}}{3}\cdot\frac{(\mathrm{R} + r)^3 - r^3}{(\mathrm{R}_1 + \mathrm{R} + r)(\mathrm{R} + r)}. \end{aligned} \qquad [2]$$

Now besides the quantity q_1 there will be a quantity q_2 flowing out at B, due to the charge represented by the area C D H I. Let this charge be q', then

$$q' = \mathrm{v}\,p;$$

but

$$\mathrm{V} : \mathrm{v} :: \mathrm{R} + r : r,$$

therefore

$$\mathrm{v} = \mathrm{V}\,\frac{r}{\mathrm{R} + r};$$

therefore

$$q' = \mathrm{V}\,\frac{r\,p}{\mathrm{R} + r}.$$

This quantity q' in flowing out at D will divide, the portion q_2 flowing along R and out at B, being

$$q_2 = q' \frac{r}{R_1 + R + r} = V \frac{r^2 p}{(R_1 + R + r)(R + r)};$$

consequently the *total* quantity flowing out at B will be

$$\begin{aligned} q_1 + q_2 &= \frac{V}{3} \cdot \frac{(R + r)^3 - r^3}{(R_1 + R + r)(R + r)} + V \frac{r^2 p}{(R_1 + R + r)(R + r)} \\ &= \frac{V}{3} \cdot \frac{(R + r)^3 - r^3 + 3 r^2 p}{(R_1 + R + r)(R + r)} = Q. \qquad [1] \end{aligned}$$

Now, if the cable had no fault in it, and its further end were insulated, and if it had been charged to the potential V, then the quantity Q_1, which the length B D would contain, would be represented by the equation

$$Q_1 = V R.$$

Again, if f be the capacity in microfarads of such a length of the cable as would have a conductor resistance of 1 unit, then Rf will be the actual total capacity of the length BD; and if Q_2 be the charge held by a condenser of F microfarads capacity also charged to the potential V, then

$$Q_1 : Q_2 :: Rf : F;$$

therefore

$$Q_1 = \frac{Q_2 R f}{F} = V R,$$

or

$$V = \frac{Q_2 f}{F}.$$

Substituting this value of V in equation [1] we get

$$Q = \frac{Q_2 f}{3 F} \cdot \frac{(R + r)^3 - r^3 + 3 r^2 p}{(R_1 + R + r)(R + r)}, \qquad [A]$$

or

$$(R + r)^3 - r^3 + 3 r^2 p = \frac{3 Q F (R_1 + R + r)(R + r)}{Q_2 f}.$$

Let

$$R + r = L$$

and

$$R + p = L_1;$$

therefore

$$r = L - R$$

and

$$p - r = L_1 - L, \quad \text{or,} \quad p = L_1 - L + r;$$

therefore

$$\begin{aligned}(R + r)^3 - r^3 + 3r^2 p &= L^3 - r^3 + 3r^2(L_1 - L) + 3r^3 \\ &= L^3 + 3r^2(L_1 - L) + 2r^3 \\ &= L^3 + 3(L - R)^2(L_1 - L) + 2(L - R)^3,\end{aligned}$$

therefore

$$\begin{aligned}(L - R)^3 + \frac{3(L_1 - L)}{2}(L - R)^2 &= \frac{3\,Q\,F\,(R_1 + L)\,L}{2\,Q_2 f} - \frac{L^3}{2} \\ &= \frac{C}{2}, \text{ say.} \qquad [1]\end{aligned}$$

Also if Q_3 equals the quantity discharged through the galvanometer, then by substituting this quantity and the combined values of a, b, c, and g, to which R_1 is equal, in the manner shown on page 493, in the last chapter, we shall have

$$C = \frac{3\,Q_3\,F\,[g(a + b) + a(b + c)]\,L}{b\,Q_2 f} - L^3.$$

If in making the test it is found necessary to employ a shunt with the galvanometer when taking the discharge, then if the value of this shunt be $\frac{1}{n}$th, we must multiply the observed deflection by n in order to obtain Q_3; also the value of g in the above equation will be $\frac{1}{n}$th of the actual resistance of the galvanometer.

From the *cubic* equation [1] R has now to be determined; this can be done in the following manner:—

Dividing each side by $(L_1 - L)^3$, we get

$$\left(\frac{L - R}{L_1 - L}\right)^3 + \frac{3}{2}\left(\frac{L - R}{L_1 - L}\right)^2 - \frac{C}{2(L_1 - L)^3} = 0;$$

therefore

$$\left(\frac{L - R}{L_1 - L} + \frac{1}{2}\right)^3 - \frac{3}{4}\left(\frac{L - R}{L_1 - L}\right) - \frac{1}{8} - \frac{C}{2(L_1 - L)^3} = 0;$$

therefore

$$\left(\frac{L - R}{L_1 - L} - \frac{1}{2}\right)^3 - \frac{3}{4}\left(\frac{L - R}{L_1 - L} + \frac{1}{2}\right) + \frac{1}{4} - \frac{C}{2(L_1 - L)^3} = 0;$$

that is

$$4\left(\frac{L-R}{L_1-L}-\frac{1}{2}\right)^3=3\left(\frac{L-R}{L_1-L}+\frac{1}{2}\right)+1-\frac{2\,C}{(L_1-L)^3}=0.\quad[2]$$

Now this equation is of the same form as the identity

$$4\cos^3 a-3\cos a-\cos 3\,a=0;$$

if then we put

$$\frac{2\,C}{(L_1-L)^3}-1=\cos 3\,a,\qquad[3]$$

we shall have

$$\frac{L-R}{L_1-L}+\frac{1}{2}=\cos a,$$

or

$$L-R=(L_1-L)\,(\cos a-\tfrac{1}{2});$$

that is

$$R=L-(L_1-L)\,(\cos a-\tfrac{1}{2}).\qquad[4]$$

So that having worked out the numerical value of $\frac{2\,C}{(L_1-L)^3}-1$, and ascertained in a table of cosines to what angle this corresponds, then the cosine of $\frac{1}{3}$rd of this angle gives cos a, which value inserted in equation [4] enables the value of R to be obtained.

For example.

In localising a fault by the foregoing test, the two arms a and b of the bridge were made 100 ohms each, and balance was obtained on g when c was adjusted to 700 ohms; therefore L = 700 ohms.

The resistance of the galvanometer was 5000 ohms, but when the discharge was noted on it the $\frac{1}{10}$th shunt was inserted, so that $g=\frac{5000}{10}=500$ ohms.

The discharge deflection observed on depressing the key was 350 divisions; therefore $Q_3 = 338 \times 10 = 3380$. The discharge deflection Q_2 obtained from a condenser of 1 microfarad capacity (F) charged to the potential V was 106 divisions with the $\frac{1}{10}$th shunt; therefore $Q_2 = 106 \times 10 = 1060$. The capacity f of such a length of the cable as would have a conductor resistance of 1 ohm was ·03 microfarad; lastly, the total conductor

resistance L_1 of the cable when sound was 1100 ohms. Thus we have

$$
\begin{aligned}
a &= 100\\
b &= 100\\
g &= 500\\
c &= 700\\
L &= 700\\
L_1 &= 1100\\
Q_2 &= 1060\\
Q_3 &= 3380\\
F &= 1\\
f &= \cdot 03
\end{aligned}
$$

we then get

$$C = \frac{3 \times 3380 \times 1\,[500\,(100 + 100) + 100\,(100 + 700)]\,700}{100 \times 1060 \times \cdot 03}$$

$$- 700^3 = 401{,}770{,}000 - 343{,}000{,}000 = 58{,}770{,}000;$$

therefore

$$\frac{2\,C}{(L_1 - L)^3} - 1 = \frac{2 \times 58{,}770{,}000}{(1100 - 700)^3} - 1 = \cdot 8366 = \cos 3\,a$$
$$= \cos \text{ of } 33^\circ\ 13'$$

therefore

$$a = \frac{33^\circ\ 13'}{3} = 11^\circ\ 4',$$

the cosine of which is $\cdot 9814$; therefore

$$R = 700 - (1100 - 700)\,(\cdot 9814 - \tfrac{1}{2}) = 507 \text{ ohms},$$

which gives the distance of the fault.

574. It may be remarked that the foregoing test is an excellent example of one of those rare cases in which the solution of an equation of the third degree is practically required, and in which the application of trigonometrical formulæ for the purpose is useful.*

575. Now the cosine of an angle can never exceed 1, and it will sometimes be found, on working out the value of $\frac{2\,C}{(L_1 - L)^3} - 1$, that its value will exceed unity; consequently in such a case R cannot be determined by the help of a cosine table, but some other method must be adopted. Let us determine this method.

* See Todhunter's Trigonometry, Third Edition, Chapter XVII. page 202.

In equation [2] (page 501) let

$$\frac{L - R}{L_1 - L} + \frac{1}{2} = y + \frac{1}{4y};$$

we then have

$$4y^3 + 3y + \frac{3}{4y} + \frac{1}{16y^3} - 3y - \frac{3}{4y} + 1 - \frac{2C}{(L_1 - L)^3} = 0,$$

or

$$y^3 + \frac{1}{64y^3} + \frac{1}{4}\left(1 - \frac{2C}{(L_1 - L)^3}\right) = 0.$$

Let

$$\frac{2C}{(L_1 - L)^3} - 1 = K;$$

therefore

$$y^6 - \frac{K}{4}y^3 + \frac{1}{64} = 0;$$

a *quadratic* equation, from which y^3 can be determined in the ordinary manner. Thus

$$y^6 - \frac{K}{4}y^3 + \left(\frac{K}{8}\right)^2 = \frac{K^2}{64} - \frac{1}{64};$$

therefore

$$y^3 - \frac{K}{8} = \pm \tfrac{1}{8}\sqrt{K^2 - 1};$$

or

$$y = \tfrac{1}{2}[K \pm \sqrt{K^2 - 1}]^{\frac{1}{3}},$$

and

$$y + \frac{1}{4y} = \tfrac{1}{2}\{[K + \sqrt{K^2 - 1}]^{\frac{1}{3}} + [K + \sqrt{K^2 - 1}]^{-\frac{1}{3}}\}$$
$$= \tfrac{1}{2}\{[K + \sqrt{K^2 - 1}]^{\frac{1}{3}} + [K - \sqrt{K^2 - 1}]^{\frac{1}{3}}\};$$

so that we get

$$R = L(L_1 - L)\tfrac{1}{2}\{[K + \sqrt{K^2 - 1}]^{\frac{1}{3}} + [K - \sqrt{K^2 - 1}]^{\frac{1}{3}} - 1\},$$

in which

$$K = \frac{2C}{(L_1 - L)^3} - 1,$$

and

$$C = \frac{3Q_3F[g(a + b) + a(b + c)]L}{bQ_2f} - L^3.$$

For example.

In making the test, suppose the following to have been the numerical values of the different quantities:—

$$
\begin{aligned}
a &= 100 \\
b &= 100 \\
g &= 500 \\
c &= 900 \\
L &= 900 \\
L_1 &= 1100 \\
Q_2 &= 300 \\
Q_3 &= 1230 \\
F &= 1 \\
f &= \cdot 03
\end{aligned}
$$

therefore

$$C = \frac{3 \times 1230 \times 1\,[500\,(100 + 100) + 100\,(100 + 900)]\,900}{100 \times 300 \times \cdot 03}$$

$$- 900^3 = 538{,}000{,}000 - 729{,}000{,}000 = 9{,}000{,}000;$$

therefore

$$K = \frac{2 \times 9{,}000{,}000}{(1100 - 900)^3} - 1 = 2 \cdot 25 - 1 = 1 \cdot 25;$$

therefore

$$\sqrt{K^2 - 1} = \sqrt{1 \cdot 25^2 - 1} = \cdot 75;$$

for this we get

$$R = 900 - (1100 - 900)\,\tfrac{1}{2}\,\{2^{\frac{1}{3}} + \cdot 5^{\frac{1}{3}} - 1\}$$

$$= 900 - \frac{200}{2}\{1 \cdot 2599 + \cdot 7937 - 1\} = 795 \text{ ohms.}$$

Localisation of Fault when Cable is broken.

576. In this case, referring to page 499, the quantity discharged at B when the key is depressed will be only q_1 instead of $q_1 + q_2$; consequently equation [A], on the same page, will become

$$Q = \frac{Q_2 f}{3\,F} \cdot \frac{R^3 + 3\,R^2 r + 3\,R\,r^2}{(R_1 + R + r)\,(R + r)},$$

or

$$(R + r)^3 - r^3 = \frac{3\,Q\,F\,(R_1 + R + r)\,(R + r)}{Q_2 f};$$

and putting

$$R + r = L, \text{ and } r = L - R,$$

we get

$$L^3 - (L - R)^3 = \frac{3\,Q\,F\,(R_1 + L)\,L}{Q_2 f};$$

therefore

$$(L - R)^3 = L^3 - \frac{3\,Q\,F\,(R_1 + L)\,L}{Q_2 f};$$

therefore

$$L - R = \sqrt[3]{L^3 - \frac{3\,Q\,F\,(R_1 + L)\,L}{Q_2 f}},$$

or

$$R = L - \sqrt[3]{L^3 - \frac{3\,Q\,F\,(R_1 + L)\,L}{Q_2 f}};$$

and by substituting a, b, c, g, and Q_3, in the manner shown on page 493, we get

$$R = L - \sqrt[3]{L^3 - \frac{3\,Q_3\,F\,[g\,(a + b) + a\,(b + c)]\,L}{b\,Q_2 f}}.$$

For example.

In localising a fracture in a submarine cable by the foregoing test, a and b were made 100 ohms each, and balance was obtained on g when c was adjusted to 700 ohms.

The resistance of the galvanometer was 5000 ohms, but when the discharge was noted, the $\frac{1}{10}$th shunt was inserted, therefore $g = \frac{5000}{10} = 500$ ohms.

The discharge deflection observed on depressing the key was 186 divisions, therefore $Q_3 = 116 \times 10 = 1860$. The discharge deflection Q_2 obtained from a condenser of 1 microfarad capacity (F) charged to the potential V was 120 divisions with the $\frac{1}{10}$th shunt, therefore $Q_2 = 120 \times 10 = 1200$. The capacity f of such a length of the cable as would have a conductor resistance of 1 ohm was ·03 microfarad, then

$$R = 700 - \sqrt[3]{700^3 - \frac{3 \times 1860 \times 1}{100 \times 1200 \times \cdot 03}[500(100 + 100) + 100(100 + 700)]700}$$

$$= 700 - 529 = 171 \text{ ohms}.$$

577. A great merit in the foregoing methods of testing for faults lies in the fact that the two cable measurements can be made almost simultaneously; thus the moment balance is obtained on g_1 by adjusting c, at that moment the key is depressed, and the discharge deflection Q_3 noted on the galvanometer g. The other measurement, viz. that from the condenser, can be made at leisure. Thus after c and Q_3 are obtained, the cable must be disconnected from the bridge, and a resistance equal to c be connected between B and F, the potential at the point B will then still be V, and further, the galvanometer g can be removed without altering this potential; the condenser and galvanometer must then be joined up in the manner shown by Fig. 142, page 313. The wires, however, which in the latter figure are shown as connected to the battery, must in the present case be connected to the points B and F, Fig. 198 (page 468); then the discharge obtained, multiplied by the shunt (if one is employed), gives Q_2.

578. Should earth currents be present when the test is about to be made, they may be neutralised in the manner explained on page 293, § 309, in Chapter IX., and also at the end of the last chapter (§ 571, page 495).

579. With reference to the foregoing test it should be mentioned that Mr. J. Gott states that it is often possible to increase the resistance of the fault at the end of a broken cable to such an extent that practically the whole of the discharge may be obtained at the nearer end. For this purpose the charging battery should be of from 7 to 10 volts electromotive force, the zinc pole being connected to earth; the battery should be applied to the cable for some time before taking the discharge. The lower the resistance of the galvanometer consistent with a sufficiently high figure of merit (page 85), the better, as must be obvious.

CHAPTER XXV.

GALVANOMETER RESISTANCE.

580. THE question of what resistance a galvanometer should have in order that its figure of merit (page 85) may be high, involves several points, such as the "shape of the coil," "the diameter of the wire," &c. The determination of all these points, however, would be more useful for the purpose of finding what are the most economical conditions under which a galvanometer can be made, than (what is more to the purpose of the practical electrician) for showing how any particular galvanometer can be arranged so as to enable any particular test to be made with accuracy.

The problem we have to solve in the latter case is as follows:—Having given a galvanometer with a coil of a certain size, should thin or thick wire be on it in order that any particular test may be made under the most favourable conditions? Or supposing the coil to be divided into several sections, how should the latter be coupled up?

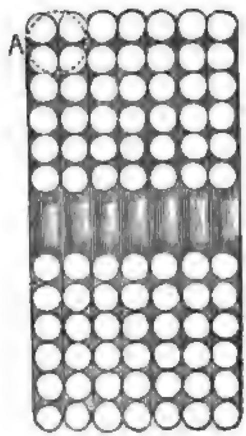


FIG. 199.

Referring to Fig. 199, which represents a section of a galvanometer coil, let us direct our attention to the 4 turns of wire at A. If these 4 turns be joined up in one continuous length, then calling the resistance of each turn 4, their total resistance will be 4×4, or 4^2. If, however, the 4 turns be coupled up for "quantity," then their joint resistance will be 1.

If we suppose the *total* current flowing to be constant, then in the case where the 4 wires are joined up in one continuous length, the current makes 4 turns round the needle of the galvanometer, its effect will therefore be equal to 4; but in the second case, where the turns of wire are coupled up for "quantity," the same

current only makes 1 turn round the needle, hence its effect can only be equal to 1.

If instead of 4 turns we have 9 turns, then the relative values of the resistances when joined up in one continuous length, and when joined up for "quantity," will be as 1 to 9×9, or 9^2, whilst the relative effect of the current on the galvanometer needle will be as 1 to 9.

In the first case then, where the resistance was reduced 4^2 times, the effect on the needle was only reduced 4 times; and in the second case, where the resistance was reduced 9^2 times, the effect was only reduced 9 times; or, in other words, the effect varied directly as the *square root* of the resistance, consequently for the whole of the galvanometer the effect varies directly as the square root of its resistance.

If we replace the 4 wires at A by a solid wire of twice their diameter, then this wire (shown by the dotted lines) will have the same resistance as these 4 wires coupled up for "quantity," and its influence on the magnetic needle will be very nearly the same. As a matter of fact, the effect will be rather less, in consequence of the metal being differently distributed over the area which the 4 wires occupy. But inasmuch as the silk covering with which the wires are insulated is practically of the same thickness for large as for small wires, therefore if the thick wire were wound on the coil, the sectional area of that wire would actually be rather larger than the area of the small wires which it takes the place of, consequently we may without any considerable error say that the effect varies directly as $\sqrt{g}$.

581. This fact enables us to determine what should be the resistance of the galvanometer in order that any particular test may be made under the best possible conditions. Let us take the case of the Wheatstone Bridge.

On page 216 we obtained an equation which gave the strength of the current flowing through the galvanometer when equilibrium was very nearly produced, viz.:

$$c_6 = \frac{E\,x\,(a\,d_1 - b\,x)}{\{g\,(a + x) + a\,(d + x)\}\;\{r\,(d + x) + d\,(a + x)\}}.$$

This equation may be written

$$c_6 = \frac{1}{\left\{g + \dfrac{a\,(d + x)}{a + x}\right\}} \times \frac{E\,x\,(a\,d_1 - b\,x)}{(a + x)\;\{r\,(d + x) + d\,(a + x)\}}$$

$$= \frac{1}{(g + k)} \times K.$$

We have shown that the effect of the galvanometer coil on the needle varies directly as the square root of the resistance of the former. Its effect must also vary directly as the current passing through the coils, consequently the total effect M will be

$$M = \frac{\sqrt{g}}{(g+k)} \times K\kappa = \frac{K\kappa}{\sqrt{g} + \frac{k}{\sqrt{g}}}$$

where κ is a constant dependent upon the shape of the coil, the magnetic strength of the needle, &c.

We have to find what value of g will make M as *large* as possible, and this we shall do, since $K\kappa$ is constant, by finding what value of g will make $\sqrt{g} + \frac{k}{\sqrt{g}}$ as *small* as possible.

Now

$$\sqrt{g} + \frac{k}{\sqrt{g}} = \left(\sqrt[4]{g} - \frac{\sqrt{k}}{\sqrt[4]{g}}\right)^2 + 2\sqrt{k},$$

and this will be made a minimum by making $\sqrt{g} - \frac{\sqrt{k}}{\sqrt[4]{g}}$ a minimum, that is, by making

$$\sqrt[4]{g} - \frac{\sqrt{k}}{\sqrt[4]{g}} = 0,$$

therefore

$$\sqrt{g} = \sqrt{k}, \quad \text{or,} \quad g = k;$$

but

$$k = \frac{a(d+x)}{a+x}$$

and $\frac{a(d+x)}{a+x}$ is the same as $\frac{(a+b)(d+x)}{a+b+d+x}$, when $bx = ad$, and this expression is the joint resistance of the resistances on either side of the galvanometer; theoretically therefore we should make g equal to this quantity if we wish M to be as large as possible.

This rule, however, although it shows what value g should have in order to make M an absolute maximum, is one which cannot well be strictly followed out. We should rather seek to determine to what extent the exact rule may be departed from without seriously diminishing M.

Let us suppose g to be n times k, then we have

$$M = \frac{\sqrt{n\,k}}{n\,k + k} \times K\,\kappa = \frac{\sqrt{n}}{n + 1} \times \frac{K\,\kappa}{\sqrt{k}};$$

for an absolute maximum $n = 1$, that is

$$M = \frac{1}{2} \times \frac{K\,\kappa}{\sqrt{k}}.$$

Suppose, now, we make g nine times as large as k, that is, make $n = 9$, then we have

$$M = \frac{\sqrt{9}}{9 + 1} \times \frac{K\,\kappa}{\sqrt{k}} = \frac{1}{3\cdot 3} \times \frac{K\,\kappa}{\sqrt{k}}.$$

In other words, although g is *nine* times as great as it should be for making M a maximum, yet M has only been reduced from $\frac{1}{2}$ down to $\frac{1}{3\cdot 3}$. Or, to put it in another way: supposing we were making a bridge test, employing a galvanometer of the exact theoretical value for obtaining a maximum deflection, and supposing that having nearly obtained equilibrium, the deflection of the galvanometer needle was 3·3 divisions, then, if the resistance of the galvanometer had been 9 times the theoretical value, the deflection would only have been reduced down to 2 divisions.

It must therefore be evident that, unless we employ a galvanometer whose resistance *very* much exceeds the theoretical value, this resistance will practically be the one required. If it is necessary to draw a limit, we may say—avoid making the resistance more than 10 times as great (or as small, as can also be shown) as the theoretical value.

582. It will be found that in all tests in which g has a particular best value, an equation of the form

$$M = \frac{\sqrt{g}}{g + k} \times K\,\kappa$$

can be obtained. k in fact is in reality the resistance external to the galvanometer, so that we have simply to find what this resistance is, and then make g as nearly as possible equal to it.

CHAPTER XXVI.

SPECIFICATION FOR MANUFACTURE OF CABLE—SYSTEM OF TESTING CABLE DURING MANUFACTURE.

583. As soon as the laying of a new cable has been decided upon, and the route which it is to take has been selected, &c., the manufacture has to be commenced. The choice of the types of cable to be adopted, the lengths of the "shore ends," "intermediate," and "deep sea" sections are purely matters of experience and discretion with the engineers in charge of the work, and no satisfactory rules for general guidance can be laid down.

When the description of cable has been settled upon, a specification has to be drawn up, of which the following is a general specimen.

584. The ________________ Telegraph Company and

________________ Telegraph Works.

Contract Specification for the manufacture of the Submarine Telegraph Cable of the ________________ Telegraph Company, to be laid between the coast of ________________, near ________, and the Island of ________.

The following lengths of cable will be required:—

Actual distance, 480 knots (each being 2029 yards), or, including 10 per cent. slack,* 528 knots.

A. Main cable	.. ..	500 knots
B. Intermediate cable	.. ..	11 "
C. Shore-end cable	.. ..	17 "

Core.

The core of the entire length of cable to be as follows:—

Conductor.—The conductor of each coil to be formed of a strand of seven copper wires all of equal diameter and of a total weight

* The amount of slack required will vary with the length of the cable and with the depth of water in which it is laid.

of 107 lbs. per nautical mile, and shall at a temperature of 75° F. have a resistance not higher than 11·145 standard ohms per nautical mile; and the weight per knot multiplied by the resistance per knot at 75° F. shall not exceed 1192·6.*

Insulator or Dielectric.—The conductor of each coil shall be insulated by being covered with three alternate layers of Chatterton's compound and gutta-percha, beginning with a layer of the said compound, and no more compound shall be used than may be necessary to secure adhesion between the conductor and the layers of gutta-percha. The dielectric on the conductor of each coil shall weigh 150 lbs. per nautical mile, making the total weight of the conductor of each coil, when covered with the dielectric, 257 lbs. per nautical mile.

Inductive Capacity.—The inductive capacity of each coil of such insulated conductor shall not exceed ·3333 microfarad per nautical mile, and this shall equally apply to the completed cable.

Labelling.—Each coil of core before being placed in the temperature-tank for testing shall be carefully labelled with the exact length of conductor and the exact weight of copper and dielectric respectively which it contains.

Insulation Resistance.—The insulation resistance of each coil of core, after such coil shall have been kept in water maintained at a temperature of 75° F. for not less than 24 consecutive hours immediately preceding the test, shall be not less than 500 megohms per nautical mile, when tested at that actual temperature and after electrification during one minute. The electrification to be perfectly steady.

Preservation.—The core during manufacture to be carefully protected from sun and heat, and kept under water.

Joints.—Every joint to be tested by accumulation, and the leakage from any joint during one minute not to be more than double that from an equal length of the perfect core. Notice to be given to the inspecting officer of the company when a joint is about to be made, so that he may be able to test it.

Serving and Sheathing.

Main Cable A.

Serving.—The insulated conductor to be served with the best wet-tanned Russian hemp to receive the sheathing as specified, and to be then kept in tanned water and not allowed to be out of water more than is necessary to feed the closing machine.

* Corresponding to 99 per cent. of pure copper; see Table II.

Sheathing.—The served core to be sheathed with fifteen galvanized iron wires, each ·120 of an inch in diameter.

The lay to be 10 inches, no loose threads of hemp to be run through the closing machine, and no weld in any one iron wire to be within six feet of a weld in any other wire. The sheathing core to be finally covered with three coatings of an approved bituminous compound, a serving of tarred yarn made from the best Russian hemp being placed between each layer of compound, each serving of yarn being laid on in contrary directions.

Intermediate Cable B.

Serving to be similar in every respect to that on the Main Cable A.

Sheathing to be generally similar to that specified for the Main Cable A, but the iron covering to consist of ten galvanised iron wires, each ·180 of an inch in diameter. The lay to be 10 inches.

Shore-end Cable C.

The shore-end cable to consist of Cable A complete, and further well served with the best wet-tanned Russian hemp, and then sheathed with twelve galvanised iron wires, ·300 of an inch in diameter.

The lay to be 17 inches, no loose threads of hemp to be run through the closing machine, and no weld in any one iron wire to be within six feet of a weld in any other wire. The sheathed core to be finally covered with three coatings of an approved bituminous compound, a serving of tarred yarn made from the best Russian long dressed hemp being placed between each layer of compound, each serving of yarn being laid on in contrary directions.

The completed cable as fast as it is made, to be passed into a tank of water and kept covered with water until shipped. A correct indicator to be attached to the closing machine, and the length of cable to be marked as agreed.

Quality of Materials.

The wire used in the Main Cable A to be of the best quality of homogeneous wire, well and smoothly galvanised, and having a tensile strength of 50 tons per square inch area, and 850 lbs. as a minimum breaking strain on a length of 12 inches between the

clamps. The wire must elongate not less than $\frac{3}{4}$ per cent. before breaking. It shall bend round itself and unbend without breaking. The joints in the homogeneous wires to be of the form decided upon by the company's and contractor's engineers, and, as far as practicable, no one joint to be within six feet of any other joint.

The iron wire to be used in Cables B and C is to be of the quality known as Best Best, free from inequalities, well and smoothly galvanised, and annealed, and having a tensile strength of 25 tons per square inch of area. A margin of 5 per cent. will be allowed in weight, provided the average weight is as specified above. The wire for Cables B and C to be capable of being bent round a cylinder four times its own diameter and unbent without breaking. No wire of brittle quality shall be put into the cables, and the engineers or their assistants shall have power to reject any hanks which break frequently in the closing machine, or are of unsatisfactory quality. No weld shall be made in the B and C cables within six feet of any other weld.

The galvanising of the iron to bear four dips of one minute each in a solution of one part by weight of sulphate of copper and five parts of water.

Before being used for the sheathing of the cable, the wire shall be heated in a kiln or oven just sufficient to drive off all moisture, and whilst warm shall be dipped into pure hot gas-tar (freed from naphtha). The wire so dipped shall not be used for sheathing the cable until the coating of gas-tar is throughly set.

Each intermediate cable to be finished off with suitable tapers to be arranged to the satisfaction of the engineer of the company.

Testing Accommodation.

A proper room and all necessary batteries and leading wires to be provided for testing the cable during the whole manufacture.

Inspection.

The engineer of the company or his agents to have access to the works for inspecting and testing cable and all materials employed, and may reject all materials which are unsatisfactory.

Penalty.

The whole of the cable to be completed on or before the time stated in the tender under a penalty of _______________ per cent.

on the price for each day, or fraction of a day, after the said time until the day the cable may be actually completed and ready for shipment.

The manufacture may not be carried on at night without the written consent of the engineer of the company or his agent.

The cable ship or ships are not to leave the wharf with cable on board until the cable has been thoroughly tested in all respects, by the engineers, or their assistants, from the shore, and ample time after the shipment of the last mile to be allowed for this purpose.

System of Testing Cable during Manufacture.

585. The tests made by the cable manufacturers, although systematic, are not as a rule quite so exact or lengthy as those made by the electrician representing the company for whom the cable is being made. The cable once manufactured, passes out of the hands of the manufacturer, and the latter has no further interest in the matter; whereas the company may require at any time to localise a fault, and the more precise the data they possess the more closely will they be able to determine the position of the defect. Besides, when a large number of cables are being made at once at the factory it would be impossible, without a very large staff, to make an elaborate series of tests for each cable; whereas these can easily be made by the electrician and his assistants when there is only one cable to look after.

The methods of working out the tests, and the forms employed for entering down the same, depend upon the individual opinion of the electrician in charge of the work, but the following will give a general idea of the course to be pursued:—

Tests of the Coils.

586. The core of the cable is usually made in 2-knot lengths approximately, which are coiled upon wooden drums as manufactured, and then placed in tanks of water heated to a temperature of 75° F. to be tested.*

After being placed in the tank, the coils should remain there

* At the works of Messrs. Siemens & Co., Charlton, the coils are tested at two different temperatures, viz. at 75° and 50° F.

for at least twenty-four hours, so that they may acquire throughout their mass the necessary uniform temperature. At the end of this time the tests may be taken.

Sheets A, B, C, and D are employed for entering all the details of the tests as they are made. The working out of the tests of the coils (and cable) is shown on corresponding pages.

The figures given are such as are often obtained in actual practice. The insulation resistances of the coils are very often considerably higher than those shown, but this depends upon the time which elapses after the manufacture, and also upon the quality of the gutta-percha.

(A)

THE ______________ TELEGRAPH COMPANY.

DETAILS OF COILS FORMING THE CORE OF ______________ CABLE.

Date.	No. of Coils.	Length of Coils.		Total Weight.			Weight per Knot.			Difference from Contract Weight in lbs. per Knot.				Remarks.
										Copper.		Gutta-percha.		
		In Yards.	In Knots.	Copper.	Gutta-percha.	Total.	Copper.	Gutta-percha.	Total.	+	−	+	−	
				lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	
1899. April 3	1	4047	1·9946	213	298	511	106·79	149·40	256·19	..	·21	..	·60	
„	2	4073	2·0074	214	302	516	106·60	150·44	257·04	..	·40	·44	..	
„	3	4072	2·0069	215	304	519	107·13	151·48	258·61	·13	..	1·48	..	
„	4	4056	1·9990	214	299	513	107·05	149·57	256·62	·05	..	..	·43	
„	5	4056	1·9990	212	296	508	106·05	148·07	254·12	..	·95	..	1·93	

Signature ______________

CALCULATIONS FOR SHEET (A).

April 3rd.

Copper.

No. 1 Coil.

$$\begin{array}{lr} \log\ 213 = & 2 \cdot 3283796 \\ \log\ 1 \cdot 9946 = & \cdot 2998558 \\ \hline & 2 \cdot 0285238 \end{array}$$

$= \log$ of $\underline{106 \cdot 79}$

No. 2 Coil.

$$\begin{array}{lr} \log\ 214 = & 2 \cdot 3304138 \\ \log\ 2 \cdot 0074 = & \cdot 3026339 \\ \hline & 2 \cdot 0277799 \end{array}$$

$= \log$ of $\underline{106 \cdot 60}$

No. 3 Coil.

$$\begin{array}{lr} \log\ 215 = & 2 \cdot 3324385 \\ \log\ 2 \cdot 0069 = & \cdot 3025257 \\ \hline & 2 \cdot 0299128 \end{array}$$

$= \log$ of $\underline{107 \cdot 13}$

No. 4 Coil.

$$\begin{array}{lr} \log\ 214 = & 2 \cdot 3304138 \\ \log\ 1 \cdot 9990 = & \cdot 3008128 \\ \hline & 2 \cdot 0296010 \end{array}$$

$= \log$ of $\underline{107 \cdot 05}$

No. 5 Coil.

$$\begin{array}{lr} \log\ 212 = & 2 \cdot 3263359 \\ \log\ 1 \cdot 9990 = & \cdot 3008128 \\ \hline & 2 \cdot 0255231 \end{array}$$

log of $\underline{106 \cdot 05}$

Gutta-percha.

No. 1 Coil.

$$\begin{array}{lr} \log\ 298 = & 2 \cdot 4742163 \\ \log\ 1 \cdot 9946 = & \cdot 2998558 \\ \hline & 2 \cdot 1743605 \end{array}$$

$= \log$ of $\underline{149 \cdot 40}$

No. 2 Coil.

$$\begin{array}{lr} \log\ 302 = & 2 \cdot 4800069 \\ \log\ 2 \cdot 0074 = & \cdot 3026339 \\ \hline & 2 \cdot 1773730 \end{array}$$

$= \log$ of $\underline{150 \cdot 44}$

No. 3 Coil.

$$\begin{array}{lr} \log\ 304 = & 2 \cdot 4828736 \\ \log\ 2 \cdot 0069 = & \cdot 3025257 \\ \hline & 2 \cdot 1803479 \end{array}$$

$= \log$ of $\underline{151 \cdot 48}$

No. 4 Coil.

$$\begin{array}{lr} \log\ 299 = & 2 \cdot 4756712 \\ \log\ 1 \cdot 9990 = & \cdot 3008128 \\ \hline & 2 \cdot 1748584 \end{array}$$

$= \log$ of $\underline{149 \cdot 57}$

No. 5 Coil.

$$\begin{array}{lr} \log\ 296 = & 2 \cdot 4712917 \\ \log\ 1 \cdot 9990 = & \cdot 3008128 \\ \hline & 2 \cdot 0174789 \end{array}$$

$= \log$ of $\underline{148 \cdot 07}$

(B)

THE ______________________ TELEGRAPH COMPANY.

CONDUCTOR RESISTANCE TESTS OF COILS AT 75° FAHR.

Date.	No. of Coils.	Length of Coils.	Resistance of Leads.	Total Resistance of Conductor and Leads.	Total Resistance of Conductor.	Resistance per Knot of Conductor.	Percentage of Conductivity compared with Pure Copper.	Remarks.
1899.		knots.	ohms.	ohms.	ohms.	ohms.		
April 3	1	1·9946	1·46	23·56	22·10	11·08	99·8	
„	2	2·0074	„	23·48	22·02	10·97	100·9	
„	3	2·0069	„	23·51	22·05	10·99	100·3	
„	4	1·9990	„	23·66	22·20	11·10	99·3	
„	5	1·9990	„	23·82	22·36	11·18	99·5	

Signature ______________________

Calculations for Sheet (B).

April 3rd.

Conductor Resistance.

No. 1 Coil.

log 22·10 = 1·3443923
log 1·9946 = ·2998558

1·0445365 = log of 11·08
log 106·79 = 2·0285238

3·0730603 = log of 1183·2
= 99·8 per cent. pure copper *

No. 2 Coil.

log 22·02 = 1·3428173
log 2·0074 = ·3026339

1·0401834 = log of 10·97
log 106·60 = 2·0277799

3·0679633 = log of 1169·4
= 100·9 per cent. pure copper *

No. 3 Coil.

log 22·05 = 1·3434086
log 2·0069 = ·3025257

1·0408829 = log of 10·99
log 107·13 = 2·0299128

3·0707957 = log of 1177·1
= 100·3 per cent. pure copper *

No. 4 Coil.

log 22·20 = 1·3463530
log 1·9990 = ·3008128

1·0455402 = log of 11·10
log 107·05 = 2·0296010

3·0751412 = log of 1188·9
= 99·3 per cent. pure copper *

No. 5 Coil.

log 22·36 = 1·3494718
log 1·9990 = ·3008128

1·0486590 = log of 11·18
log 106·05 = 2·0255231

3·0741821 = log of 1186·3
= 99·5 per cent. pure copper *

---

* Table II. See also page 459, § 522.

(C)

THE ______________ TELEGRAPH COMPANY.

INDUCTIVE CAPACITY TESTS OF COILS AT 75° FAHR.

Date.	No. of Coil.	Length of Coil.	Resistance of Galvanometer.	Condenser. ⅓ Microfarad. Shunt.	Condenser. ⅓ Microfarad. Discharge	Coils. Shunt.	Coils. Immediate Discharge after 10 secs. Electrification (lead deducted).	Coils. Total Capacity.	Coils. Capacity per Knot.	Remarks.
1899. April 3	1	knots. 1·9946	ohms. 5460	$\frac{1}{10}$th	divisions 151	ohms. 330	divisions. 167·5	microfarads. ·6488	microfarads. ·3253	
„	2	2·0074	„	„	„	„	169·5	·6565	·3270	
„	3	2·0069	„	„	„	„	169·5	·6565	·3271	
„	4	1·9990	„	„	„	„	171·5	·6642	·3323	
„	5	1·9990	„	„	„	„	168·5	·6526	·3265	

Signature ______________

CALCULATIONS FOR SHEET (C).

April 3rd.

Inductive Capacity.

log 3 = ·4771213
log 1510 = 3·1789769
log 330 = 2·5185139

6·1746121
log 5790 = 3·7626786

2·4119335

$$\frac{G+S}{S} = \frac{5460+330}{330} = \frac{5790}{330}$$

No. 1 Coil.

log 167·5 = 2·2240148
2·4119335

$\bar{1}$·8120813 = log of ·6488
log 1·9946 = ·2998558

$\bar{1}$·5122255 = log of ·3253

No. 2 Coil.

log 169·5 = 2·2291697
2·4119335

$\bar{1}$·8172362 = log of ·6565
log 2·0074 = ·3026339

$\bar{1}$·5146023 = log of ·3270

No. 3 Coil.

log 169·5 = 2·2291697
2·4119335

$\bar{1}$·8172362 = log of ·6565
log 2·0069 = ·3025257

·5147105 = log of ·3271

No. 4 Coil.

log 171·5 = 2·2342641
2·4119335

$\bar{1}$·8223306 = log of ·6642
log 1·9990 = ·3008128

$\bar{1}$·5215178 = log of ·3323

No. 5 Coil.

log 168·5 = 2·2265999
2·4119335

$\bar{1}$·8146664 = log of ·6526
log 1·9990 = ·3008128

$\bar{1}$·5138536 = log of ·3265

(D)

THE ______________________ TELEGRAPH COMPANY.

INSULATION TESTS OF COILS AT 75° FAHR.

Date.	No. of Coil.	Length of Coil.	Resistance of Galvanometer.	Constant.			Coils.						
				Shunt.	Deflection from Battery through 1 megohm.	Value of Constant.	Shunt.	Deflection after Electrification (lead deducted). 1 min.	Deflection after Electrification (lead deducted). 2 min.	Percentage of Electrification during 2 min.	Total Resistance after 1 min. Electrification.	Resistance per knot.	Remarks.
1899 April 3	1	..	ohms. 5460	$\frac{1}{1000}$th	divisions. 152	64271	ohms. 4000	divisions. 148	divisions. 139	6·08	megs. 434·3	megs. 866·2	
,,	2	..	,,	,,	,,	,,	,,	142	132	7·42	452·6	908·6	
,,	3	..	,,	,,	,,	,,	,,	144·5	136	5·54	444·8	892·6	
,,	4	..	,,	,,	,,	,,	,,	140·5	133·5	4·98	457·4	914·4	
,,	5	..	,,	,,	,,	,,	,,	138	129·5	5·80	465·7	931·0	

Signature ______________________

Calculations for Sheet (D).

April 6th.

Insulation Resistance.

$$
\begin{array}{rl}
\log 152{,}000 = & 5\cdot1818436 \\
\log 4000 = & 3\cdot6020600 \\
\hline
 & 8\cdot7839036 \\
\log 9460 = & 3\cdot9758911 \\
\hline
 & 4\cdot8080125 = \log \text{ of } \underline{64271}
\end{array}
\qquad
\frac{5460 + 4000}{4000} = \frac{9460}{4000}
$$

No. 1 Coil.

$$
\begin{array}{rl}
 & 4\cdot8080125 \\
\log 148 = & 2\cdot1702617 \\
\hline
 & 2\cdot6377508 = \log \text{ of } \underline{434\cdot3} \\
\log 1\cdot9946 = & \cdot2998558 \\
\hline
 & 2\cdot9376066 = \log \text{ of } \underline{866\cdot2}
\end{array}
$$

No. 2 Coil.

$$
\begin{array}{rl}
 & 4\cdot8080125 \\
\log 142 = & 2\cdot1522883 \\
\hline
 & 2\cdot6557242 = \log \text{ of } \underline{452\cdot6} \\
\log 2\cdot0074 = & \cdot3026339 \\
\hline
 & 2\cdot9583581 = \log \text{ of } \underline{908\cdot6}
\end{array}
$$

No. 3 Coil.

$$
\begin{array}{rl}
 & 4\cdot8080125 \\
\log 144\cdot5 = & 2\cdot1598678 \\
\hline
 & 2\cdot6481447 = \log \text{ of } \underline{444\cdot8} \\
\log 2\cdot0069 = & \cdot3025257 \\
\hline
 & 2\cdot9506704 = \log \text{ of } \underline{892\cdot6}
\end{array}
$$

No. 4 Coil.

$$
\begin{array}{rl}
 & 4\cdot8080125 \\
\log 140\cdot5 = & 2\cdot1476763 \\
\hline
 & 2\cdot6603362 = \log \text{ of } \underline{457\cdot4} \\
\log 1\cdot9990 = & \cdot3008128 \\
\hline
 & 2\cdot9611490 = \log \text{ of } \underline{914\cdot4}
\end{array}
$$

No. 5 Coil.

$$
\begin{array}{rl}
 & 4\cdot8080125 \\
\log 138 = & 2\cdot1398791 \\
\hline
 & 2\cdot6681334 = \log \text{ of } \underline{465\cdot7} \\
\log 1\cdot9990 = & \cdot3008128 \\
\hline
 & 2\cdot9689462 = \log \text{ of } \underline{931\cdot0}
\end{array}
$$

Calculations for Sheet (D)—*continued.*

Percentage of Electrification.

No. 1 Coil.

148
139

$$\begin{aligned} \log\ 9 &= \cdot 9542425 \\ \log\ 100 &= 2\cdot \\ \hline &\ \ 2\cdot 9542425 \\ \log\ 148 &= 2\cdot 1702617 \\ \hline &\ \ \cdot 7839808 \\ &= \log \text{ of } \underline{6\cdot 08} \end{aligned}$$

No. 2 Coil.

142
132

$$\begin{aligned} \log\ 10 &= 1\cdot 0000000 \\ \log\ 100 &= 2\cdot \\ \hline &\ \ 3\cdot 0000000 \\ \log\ 142 &= 2\cdot 1522883 \\ \hline &\ \ \cdot 8477117 \\ &= \log \text{ of } \underline{7\cdot 42} \end{aligned}$$

No. 3 Coil.

144·5
136

$$\begin{aligned} \log\ 8\cdot 5 &= \cdot 9030900 \\ \log\ 100 &= 2\cdot \\ \hline &\ \ 2\cdot 9030900 \\ \log\ 144\cdot 5 &= 2\cdot 1598678 \\ \hline &\ \ \cdot 7432222 \\ &= \log \text{ of } \underline{5\cdot 54} \end{aligned}$$

No. 4 Coil.

140·5
133·5

$$\begin{aligned} \log\ 7\cdot 0 &= \cdot 8450980 \\ \log\ 100 &= 2\cdot \\ \hline &\ \ 2\cdot 8450980 \\ \log\ 140\cdot 5 &= 2\cdot 1476763 \\ \hline &\ \ \cdot 6974217 \\ &= \log \text{ of } \underline{4\cdot 98} \end{aligned}$$

No. 5 Coil.

138
129·5

$$\begin{aligned} \log\ 8\cdot 5 &= \cdot 9030900 \\ \log\ 100 &= 2\cdot \\ \hline &\ \ 2\cdot 9030900 \\ \log\ 138 &= 2\cdot 1398791 \\ \hline &\ \ \cdot 7631209 \\ &= \log \text{ of } \underline{5\cdot 80} \end{aligned}$$

Tests of the Cable.

587. As soon as one or more coils have been tested, the manufacture of the cable is commenced; and as each coil is passed through the covering or "closing" machine, another is jointed on, the joint being made at such a time that at least twenty-four hours can elapse between the making and testing of the same. To ensure this necessary time intervening, as soon as one joint is passed through the closing machine the next should be made, so that there is a length of two knots of coil to be sheathed before the new joint is reached.

The system of testing joints has been described in Chapter XIX. A form for entering the results of the tests is shown by Sheet E.

In making a joint it is necessary to cut off a certain length from each coil. The amount of this length varies according to circumstances, but it is seldom more than a few yards.

The order in which the coils are jointed together does not always correspond to the order in which they are tested at 75°, and therefore it is necessary to note down their consecutive order in a column provided on the test sheets for the purpose. In the case of a fault occurring in the cable, this information is of use in enabling an accurate measurement to be made.

Sheets F, G, H, I, and J, show the system of entering the tests as they are taken each day. The method of working out and entering the results will be understood from the examples given.

Sheet J shows the values which the insulation, &c., of the cable would have if no change took place during the course of manufacture; the insulation values, therefore, compared with the measured results during manufacture, indicate whether (as should be the case) the insulation is improving. The conductor resistance values are used for the purpose of calculating the temperature of the cable (page 476, § 554).

With reference to the 4th, 5th, 6th, and 7th columns on Sheet J, as has been explained on page 260, § 275, the joint insulation resistance of a number of wires is equal to the reciprocal of the sum of the reciprocals of their respective insulation resistances. The 5th column contains, therefore, the reciprocals * of the values in the 4th column. These reciprocals are added together, and the results noted in the 6th column: the reciprocals of these numbers give the values in the 7th column.

* These are best obtained from tables (Barlow's are generally used).

588. When a cable is of a considerable length it is usual, in order to save time, to manufacture the same in several lengths or "sections," so that several machines can be running at the same time. When the sections are completed they are spliced together so as to form one continuous length. The examples of tests given represent the tests of one section of the Main Cable.

Final Tests.

589. On the completion of the cable special tests for insulation (page 411, § 459) are made.

The general method of recording these special insulation and other tests, is shown on page 536, by Sheet K.

590. In the case of a cable whose core is insulated with gutta-percha the insulation goes on improving during the course of manufacture, so that the final test will show (or ought to show) the "Resistance per knot reduced to 75°" (see bottom of Sheet K) to be greater than the "Estimated resistance per knot from tests of coils at 75°" (Sheet J, col. 8, and bottom of Sheet K). This however, it should be mentioned, is not the case when the cor employed is insulated with indiarubber; in this case the insu lation almost always (if not always) falls to some extent as the manufacture proceeds, and this fall must not be assumed to be an indication of any deterioration taking place.

591. Most of the calculations can be done with great facility and advantage by means of a "Fuller's" slide-rule, instead of by logarithms, as in the examples given.

(E)

THE ______________________ TELEGRAPH COMPANY.

SECTION A.—MAIN CABLE.

JOINT TESTS OF COILS FOR ______________ CABLE.

Con-secutive Order of Coils.	Joint Made.		Joint Tested.		Time elapsing between Making and Testing Joint.	Leakage from Trough.			Solid Core.			Joint.			Remarks.
	Date.	Time.	Date.	Time.		Full Potential.	Reduced Potential after 1 minute.	Percentage of Loss.	Induced Discharge.	Accumulation by Leakage.		Induced Discharge.	Accumulation by Leakage.		Number of Cells for Testing Joint, 500 Leclanchés.
										1st min.	2nd min.		1st min.	2nd min.	
	1899.	P.M.		P.M.	hours.	divisions.	divisions.		divisions.	divisions.		divisions.	divisions.		
1 } 2	Apr. 5	2·0	Apr. 6	4·0	26	200	194	3·0	150	8	15	145	10	20	
2 } 7	„ 6	3·30	„ 7	5·0	26½	202	196	3·0	147	9	16	145	13	22	

Signature ______________________

Part of Calculations for Sheet (F).

Length Manufactured.

April 8th.

$$
\begin{array}{rl}
\log 1404 & = 3\cdot1473671 \\
\log 274\cdot25 & = 2\cdot4381466 \\
\hline
 & \cdot7092205 = \log \text{ of } 5\cdot1194
\end{array}
\qquad
\begin{array}{r}
5\cdot9926 \\
5\cdot1194 \\
\hline
\cdot8732
\end{array}
$$

Part of Calculations for Sheet (G).

Estimated Temperature.

$$
\begin{array}{rl}
\log 66\cdot26 & = 1\cdot8212514 \\
\log 63\cdot90 & = 1\cdot8055009 \\
\hline
 & \cdot0157505 = \log. \text{ of } 1\cdot0369 \\
 & \qquad = \text{coeff. for } 58\tfrac{1}{2}.^{*}
\end{array}
$$

Part of Calculations for Sheet (H).

Inductive Capacity.

$$
\begin{array}{rl}
\log 3 & = \cdot4771213 \\
\log 152 & = 3\cdot1818436 \\
\log 100 & = 2\cdot0000000 \\
\hline
 & 5\cdot6589649 \\
\log 5560 & = 3\cdot7450748 \\
\hline
 & 1\cdot9138901
\end{array}
\qquad
\frac{G+S}{S} = \frac{5460+100}{100} = \frac{5560}{100}
$$

$$
\begin{array}{rl}
\log 159\cdot5 & = 2\cdot2027607 \\
 & \ 1\cdot9138901 \\
\hline
 & \cdot2888706 = \log \text{ of } 1\cdot9448 \\
\log 5\cdot9926 & = \cdot7776153 \\
\hline
 & \bar{1}\cdot5112553 = \log \text{ of } \underline{\cdot3245}
\end{array}
$$

---

* Table IV.

Part of Calculations for Sheet (H)—*continued.*

Percentage of Loss.

$$
\begin{array}{lrl}
 & 159\cdot5 & \\
 & 129 & \\
\hline
\log & 30\cdot5 & = 1\cdot4842998 \\
\log & 100 & = 2\cdot \\
\hline
 & & \ 3\cdot4842998 \\
\log & 159\cdot5 & = 2\cdot2027607 \\
\hline
 & & \ 1\cdot2815391 = \log \text{ of } \underline{19\cdot1}
\end{array}
$$

Part of Calculations for Sheet (I).

Insulation Resistance.

April 8th.

$$
\begin{array}{lrl}
\log & 152{,}000 & = 5\cdot1818436 \\
\log & 8000 & = 3\cdot9030900 \\
\hline
 & & \ 9\cdot0849336 \\
\log & 13460 & = 4\cdot1290451 \\
\hline
 & & \ 4\cdot9558885 = \log \text{ of } \underline{90{,}342} \\
\log & 154 & = 2\cdot1875207 \\
\hline
 & & \ 2\cdot7683678 \\
\log \text{ coeff.}^* & 58\tfrac{1}{2}^\circ & = \ \cdot5448531 \\
\hline
 & & \ 2\cdot2235147 = \log \text{ of } \underline{1\cdot673} \\
\log & 5\cdot9926 & = \ \cdot7776153 \\
\hline
 & & \ 3\cdot0011300 = \log \text{ of } \underline{1003}
\end{array}
$$

$$\frac{5460 + 8000}{8000} = \frac{13460}{8000}$$

* Table V.

(F)

THE ________________________ TELEGRAPH COMPANY.

SECTION A.—MAIN CABLE.

DETAILS OF CONSECUTIVE ORDER OF COILS, LENGTH OF COMPLETED CABLE, ETC.

Date.	Consecutive Order of Coils.	Original Lengths of Coils.	Lengths cut off in making Joints.*	Corrected Lengths of Coils.		Length of Circuit.					Remarks.
						Total Length of Core in Circuit.		Revolutions of Drum. 274·25 Revolutions = 1 Knot.	Length of Cable Sheathed.	Length of Cable Unsheathed.	
		yards.	yards.	yards.	knots.	yards.	knots.		knots.	knots.	
1899. April 6	1	4047	..	..	..	4047	1·9946	286	1·0419	·9527	
	1	4047	7	4040	1·9911	..	..	..	..	..	
„ 7	2	4043	2	4041	2·0064	8111	3·9975	920	3·3546	·6429	
„ 8	7	4051	3	4048	1·9952	12159	5·9926	1404	5·1194	·8732	

Signature ________________________

* See note next page.

(G)

THE ______________________ TELEGRAPH COMPANY.

SECTION A.—MAIN CABLE.

CONTROL CONDUCTOR RESISTANCE TESTS OF CABLE.

Date.	Consecutive Order of Coils.	Total Length of Circuit.	Observed Temperature in Tanks.		Resistance of Leads.	Measured Total Resistance of Conductor and Leads.	Measured Total Resistance of Conductor.	Total Resistance of Cut Coils from Tests at 75° Fahr.*	Calculated Mean Temperature.	Remarks.
			Cable (sheathed).	Core (unsheathed).						
1899. April 6	No. 1	knots. 1·9946	deg. Fahr. 58	deg. Fahr. 63	ohms. 1·43	ohms. 23·85	ohms. 21·42	ohms. 22·10	deg. Fahr. 60	
„ 7	2	3·9975	58	62	1·42	43·99	42·57	44·07	58¼	
„ 8	7	5·9926	56	67	1·42	65·32	63·90	66·26	57¼	

Signature ______________________

* In the case of a Single Wire Cable the amount cut off from the original lengths of the coils when making a joint is usually insignificant, and does not require to be taken into account; but in the case of a Multiple Cable (that is, a cable with several cores) the coils have to be cut down to the length of the shortest coil, and then the resistances of the cut coils are estimated from the 75° tests by direct proportion—i.e. the original resistance is divided by the original length in yards and multiplied by the cut length in yards; it is assumed in the present case that it has been necessary to cut the coils, so that the method of correction may be explained.

(H)

THE ______________ TELEGRAPH COMPANY.

SECTION A.—MAIN CABLE.

CONTROL INDUCTIVE CAPACITY TESTS OF CABLE.

Date.	Consecutive Order of Coils.	Total Length of Circuit.	Resistance of Galvanometer.	Condenser. ½ Microfarad. Shunt.	Condenser. ½ Microfarad. Discharge.	Cable. Shunt.	Cable. Immediate Discharge after 10 secs. Electrification (lead deducted).	Cable. Discharge after 10 secs. Electrification and 60 secs. Insulation (lead deducted).	Cable. Percentage of Loss.*	Cable. Total Capacity.	Cable. Capacity per Knot.	Remarks.
		knots.	ohms.		divisions.	ohms.	divisions.	divisions.		m. farads.	m. farads.	
1899. April 6	1	1·9946	5450	$\frac{1}{10}$th	152	330	169	135	20·1	·6491	·3254	
„ 7	2	3·9975	„	„	151·5	160	„	136	19·5	1·3036	·3261	
„ 8	7	5·9926	5460	„	151	100	159·5	129·0	19·1	1·9448	·3245	

Signature ______________

* See p. 430, § 493.

(I)

THE ____________________ TELEGRAPH COMPANY.

SECTION A.—MAIN CABLE.

CONTROL INSULATION TESTS OF CABLE.

Date.	Consecutive Order of Coils.	Total Length of Circuit.	Resistance of Galvanometer.	Constant.			Cable.							Remarks.
				Shunt.	Deflection from Battery through 1 megohm.	Value of Constant.	Shunt.	Deflection after Electrification (Lead deducted). 1 min.	Deflection after Electrification (Lead deducted). 2 min.	Percentage of Electrification during 2nd min.	Total Resistance reduced to 75°.	Resistance per Knot reduced to 75°.		
1899.		knots	ohms.		divisions.		ohms.	divisions.	divisions.		megohms.	megohms.		
April 6	1	1·9946	5450	$\frac{1}{1000}$	154	154,000	∞	110	95·5	13·2	447·5	893		
„ 7	2	3·9975	5450	„	154	„	„	191	164·5	14·0	230·0	919		
„ 8	7	5·9926	5460	„	152	90,842	8000	154	134	13·0	167·3	1002		

Signature ____________________

(J)

THE ______________ TELEGRAPH COMPANY.

SECTION A.—MAIN CABLE.

ESTIMATED RESISTANCE AND CAPACITY OF CABLE FROM TESTS OF COILS AT 75°.

Date.	Consecutive Order of Coils.	Total Length of Circuit.	Insulation.					Capacity.			Conductor.		Remarks.
			Total of each Coil when cut for jointing.*	Reciprocals.	Sums of Reciprocals.	Total of Circuit.	Per Knot.	Total of each Coil when cut for jointing.*	Total of Circuit.	Per Knot.	Total of each Coil when cut for jointing.*	Total of Circuit.	
1899. April 6	1	1·9911	435·1	·002298322	..	..	..	·6477	..	..	22·06	..	
„ 7	2	3·9975	452·8	·002208481	·004506803	221·9	887·1	·6561	1·3038	·3254	22·01	44·07	
„ 8	7	5·9926	448·0	·002232143	·006738946	148·4	889·3	·6499	1·9537	·3260	22·19	66·26	

Signature ______________

* See note on page 532. In the case of the Insulation Resistance, the original resistance is *multiplied* by the original length and *divided* by the cut-length.

(K)

THE ________________ TELEGRAPH COMPANY.

MANUFACTURE OF ________________ SUBMARINE CABLE AT ________________
CABLE WORKS.

SECTION A.—MAIN CABLE.

FINAL TEST.

Length, 40·82 knots. *May 3rd,* 1899.

Conductor Resistance.

Total Observed.	Total of Coils at 75°.	Temperature. Observed.	Temperature. Calculated.	Per Mile from Coils at 75°.
483·93 ohms	447·55 ohms	$61\frac{1}{2}$° Fahr.	61° Fahr.	11·10

Inductive Capacity.

$\frac{1}{3}$ m.f. Condenser Dis.	Cable. Immediate.	Cable. After 1 min.	Percentage of Loss.	Total.	Per Knot.
151 × 10	$162 \times \frac{5520 + 15}{15}$	$144 \times \frac{5520 + 15}{15}$	11·1	13·196 m.f.	·3273 m.f.

Insulation Resistance.

Constant. Battery 300 Leclanchés. G = 5520.
Battery through 1 megohm, $S = \frac{1}{1000}$, 220 def. S. on Cable, 650 ohms.

Time.	Zinc to Line.	Earth Reading.	Copper to Line.	Earth Reading.
After 1 min.	267	82	300	66
„ 2 „	233	53	264	40
„ 3 „	219	42	250	28
„ 4 „	213	35	241	22
„ 5 „	207	30	234	18
„ 6 „	204		229	
„ 7 „	201		225	
„ 8 „	199		222	
„ 9 „	197		218	
„ 10 „	195		215	
„ 11 „	193		212	
„ 12 „	191		210	
„ 13 „	189		208	
„ 14 „	188		206	
„ 15 „	187		205	All readings steady.

Resistance per knot—at normal temp. at end of 1st min. Zinc to Line	3500 megs.
Do. reduced to 75°	1207 „
Do. from tests of Coils at 75°	838 „

	Zinc to Line.	Copper to Line.
Percentage of Electrification between 1st and 2nd min.	13·1	12·5
Do. 1st and 15th min.	43·5	48·5

Signature ________________

APPENDIX.

CHAPTER XXVII.

ELECTRICAL UNITS.

592. THE values of the electrical units as defined by the Board of Trade, are as follows :—

1. The *Ohm*, which has the value 10^9 in terms of the centimetre and the second of time, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14·4521 grammes in mass, of a constant cross sectional area, and of a length of 106·3 centimetres.

2. The *Ampère*, which has the value $\frac{1}{10}$th in terms of the centimetre, the gramme, and the second of time, and which is represented by the unvarying electric current which, when passed through a solution of nitrate of silver in water in accordance with the specification, deposits silver at the rate of 0·001118 of a gramme per second.

3. The *Volt*, which has the value 10^8 in terms of the centimetre, the gramme, and the second of time, being the electrical pressure that if steadily applied to a conductor whose resistance is 1 ohm will produce a current of 1 ampère, and which is represented by ·6974 ($\frac{1000}{1434}$) of the electrical pressure at a temperature of 15° C. between the poles of the voltaic cell known as Clark's cell set up in accordance with the specification (see page 160).

SPECIFICATION OF CURRENT VOLTAMETER.

In the following specification the term silver voltameter means the arrangement of apparatus by means of which an electric current is passed through a solution of nitrate of silver in water. The silver voltameter measures the total electrical quantity which has passed during the time of the experiment, and by noting this time the time average of the current, or if the current has been kept constant the current itself, can be deduced.

In employing the silver voltameter to measure currents of about 1 ampère the following arrangements should be adopted. The cathode on which the silver is to be deposited should take the form of a platinum bowl not less than 10 centimetres in diameter, and from 4 to 5 centimetres in depth.

The anode should be a plate of pure silver some 30 square centimetres in area and 2 or 3 millimetres in thickness.

This is supported horizontally in the liquid near the top of the solution by a platinum wire passed through holes in the plate at opposite corners. To prevent the disintegrated silver which is formed on the anode from falling on to the cathode, the anode should be wrapped round with pure filter paper, secured at the back with sealing wax.

The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the nitrate to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current, some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

Method of Making a Measurement.

The platinum bowl is washed with nitric acid and distilled water, dried by heat, and then left to cool in a desiccator. When thoroughly dry it is weighed carefully.

It is nearly filled with the solution, and connected to the rest of the circuit by being placed on a clean copper support, to which a binding screw is attached. This copper support must be insulated.

The anode is then immersed in the solution so as to be well covered by it and supported in that position; the connections to the rest of the circuit are made.

Contact is made at the key, noting the time of contact. The current is allowed to pass for not less than half-an-hour, and the time at which contact is broken is observed. Care must be taken that the clock used is keeping correct time during this interval.

The solution is now removed from the bowl, and the deposit is washed with distilled water and left to soak for at least six hours. It is then rinsed successively with distilled water and absolute alcohol, and dried in a hot-air bath at a temperature of about 160° C. After cooling in a desiccator it is weighed again. The gain in weight gives the silver deposited.

To find the current in ampères, this weight, expressed in grammes, must be divided by the number of seconds during which the current has been passed, and by 0·001118.

The result will be the time-average of the current, if during the interval the current has varied.

In determining by this method the constant of an instrument, the current should be kept as nearly constant as possible, and the readings of the instrument observed at frequent intervals of time. These observations give a curve from which the reading corresponding to the mean current (time-average of the current), can be found. The current, as calculated by the voltameter, corresponds to this reading.

To determine the True Insulation and Conductor Resistances of a Uniformly Insulated Telegraph Line.

593. On page 7 (§ 11) it was pointed out that the rule of multiplying the total insulation by the mileage of the wire to get the insulation per mile was not strictly correct. Now, although the leakage on a telegraph line insulated on poles is really a leakage at a series of detached points, and not a uniform leakage, as in a cable, yet practically, and especially in the case of long lines, it may be considered as taking place uniformly, and consequently the solutions of problems dealing with cables also apply with considerable accuracy to land lines. We may therefore consider the case in question by the help of the equations we have obtained in the investigations made in Chapter XXII.

On page 484 we have an equation [12]

$$\frac{1}{2m} = \frac{l}{\log_e \frac{\sqrt{R_i} + \sqrt{R_e}}{\sqrt{R_i} - \sqrt{R_e}}},$$

and on the same page an equation [10]

$$\frac{m}{r} = \frac{1}{\sqrt{R_e R_i}};$$

therefore

$$\frac{1}{2m} = \frac{\sqrt{R_e R_i}}{2r};$$

by substitution and transposition we get

$$l\,r = \frac{\sqrt{R_e R_i}}{2} \cdot \log_e \frac{\sqrt{R_i} + \sqrt{R_e}}{\sqrt{R_i} - \sqrt{R_e}}. \qquad \text{[A]}$$

Since l is the length of the line, and r is the *Conductor* resistance per unit length, $l\,r$ is the *Total Conductor Resistance* of the line, R_e and R_i being the respective total resistances of the line when the further end is to earth and when it is insulated.

Again we have (page 480)

$$m^2 = \frac{r}{i};$$

therefore

$$\frac{1}{m\,i} = \frac{m}{r} = \frac{1}{\sqrt{R_e\,R_i}};$$

therefore

$$\frac{1}{2\,m} = \frac{i}{2\,\sqrt{R_e\,R_i}};$$

by substitution and transposition we get

$$\frac{i}{l} = \frac{\sqrt{2\,R_e\,R_i}}{\log_e \dfrac{\sqrt{R_i} + \sqrt{R_e}}{\sqrt{R_i} - \sqrt{R_e}}}. \qquad \text{[B]}$$

Since l is the length of the line, and i is the *Insulation* resistance per unit length, $\frac{i}{l}$ is the *Total Insulation Resistance* of the line.

To get the *per mile* results, we must, of course, in the first case *divide* the total by the mileage, and in the second *multiply* it by the mileage.

By expanding the logarithm we may obtain approximate simplifications of the foregoing formulæ.

We have

$$\log_e \frac{\sqrt{R_i} + \sqrt{R_e}}{\sqrt{R_i} - \sqrt{R_e}} = \log_e \frac{1 + \dfrac{\sqrt{R_e}}{\sqrt{R_i}}}{1 - \dfrac{\sqrt{R_e}}{\sqrt{R_i}}} = \log_e \frac{1 + x}{1 - x}, \text{ if } x = \frac{\sqrt{R_e}}{\sqrt{R_i}};$$

but

$$\log_e \frac{1 + x}{1 - x} = 2\left\{x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots\right\};^*$$

therefore

$$\log_e \frac{\sqrt{R_i} + \sqrt{R_e}}{\sqrt{R_i} - \sqrt{R_e}} = 2\,\frac{\sqrt{R_e}}{\sqrt{R_i}}\left\{1 + \frac{1}{3}\cdot\frac{R_e}{R_i} + \frac{1}{5}\cdot\left(\frac{R_e}{R_i}\right)^2 + \ldots\ldots\right\};$$

* Todhunter's Algebra, 5th Edition, page 337.

therefore from equation [A] (page 539)

$$l\,r = \mathrm{R}_e \left\{1 + \frac{1}{3} \cdot \frac{\mathrm{R}_e}{\mathrm{R}_i} + \frac{1}{5}\left(\frac{\mathrm{R}_e}{\mathrm{R}_i}\right)^2 + \frac{1}{7}\left(\frac{\mathrm{R}_e}{\mathrm{R}_i}\right)^3 + \cdot \quad \right\};$$

and from equation [B] (page 540)

$$\frac{i}{l} = \frac{\mathrm{R}_i}{1 + \frac{1}{3} \cdot \frac{\mathrm{R}_e}{\mathrm{R}_i} + \frac{1}{5}\left(\frac{\mathrm{R}_e}{\mathrm{R}_i}\right)^2 + \frac{1}{7}\left(\frac{\mathrm{R}_e}{\mathrm{R}_i}\right)^3 + \cdot\,\cdot}$$

$$= \mathrm{R}_i \left\{1 - \frac{1}{3} \cdot \frac{\mathrm{R}_e}{\mathrm{R}_i} - \frac{4}{45} \cdot \left(\frac{\mathrm{R}_e}{\mathrm{R}_i}\right)^2 - \frac{44}{945}\left(\frac{\mathrm{R}_e}{\mathrm{R}_i}\right)^3 - \cdot\,\cdot \right\}.$$

If R_i is not less than 5 times R_e, then the abbreviated formulæ

$$l\,r = \mathrm{R}_e \left\{1 + \frac{1}{3} \cdot \frac{\mathrm{R}_e}{\mathrm{R}_i}\right\}, \quad \text{and } \frac{i}{l} = \mathrm{R}_i \left\{1 - \frac{1}{3} \cdot \frac{\mathrm{R}_e}{\mathrm{R}_i}\right\}$$

are correct within 1 per cent.

If, however, R_i is not more than 2½ times R_e, then it would be necessary to take three of the terms given above in order to be correct within 1 per cent. In such cases the logarithmic formulæ would probably be but little more laborious to work out, and would, of course, give exact results.

594. A direct means of ascertaining the *Insulation Resistance per mile* of an insulated wire is the following:—

As has been pointed out, on page 480, we have an equation

$$m^2 = \frac{r}{i},$$

where r is the conductivity resistance per unit length, and i the insulation resistance per unit length, of the line. Also on page 484 we have an equation

$$\mathrm{R}_e\,\mathrm{R}_i = \frac{r^2}{m^2},$$

where, as before, R_e is the total resistance of the line when the further end is to earth, and R_i the total resistance when the end is insulated. By combining these two equations we have

$$\mathrm{R}_e\,\mathrm{R}_i = r^2\,\frac{i}{r} = r\,i,$$

or

$$i = \mathrm{R}_i\,\frac{\mathrm{R}_e}{r}. \qquad \text{[A]}$$

If we take the unit length to be a mile, then r being the true conductor resistance per mile, i will be the *Insulation Resistance per mile.*

It will be seen that the mileage of the line does not come into the equation, this quantity being represented by

$$\frac{R_e}{r}.$$

What we do, in fact, in order to obtain the *true Insulation per mile* of a line, is to multiply the total resistance of the line when its end is insulated, not by the *absolute* total conductivity divided by the *absolute* conductivity per mile, which is the same thing as the mileage, but by the *observed* total conductivity (i.e. the total resistance of the line when its end is to earth) divided by the *true* conductivity per mile.

For example.

The resistance of a line, 200 miles long, when the further end was insulated, was 4000 ohms. When the end was to earth the resistance was 2400 ohms. The absolute conductor resistance of the wire, at the time the measurements were being made, was known to be 16 ohms per mile. What was the true insulation per mile of the line?

$$i = 4000 \times \frac{2400}{16} = 600{,}000 \text{ ohms.}$$

The value of i given by the ordinary rule would be

$$i = 4000 \times 200 = 800{,}000 \text{ ohms,}$$

a result 200,000 ohms, or 33 per cent., too high.

595. It must be evident that what is ordinarily called the conductor resistance of a line is really the true conductivity resistance diminished by the conducting power of the insulators. In the case of a land line, therefore, to obtain the value of r from equation [A] (page 541) it would be necessary to take a conductivity test in fine weather when the insulation is very high, and to note the temperature at that time; and then when an insulation test is made in wet weather, to observe the temperature, and from this correct the value of r previously obtained in the fine weather.

596. In the case of a short submarine cable, the insulation resistance (when the cable is in good condition) is always so greatly in excess of the conductivity resistance that the true value of the latter is obtained at once by measuring the resistance of the cable

when its end is to earth. Also the insulation per mile is practically equal to the total resistance when the end is insulated, multiplied by the mileage.

The Interpretation and Correction for Leakage of Conductor-Resistance Tests on Submarine Cables.*

597. If a long cable were cut into by a repairing ship at a point at which the true conductor resistance to shore was 8000 ohms, and the leakage through the dielectric such as to be equivalent to a small fault of 1 megohm resistance acting at a spot 2000 ohms from the ship, the conductor resistance observed on board would be 7964 ohms, while shore would obtain 7996 ohms. The ship's result would thus fall short of the true conductor resistance by 36 ohms, equivalent to an error of, say, 6 knots in length of cable. If, at some subsequent date, the ship lifted and tested the cable again at the same place, and the dielectric resistance had in the interval fallen to 0·5 megohm, its resultant being still 2000 ohms distant, the conductor resistances observed by ship and shore would be 7928 ohms and 7992 ohms respectively, the ship's error now being 72 ohms, or 12 knots.

The dielectric resistance mentioned in the first instance might be the normal insulation of a long cable, and the eccentricity of the resultant due to shallow water and a high bottom temperature towards one end of the line. The subsequent fall to half value might probably be caused by the weakening of a bad joint at about the position of the resultant leak.

From these examples it will be evident that the normal observed conductor resistance of even a perfect cable may differ materially from the true, and that in consequence of changes in the dielectric resistance along the line, whether due to variations of bottom temperature or to other causes, the conductor resistance of the whole or of any portion of the cable may apparently alter from time to time.

Conductor resistance tests cannot therefore be properly compared until they have been corrected for leakage as well as temperature. Nevertheless, it is a general custom to accept the observed conductor resistances as true measurements of the lengths of cables, and of their sections, and errors have accumulated in splice lists and charts in consequence, which make it a matter of difficulty to determine the true distance to a fault or break with any certainty, even when a good localisation has been effected.

* Walter J. Murphy, 'The Electrician' Aug. 12, 1898.

If (Fig. 200) the leakage along the line A B be collectively represented by a leak, z, whose resistance and position are such that it has the same effect on the tests taken at either end as the

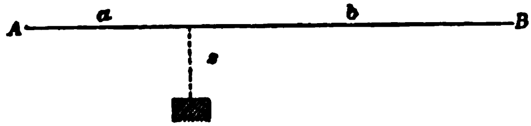


Fig. 200.

real leaks, and a and b be the true conductor resistances from A and B to the supposed position of z, then the result of a conductor-resistance test taken at A will be equal to $a + \frac{b\,z}{b+z}$. This is less than the true conductor resistance $(a + b)$ by the difference (f_A) between b and $\frac{b\,z}{b+z}$.

Calling this difference A's "fall," we have

$$f_A = b - \frac{b\,z}{b+z} = \frac{b^2}{b+z}.$$

Similarly,

$$\text{B's fall} = f_B = a - \frac{a\,z}{a+z} = \frac{a^2}{a+z}.$$

A and B will consequently only obtain similar conductor resistances when $a = b$, that is, when the resultant leakage is central, which may be the case either when the dielectric is homogeneous or when mutually balancing leaks exist. In any case, neither A nor B can obtain the true conductor resistance by observation, although in short well-insulated cables the difference may not be appreciable.

As $a + z$ and $b + z$ are, in fact, each practically equal to the insulation resistance, as observed from A and B respectively, the last equations may be written

$$f_A = \frac{b^2}{r'} \quad \text{and} \quad f_B = \frac{a^2}{r''},$$

$$\text{where } \left.\begin{aligned} r' &= b + z \\ &= \text{B's D.R.*} \end{aligned}\right\} \text{ and } \left\{\begin{aligned} r'' &= a + z \\ &= \text{A's D.R.*} \end{aligned}\right.$$

They may now be expressed as follows:—

Rule 1.—The "fall" in conductor resistance due to a leak, is directly proportional to the square of the conductor resistance

* Dielectric Resistance.

beyond the apparent position of the leak, and inversely proportional to its apparent resistance.

$$f_A = \frac{b^2}{r'}. \qquad [1]$$

Rule 2.—The apparent distance of the leak from the observer is equal to the square root of the product of the fall observed by the distant station and the apparent resistance of the leak observed by himself.

$$a = \sqrt{f_B r''}. \qquad [2]$$

Rule 3.—The insulation resistance of the cable is equal to the square of the conductor resistance beyond the leak divided by the observed fall.

$$r' = \frac{b^2}{f_A}.$$

When in equations [1] and [2] the difference between r' and r'' is negligible, as will usually be the case, we may take

$$\frac{a^2}{f_B} = \frac{b^2}{f_A};$$

and therefore

$$\frac{a}{b} = \frac{\sqrt{f_B}}{\sqrt{f_A}},$$

and since $a + b =$ true conductor resistance, therefore

$$a = \text{true conductor resistance } \frac{\sqrt{f_B}}{\sqrt{f_A} + \sqrt{f_B}},$$

and

$$b = \text{true conductor resistance } \frac{\sqrt{f_A}}{\sqrt{f_A} + \sqrt{f_B}}.$$

Hence follows :—

Rule 4.—The resultant leak is distant from each end inversely as the square roots of the falls.

The true conductor resistance of a cable should be established by correcting the usual weekly tests by the application of the above rules; but in cases in which it is not known, the following is a rapid method of approximating to its value, when $a + b$ is taken of a probable value, and r' and r'' are taken as equal and $= r$.

Since

$$f_A = \frac{b^2}{r}, \quad \text{and} \quad f_B = \frac{a^2}{r},$$

therefore

$$d = \frac{a^2 - b^2}{r},$$

where $d = f_B - f_A$, the difference between the falls, and consequently the difference between the observed conductor resistances of A and B. Then

$$d\,r = a^2 - b^2 = (a + b)(a - b).$$

If we say $(a + b)$, the true conductor resistance, $=$ L, then $b = \text{L} - a$, and $d\,r = \text{L}\,\{a - (\text{L} - a)\}$; hence we have Rule 5,

$$a = \tfrac{1}{2}\left(\text{L} \pm \frac{d\,r}{\text{L}}\right),$$

taking the $+$ or $-$ sign according as f_B or f_A is the greater, d being the *algebraic* difference between them.

The true conductor resistance, then, will be obtained by adding $\frac{a^2}{r}$ to the conductor resistance measured from B. If this result differs materially from L, the calculation is repeated, using the value so obtained as L, but as the errors tend to correct themselves this will seldom be necessary. In working all the formulæ but the last, it will be found very convenient to write, or to think of, the "falls" in ohms, the a's and b's in thousands, and the z values in megohms. Taking the example given at the commencement of § 595, the ship obtained 7964 ohms in the first case, because $z = 1$ megohm, and this was distant $8000 - 2000 = 6000$ ohms from the further station,

$$\text{therefore } f = \frac{(6000)^2}{1,000,000} = 36 \text{ ohms},$$

and the observed conductor resistance was consequently $8000 - 36 = 7964$ ohms.

The tests would in practice have been dealt with as follows:—

Conductor resistance measured by ship $= 7964$.
" " " shore $= 7996$.
$d = 32$ ohms. $z = 1$ megohm, about.

By Rule 5, assuming the ship's position, and therefore true conductor resistance, as about 7998 ohms from the shore,

$$a = \tfrac{1}{2}\left(7998 - \frac{32 \times 1,000,000}{7998}\right) = 1998 \text{ ohms},$$

and

$$b = 7998 - 1998 = 6000 \text{ ohms}.$$

From these the true conductor resistance

$$= 7996 + \frac{(1\cdot998)^2}{1} = 7996 + 4 = 8000 \text{ ohms};$$

or,

$$7964 + \frac{6^2}{1} = 7964 + 36 = 8000 \text{ ohms}.$$

Had the true conductor resistance been known, as should be the case when testing from station to station, it would have been unnecessary to observe the insulation resistance, since, by Rule 4, the leak would be distant from the ship

$$8000 \frac{\sqrt{8000 - 7996}}{\sqrt{8000 - 7964} + \sqrt{8000 - 7996}} = 8000 \frac{2}{8} = 2000 \text{ ohms},$$

and the insulation resistance would be (Rule 3)

$$= \frac{(8000 - 2000)^2}{36} = 1 \text{ megohm}.$$

The true conductor resistance is not necessarily a fixed quantity. When the cable is in shallow water, the bottom temperature may vary considerably with the season of the year. Curve sheets should therefore be kept, on which the value of the true conductor resistance and the position of the resultant leak obtained by treating the weekly tests as above, should be systematically recorded. If an account of the changes made in the true conductor resistance by repairs be kept, and applied to the results, it will furnish an interesting indication of the change of temperature along the line of cable from season to season and year to year, as well as data for the proper correction of localisations.

Postal Telegraph System of Testing Telegraph Lines.

598. The system of daily testing for insulation described in Chapter I. page 6, and which was in general use on the lines of the Postal Telegraph Department, was first superseded by a system of testing by received currents, which necessitated very exact measurements of force at the sending end of the line, and also an exact determination of the galvanometer sensitiveness at the receiving end of the line. This it was found could not be ensured with facility, and moreover the great increase in the number of

wires to be tested, necessitated the adoption of some method which would be

(*a*) Independent of the exact force of the testing batteries;

(*b*) „ „ sensitiveness of the galvanometer used;

(*c*) Capable of application to lines differing considerably in conductor resistance.

To meet these conditions the following system of testing, which has proved quite successful, was devised by Mr. A. Eden.

A current is first sent from a battery of low resistance through a fixed resistance of 20,000 ohms, which is in circuit with one of the coils of a differentially wound tangent galvanometer, so as to obtain a deflection of 110 tangent divisions.

The exact voltage of the testing battery is immaterial, as an insensitive galvanometer and a high voltage, or a sensitive galvanometer and a low voltage, may equally ensure a constant deflection of 110 divisions; but in practice, as the Post Office tangent galvanometers are adjusted (by means of an adjusting magnet) to indicate 80 divisions when a current of 1 milliampère is flowing through their coils, the constant of 110 is obtained by increasing or decreasing the number of cells (about 40 dry cells) until the resulting deflection is between 108½ and 111½ tangent divisions.

The galvanometer is then connected up in circuit with an earthed battery, the free end of which is joined so as to cause a current to pass from the battery through one coil of the differential galvanometer, a 10,000 ohm coil, two lines looped at a distant office, a second 10,000 ohm coil, and thence *viâ* the other half of

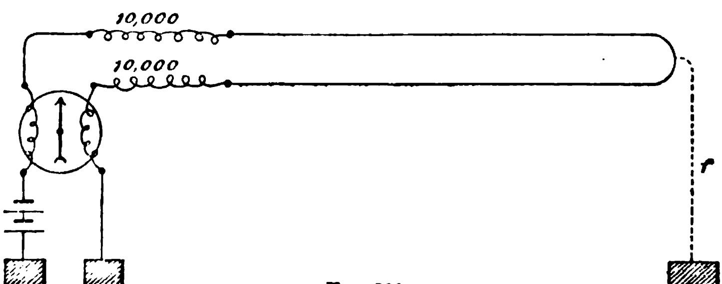


Fig. 201.

the galvanometer coils, to earth. The galvanometer coils are so connected that the outgoing and incoming currents produce a deflection in opposite directions.

These connections are shown in Fig. 201, and it will be understood that if the current sent out from the battery along one line

equals in strength that which is received back on the other line, no deflection will take place, but if there is any loss at the insulators, the current sent out will be greater than that received back, and a deflection, d, will result, which will be the exact equivalent of the total leakage on the loop.

Now from page 556 we can see that equation [A], there given, will express the value of this total leakage, $C_s - C_r$ being the difference of the two currents giving the deflection, d, and C being the current from the same battery passed through one coil of the galvanometer, and through a total resistance equivalent to that of the looped line and resistances when no fault exists. If D be this latter deflection, then we have

$$f = \frac{R' D}{2} \div d - \frac{R'}{4}.$$

Now as 110 is the indicated current through 20,000 ohms resistance + 160 ohms in the galvanometer coil, and 30 ohms in the battery, it follows that the addition of, say, a 500 ohms loop and the second coil of the galvanometer, would reduce the current to

$$\frac{(20{,}000 + 160 + 30)\,110}{20{,}000 + 160 + 30 + 500 + 160} = \frac{20{,}190 \times 110}{20{,}850} = 106 \cdot 52,$$

where no leakage existed; this then is the value of D, so that the foregoing formula becomes

$$f = \frac{20{,}850 \times 106 \cdot 52}{2} \div d - \frac{20{,}850}{4} = 1{,}110{,}471 \div d - 5213.$$

For example.

With a 500 ohms loop a leakage of 25 divisions (d) was obtained; what was the total insulation resistance of the loop?

$f = 1{,}110{,}471 \div 25 - 5213 = 44{,}418 \cdot 84 - 5213 = 39{,}205$ ohms $= \cdot 039205$ megohms, or $\cdot 04$ megohms approximately.

If the loop had been, say, 200 miles long, then the insulation per mile would have been

$$\cdot 039205 \times 200 = 7 \cdot 84 \text{ megohms.}$$

In order to save calculation, a table (A) giving the insulation resistances corresponding to various values of d, worked out from the foregoing formula, is supplied to each testing office. This table gives the results of any reading on loops of different mileages in terms of the insulation resistance in "megohms per mile."

Table (A). Morning Testing.

Showing the Equivalent Insulation Resistance in Megohms per Mile of various Amounts of Leakage on Loops of the following Lengths.

Leakage on 500 Ohm Loop.	Mileages.								
	1	5	10	20	40	50	70	80	200
Divisions.	Megohms per Mile.								
1	1·10	5·52	11·0	22·1	44·2	55·2	77·3	88·4	221
2	·55	2·75	5·50	11·0	22·0	27·5	38·5	44·0	110
3	·36	1·82	3·65	7·30	14·6	18·2	25·5	29·2	73·0
4	·27	1·36	2·72	5·44	10·8	13·6	19·1	21·8	54·4
5	·21	1·08	2·17	4·33	8·68	10·8	15·2	11·3	43·4
6	·18	·90	1·80	3·59	7·20	8·99	12·6	14·4	36·0
7	·15	·78	1·53	3·06	6·10	7·67	10·7	12·2	30·6
8	·13	·67	1·34	2·67	5·34	6·68	9·35	10·7	26·8
9	·11	·60	1·18	2·36	4·77	5·91	8·27	9·46	23·6
10	·10	·53	1·06	2·11	4·23	5·30	7·41	8·46	21·2
11	·09	·48	·96	1·92	3·92	4·80	6·71	7·66	19·1
12	·08	·44	·87	1·74	3·59	4·36	6·01	6·98	17·4
13	·08	·40	·80	1·60	3·25	4·01	5·65	6·42	16·0
14	·07	·37	·74	1·48	3·01	3·70	5·32	5·93	14·8
15	·07	·34	·69	1·37	2·75	3·44	4·90	5·50	13·7
16	·06	·32	·64	1·28	2·56	3·21	4·53	5·14	12·8
17	·06	·30	·60	1·20	2·48	3·00	4·31	4·81	12·0
18	·06	·28	·56	1·13	2·26	2·82	3·92	4·52	11·3
19	·05	·27	·53	1·06	2·13	2·66	3·66	4·26	10·6
20	·05	·25	·50	1·00	2·00	2·51	3·50	4·02	10·0
21	·05	·24	·48	·95	1·89	2·38	3·31	3·82	9·54
22	·04	·23	·45	·91	1·79	2·26	3·12	3·62	9·06
23	·04	·21	·43	·86	1·75	2·15	3·00	3·45	8·62
24	·04	·20	·41	·82	1·66	2·05	2·90	3·29	8·22
25	·04	·19	·39	·78	1·58	1·96	2·75	3·14	7·84
26	·04	·19	·37	·75	1·50	1·87	2·60	3·00	7·50
27	·03	·18	·36	·72	1·46	1·79	2·52	2·87	7·18
28	·03	·17	·34	·69	1·38	1·72	2·40	2·75	6·88
29	·03	·16	·33	·66	1·33	1·65	2·33	2·65	6·62
30	·03	·16	·32	·64	1·27	1·60	2·22	2·54	6·36
31	·03	·15	·31	·61	1·24	1·53	2·16	2·45	6·12
32	·03	·15	·29	·59	1·20	1·47	2·06	2·36	5·90
33	·03	·14	·28	·57	1·14	1·42	2·00	2·28	5·70

The above table is abbreviated from the actual table used.

TABLE (B). MORNING TESTING.

Conductor Resistance Correction.

Actual Reading.	Conductor Resistance of Loop in Thousands of Ohms.							
	1	1½	2	2½	3	3½	4	4½
	Equivalent Reading on 500 Ohm Loop.							
67	68	68	68	69	69	69	70	70
68	69	69	69	70	70	70	71	71
69	70	70	71	71	71	71	72	72
70	71	71	72	72	72	72	73	73
71	72	72	73	73	73	73	74	74
72	73	73	74	74	74	74	75	76
73	74	74	75	76	76	76	77	77
74	75	75	76	77	77	77	78	78
75	76	76	77	78	78	78	79	80
76	77	77	78	79	79	79	80	81
77	78	78	79	80	80	80	81	82
78	79	79	80	81	81	81	82	83
79	80	80	81	82	82	82	83	84
80	81	81	82	83	83	84	85	86
81	82	82	83	84	84	85	86	87
82	83	83	84	85	85	86	87	88
83	84	84	85	86	87	88	89	89
84	85	85	86	87	88	89	90	90
85	86	86	87	88	89	90	91	92
86	87	87	88	89	90	91	92	93
87	88	88	89	90	92	93	94	94
88	89	89	90	91	93	94	95	95
89	90	90	91	92	94	95	96	97
90	91	92	93	94	95	96	97	98
91	92	93	94	95	96	97	98	99
92	93	94	95	96	97	98	99	100
93	94	95	96	97	98	99	100	101
94	95	96	97	98	99	100	101	102
95	96	97	98	99	100	101	102	103
96	97	98	99	100	102	103	104	105
97	98	99	100	101	103	104	105	106
98	99	100	101	102	104	105	106	107
99	100	101	102	103	105	106	107	108

The above table is abbreviated from the actual table used.

In order to enable the insulation resistances given in Table A to be utilised for loops of more than 500 ohms resistance, a second table (B) is employed, which enables the deflections actually obtained with such higher resistance loops to be corrected to the value they would have had, had the resistance of the loop only, been 500 ohms.

In order to calculate these corrected deflections, d', let r be the extra resistance of the loop beyond 500 ohms; then we have

$$f = \frac{(20{,}850 + r)\,106 \cdot 52}{2} \div d' - \frac{20{,}850 + r}{4};$$

but since we have also

$$f = \frac{20{,}850 \times 106 \cdot 52}{2} \div d - \frac{20{,}850}{4},$$

where d is the observed deflection, therefore

$$\frac{(20{,}850 + r)\,106 \cdot 52}{2} \div d' - \frac{20{,}850 + r}{4}$$

$$= \frac{20{,}850 \times 106 \cdot 52}{2} \div d - \frac{20{,}850}{4};$$

therefore

$$\frac{(20{,}850 + r)\,106 \cdot 52}{d'} = \frac{20{,}850 \times 106 \cdot 52}{d} - \frac{20{,}850}{2}$$

$$+ \frac{20{,}850 + r}{2} = \frac{20{,}850 \times 106 \cdot 52}{d} + \frac{r}{2};$$

therefore

$$d' = \frac{(20{,}850 + r)\,106 \cdot 52}{\frac{20{,}850 \times 106 \cdot 52}{d} + \frac{r}{2}} = \frac{20{,}850 + r}{\frac{20{,}850}{d} + \frac{r}{213 \cdot 04}}$$

$$= \frac{21{,}000 + r}{\frac{21{,}000}{d} + \frac{r}{200}} \text{ approximately.}$$

For example.

What is the corrected deflection, d', for a loop of 4500 ohms resistance, the observed deflection, d, being 100 divisions?

The extra resistance, r, beyond 500 ohms, is in this case 4000 ohms, therefore

$$d' = \frac{21{,}000 + 4000}{\frac{21{,}000}{100} + \frac{4000}{200}} = \frac{25{,}000}{210 + 20} = \frac{25{,}000}{230} = 109 \text{ ohms.}$$

Table B is compiled from the foregoing formula.

Where loops cannot be obtained, the distant office, instead of looping, connects the single wire to earth through a resistance equal to the receiving half coil of the galvanometer, and also through a resistance of 10,000 ohms. In this case the calculated deflection which would be obtained on this particular wire when the insulation was perfect, is subtracted from the deflection observed at the sending office, and the result multiplied by 2 gives the leakage reading, *d*, on this particular wire. The tables then furnish the insulation resistance per mile as in the case of loops.

599. The pattern of galvanometer used for the foregoing tests is shown in general plan by Fig. 202 (and in general view by Fig. 16, page 24).

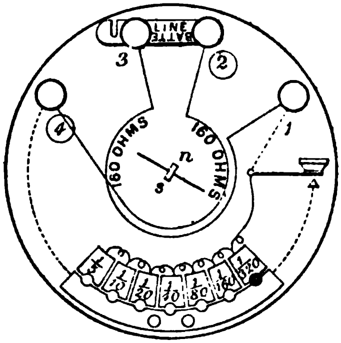


FIG. 202.

In this instrument the ring is double wound with two wires, each of the latter having a resistance of 160 ohms, so that when the two wires are joined in series the total resistance of the ring is 320 ohms.

The ends of the wires are connected to terminals 1 and 3, and to 2 and 4, respectively, the connections being such that when terminals 2 and 3 are joined together, and a current is sent from terminal 1 to terminal 4, the two coils both tend to deflect the needle in the same direction. Connected with terminals 1 and 4 (so as to embrace both the 160 ohm coils when the latter are joined in series by connecting terminals 2 and 3) are seven shunts of the respective values $\frac{1}{9}$th, $\frac{1}{19}$th, $\frac{1}{39}$th, $\frac{1}{79}$th, $\frac{1}{159}$th; $\frac{1}{319}$th, and $\frac{1}{639}$th; these shunts reduce the sensitiveness of the instrument to these values, and at the same time reduce the resistance between the terminals 1 and 4 from 320 ohms to 32, 16, 8, 4, 2, 1, and $\frac{1}{2}$ an ohm, respectively.

The winding of the two 160 ohm coils on the ring is differential, so that if necessary the instrument can be used as an instrument of this description.

To Determine the Conductor Resistance of a Line when the Strengths of the Sent and Received Currents are known.

600. If in Fig. 203, A B is a line which has high resistances R, R, placed at its ends, then it can be demonstrated mathematically that if R, R, are very great, a "resultant" fault * f (that is, the total insulation resistance of the line) will produce very nearly the same effect on the current received on the galvanometer G,

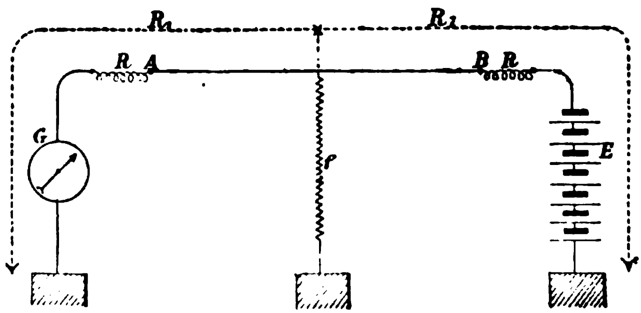


Fig. 203.

whether this fault is at the middle, at the end, or at any intermediate point on the line. As a matter of fact the fault has the greatest influence when it is at the middle of the line, and the least influence when it is at either of the ends, but when the resistances R, R, are each about 10 times the conductor resistance of the line, then the difference in the two cases is practically very small. If we assume, for convenience of calculation, that the resultant fault is at the middle of the line, then if C_s be the sent, and C_r the received, current, and R_1, R_1 the total equal resistances on either side of the fault f, we have

$$C_r = C_s \frac{f}{R_1 + f}, \quad \text{or,} \quad \frac{f}{R_1 + f} = \frac{C_r}{C_s},$$

* See page 300, § 320.

but

$$C_s = \frac{E}{R_1 + \dfrac{R_1 f}{R_1 + f}},$$

therefore

$$C_s = \frac{E}{R_1 + R_1 \dfrac{C_r}{C_s}} = \frac{E}{R_1\left(1 + \dfrac{C_r}{C_s}\right)} = \frac{E\,C_s}{R_1\,(C_s + C_r)};$$

therefore

$$R_1 = \frac{E}{C_s + C_r}. \qquad [A]$$

But R_1 is *half* the total resistance of the circuit, hence if this circuit is the conductor resistance of the line, then

$$\text{Conductor resistance} = \frac{2\,E}{C_s + C_r} = \frac{E}{\dfrac{C_s + C_r}{2}};$$

that is to say, it equals the electromotive force of the battery divided by the arithmetic mean of the sent and received currents, a fact which was first pointed out by Mr. A. Eden.*

It is also evident that since from [A]

$$C_s + C_r = \frac{E}{R_1},$$

therefore $C_s + C_r$ must be of constant value if the conductor resistance R_1 and the electromotive force E remain unaltered.

To Determine the Insulation Resistance of a Line when the Strengths of the Sent and Received Currents are known.

601. This may be arrived at as follows: Referring to Fig. 203, we have

$$C_s = \frac{E}{R_1 + \dfrac{R_1 f}{R_1 + f}} = \frac{E\,(R_1 + f)}{R_1\,(R_1 + 2f)},$$

and

$$C_r = C_s \frac{f}{R_1 + f} = \frac{E\,(R_1 + f)}{R_1\,(R_1 + 2f)} \cdot \frac{f}{R_1 + f} = \frac{E f}{R_1\,(R_1 + 2f)},$$

therefore

$$C_s - C_r = \frac{E\,(R + f)}{R_1\,(R_1 + 2f)} - \frac{E f}{R_1\,(R_1 + 2f)} = \frac{E}{R_1 + 2f},$$

* 'Electrical Review,' Jan. 15th, 1892.

therefore

$$E = R_1 (C_s - C_r) + 2f(C_s - C_r),$$

therefore

$$2f = \frac{E - R_1 (C_s - C_r)}{C_s - C_r},$$

or

$$f = \tfrac{1}{2}\left(\frac{E}{C_s - C_r} - R_1\right) = \frac{R_1}{2}\left(\frac{E}{R_1 (C_s - C_r)} - 1\right).$$

If now C is the current flowing out to line when there is no leakage, then

$$C = \frac{E}{2\,R_1}, \quad \text{or} \quad \frac{E}{R_1} = 2\,C,$$

so that

$$f = \frac{R'}{2}\left(\frac{2\,C}{C_s - C_r} - 1\right).$$

Or, if we call R′ the resistance of the whole length of circuit, then, since

$$R' = 2\,R_1,$$

we get

$$f = \frac{R'}{4}\left(\frac{2\,C}{C_s - C_r} - 1\right).$$

Or, preferably, for convenience of calculation when $C_s - C_r$ is the only variable,

$$f = \frac{R'\,C}{2} \div (C_s - C_r) - \frac{R'}{4}. \qquad [A]$$

Also since (page 554)

$$\frac{f}{R_1 + f} = \frac{C_r}{C_s},$$

therefore

$$C_s f = C_r R_1 + C_r f,$$

therefore

$$C_r R_1 = f(C_s - C_r),$$

or

$$R_1 = f\frac{C_s - C_r}{C_r};$$

but since

$$R_1 = \frac{E}{C_s + C_r},$$

therefore

$$f\frac{C_s - C_r}{C_r} = \frac{E}{C_s + C_r},$$

or

$$f = \frac{E\,C_r}{(C_s - C_r)(C_s + C_r)} = \frac{E\,C_r}{C_s^2 - C_r^2}.$$

602. Supposing now we wished to know what would be the resistance of the line (Fig. 203) measured from one end with the further end disconnected, the resistance $R_1 + f$ in fact. We have then

$$R_1 + f = \frac{E}{C_s + C_r} + \frac{E\,C_r}{(C_s - C_r)(C_s + C_r)} = \frac{E}{C_s + C_r}\left[1 + \frac{C_r}{C_s - C_r}\right]$$

$$= \frac{E}{C_s + C_r} \times \frac{C_s}{C_s - C_r} = \frac{E\,C_s}{C_s^2 - C_r^2}. \qquad [A]$$

603. This result may also be arrived at in the following manner:—

The further end of the line being to earth, and l being the length of the line, we have from equation [2], page 480, by putting $x = l$,

$$\text{Current sent} = C_s = \frac{m}{r}[A\,e^{ml} - B\,e^{-ml}];$$

and from the same equation by putting $x = 0$,

$$\text{Current received} = C_r = \frac{m}{r}[A - B];$$

therefore

$$\frac{C_s}{C_r} = \frac{A\,e^{ml} - B\,e^{-ml}}{A - B};$$

but from equation [4], page 481, we have

$$\frac{A}{B} = \frac{\sigma\frac{m}{r} + 1}{\sigma\frac{m}{r} - 1}, \quad \text{or,} \quad A\left(\sigma\frac{m}{r} - 1\right) = B\left(\sigma\frac{m}{r} + 1\right);$$

therefore

$$\frac{C_s}{C_r} = \frac{e^{ml}\left(\sigma\frac{m}{r} + 1\right) - e^{-ml}\left(\sigma\frac{m}{r} - 1\right)}{2};$$

by inserting the value of e^{ml}, e^{-ml}, and $\frac{m}{r}$, given by equations [10] and [13], page 484, we get

$$\frac{C_s}{C_r} = \frac{\sqrt{\frac{\sqrt{R_i} + \sqrt{R_e}}{\sqrt{R_i} - \sqrt{R_e}}}\left(\frac{\sigma}{\sqrt{R_e R_i}} + 1\right) - \sqrt{\frac{\sqrt{R_i} - \sqrt{R_e}}{\sqrt{R_i} + \sqrt{R_e}}}\left(\frac{\sigma}{\sqrt{R_e R_i}} + 1\right)}{2}$$

$$= \frac{(\sqrt{R_i} + \sqrt{R_e})\left(\frac{\sigma}{\sqrt{R_e R_i}} + 1\right) - (\sqrt{R_i} - \sqrt{R_e})\left(\frac{\sigma}{\sqrt{R_e R_i}} - 1\right)}{2\sqrt{R_i - R_e}}$$

$$= \frac{\sqrt{R_i} + \frac{\sigma}{\sqrt{R_i}}}{\sqrt{R_i - R_e}} = \frac{\sqrt{R_i}}{\sqrt{R_i - R_e}} \times \left(1 + \frac{\sigma}{R_i}\right).$$

The value of R_i, although it could be determined from this equation, would be represented by a somewhat complex fraction; if, however, we have $\sigma = 0$, we then get

$$\frac{C_s}{C_r} = \frac{\sqrt{R_i}}{\sqrt{R_i - R_e}}, \quad \text{or,} \quad R_i = R_e \frac{C_s^2}{C_s^2 - C_r^2}. \qquad [B]$$

In which equation, C_s and C_r (being in the form of a proportion) may be measured in ampères or milliampères, or indeed in any multiple or submultiple of an ampère.

For example.

The resistance of a line when to earth at the further end was 1500 ohms (R_e). The strengths of the sent and received currents were 2·8 and 2·6 milliampères respectively. What would be the resistance of the line if the further end were insulated?

$$R_i = 1500 \frac{2 \cdot 8^2}{2 \cdot 8^2 - 2 \cdot 6^2} = 10{,}908 \text{ ohms.}$$

The measurement of the received current would have to be made by means of a low resistance galvanometer in order to avoid the introduction of the quantity σ into the formula.

It may be remarked that the equation

$$\frac{C_s}{C_r} = \frac{\sqrt{R_i}}{\sqrt{R_i - R_e}} \times \left(1 + \frac{\sigma}{R_i}\right)$$

cannot be arrived at on the basis represented by Fig. 203.

Since

$$C_s = \frac{E}{R_s}, \quad \text{or,} \quad R_s = \frac{E}{C_s},$$

therefore from equation [B] we get

$$R_i = \frac{E}{C_s} \times \frac{C_s^2}{C_s^2 - C_r^2} = \frac{E\,C_s}{C_s^2 - C_r^2},$$

which corresponds with equation [A], page 557.

604. Having obtained R_i, the insulation per mile could be obtained in the manner shown on page 541, § 595; a simpler method of doing this is the following:—

$$C_s = \frac{E}{R_s}, \quad \text{or} \quad C_s^2 = \frac{E^2}{R_s^2};$$

by substituting this value in equation [B] (page 558) we get

$$R_i = \frac{E^2}{R_s\,(C_s^2 - C_r^2)};$$

also, from equation [A], page 541, we have

$$i = R_i \frac{R_s}{r}, \quad \text{or} \quad R_i = \frac{i\,r}{R_s},$$

where i is the true insulation resistance per mile of the line, and r its true conductor resistance per mile; therefore

$$\frac{i\,r}{R_s} = \frac{E^2}{R_s\,(C_s^2 - C_r^2)}, \quad \text{or} \quad i = \frac{E^2}{r\,(C_s^2 - C_r^2)};$$

in which C_s and C_r are in ampères, E in volts, and i and r in ohms. If C_s and C_r are measured in milliampères, then we have

$$i = \frac{(E \times 1000)^2}{r\,(C_s^2 - C_r^2)} = \frac{E^2 \times 1{,}000{,}000}{r\,(C_s^2 - C_r^2)} \text{ ohms.}$$

For example.

The strengths of the sent and received currents on a line were 12 and 10 milliampères respectively, the sending battery being a 10-cell Daniell (10 volts approximately); the line had an average estimated conductor resistance of 14 ohms per mile. What was the insulation per mile of the line?

$$i = \frac{10^2 \times 1{,}000{,}000}{14\,(12^2 - 10^2)} = 162{,}000 \text{ ohms.}$$

Kirchoff's Laws.

605. These laws are two in number; the first is:—

The algebraical sum of the current strengths in all those wires which meet in a point is equal to nothing.

The truth of this law is almost obvious; thus, if we have, say, five wires meeting in a point, as shown by Fig. 204, then

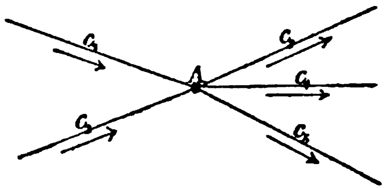


Fig. 204.

as the point A cannot be a reservoir, the sum of the currents c_1, c_2, approaching A must equal the sum of the currents c_3, c_4, c_5, receding from A, that is

$$c_1 + c_2 = c_3 + c_4 + c_5,$$

or

$$c_1 + c_2 - c_3 - c_4 - c_5 = 0.$$

It may be as well, perhaps, to point out that although the quantities c_1, c_2, c_3, c_4, c_5, are partly positive and partly negative, yet they together constitute an *algebraical* "sum," for the equation may be written

$$c_1 + c_2 + (-c_3) + (-c_4) + (-c_5) = 0;$$

the quantities c_3, c_4, and c_5, in fact, are negative because the currents they represent flow in the opposite direction to the currents c_1, c_2.*

606. The second law of Kirchoff is as follows:—

The algebraical sum of all the products of the current strengths and resistances in all the wires forming an enclosed figure, equals the algebraical sum of all the electromotive forces in the circuit.

The truth of this law follows as a consequence from the laws investigated on pages 328–331; viz.:—

(A) *The difference of the potentials at two points in a resistance* (*in*

* It is important that *algebraical* sum should not be confounded with *arithmetical* sum; the latter signifies a number of quantities connected by *plus* signs, whilst in the former the signs may be partly negative and partly positive, or, indeed, all negative. As a rule, when the word "sum" is used in stating a law, it is the *algebraical* sum which is meant.

which no electromotive force exists) is equal to the product of the current and the resistance between the two points.

(B) *The difference of the potentials at two points in a resistance in which an electromotive force exists, is equal to the product of the current and the resistance between the two points added to the electromotive force in the resistance, this electromotive force being negative if it acts with the current, and positive if it opposes it.*

If we refer to Fig. 205, and we consider any closed circuit in it, then we can see that the sum of the differences of the potentials

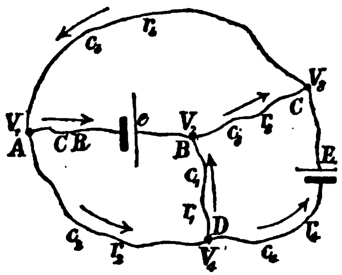


FIG. 205.

between the points in that circuit must be equal to 0; thus, for example, if we take the closed circuit formed by the sections A B, B C, C D, D A, then it is evident that

$$(V_1 - V_2) + (V_2 - V_3) + (V_3 - V_4) + (V_4 - V_1)$$

is the same as

$$V_1 - V_1 + V_2 - V_2 + V_3 - V_3 + V_4 - V_4,$$

which equals 0.

Now from laws (A) and (B) we have

$$\begin{aligned} V_1 - V_2 &= C\,R - e \\ V_2 - V_3 &= c_3 r_3 \\ V_3 - V_4 &= -(c_4 r_4 - E)^* \\ V_4 - V_1 &= -c_2 r_2^*; \end{aligned}$$

therefore, by addition, we get

$$C\,R - e + c_3 r_3 - c_4 r_4 + E - c_2 r_2 = 0;$$

or

$$C\,R + c_3 r_3 - c_4 r_4 - c_2 r_2 = e - E,$$

which proves the law.

* These quantities are *negative* because the currents c_4 and c_2 flow in the reverse direction to the currents c_1 and c_3.

As in the case of Kirchoff's first law, we have in the last equation, *algebraical* sums, for this equation may be written:

$$C R + c_3 r_3 + (- c_4 r_4) + (- c_2 r_2) = e + (- E);$$

c_4, c_2, and E, in fact, are negative, because the currents in the sections (C D and D A) in which these quantities occur are in the reverse direction to the currents in the other sections (A B and B C).

Pollard's Theorem.

607. Let E (Fig. 206) be a battery of internal resistance r, which is shunted by a shunt S and is in circuit with a resistance R, then current through battery is

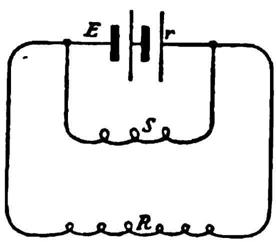


Fig. 206.

$$\frac{E}{r + \frac{S R}{S + R}} = \frac{E (S + R)}{S r + R r + S R}$$

$$= \frac{E (S + R)}{S r + R (S + r)},$$

and current, C, through R, is

$$C = \frac{E (S + R)}{S r + R (S + r)} \times \frac{S}{S + R}$$

$$= \frac{E S}{S r + R (S + r)} = \frac{E \frac{S}{S + r}}{\frac{S r}{S + r} + R}; \qquad [A]$$

that is to say, a battery E, having a resistance r and shunted by a shunt S, is equivalent to a battery of electromotive force $E \frac{S}{S + r}$, and internal resistance $\frac{S r}{S + r}$.

608. Now if we call e the electromotive force of the shunted battery, then we have

$$e = E \frac{S}{S + r},$$

or

$$e r = E \frac{S r}{S + r},$$

that is,

$$e : E :: \frac{S r}{S +} : r.$$

It follows, therefore, from the theorem, that the original electromotive force, E, is to the reduced electromotive force, e, in the ratio of the original resistance of the battery to the shunted resistance of the same.

A Method of Measuring the Resistance of, and the Current flowing through, Electric Lamps when burning.

609. This method is an adaptation of the methods given on page 348, § 383, and page 420, § 475, and is as follows:—

A resistance, R (fig. 207), is inserted in the circuit of the lamp whose resistance is to be measured, and then the potential, V, between the points, A and B, is measured. A similar measurement is then taken of the potential, V_1, between the terminals, C and D, of the lamp. We then have—

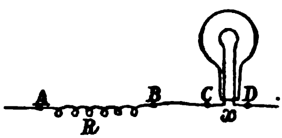


Fig. 207.

$$x = R\frac{V_1}{V}.$$

For example.

Suppose the resistance, R, were 1 ohm, and suppose that the discharge deflection obtained by the condenser from the points A and B, were 250 divisions, there being no shunt to the galvanometer; also suppose that the discharge deflection obtained from between the points, C and D, were 260 divisions, the galvanometer, whose resistance was 6100 ohms, being shunted with a shunt of 200 ohms; then we have

$$V = 250,$$

$$V_1 = 260 \times \frac{6100 + 200}{200} = 8190;$$

therefore

$$x = 1 \times \frac{8190}{250} = 32 \cdot 8 \text{ ohms.}$$

If the discharge given by a Daniell cell were 140, then we should have—

$$\text{Electromotive force between A and B} = 1 \cdot 08 \times \frac{250}{140} = 1 \cdot 93.$$

The current flowing, therefore, equals

$$\frac{1 \cdot 93}{1} = 1 \cdot 93 \text{ ampères.}$$

In cases where the current is powerful, and where it is not advisable to introduce so high a resistance as 1 ohm into the circuit, R could be made, say $\frac{1}{10}$th of an ohm.

Measurement of the Insulation Resistance of a live Electric Light Installation.

Russell's Method.

610. This is a very simple method. First measure the potential difference V between any point in the installation and earth, then connect that point to earth through a very high resistance, R_1, this resistance being preferably approximately equal to the estimated insulation resistance, R, of the installation; again measure the potential difference between the point and earth, let this be V_1.

Now the combined resistance of R and R_1 is obviously

$$\frac{R\,R_1}{R + R_1},$$

and the values of V and V_1 must be in the inverse proportion of R to $\frac{R\,R_1}{R + R_1}$, that is,

$$V : V_1 :: \frac{R\,R_1}{R + R_1} : R;$$

therefore

$$V = V_1 \frac{R_1}{R + R_1},$$

or

$$V\,R + V\,R_1 = V_1\,R_1,$$

that is,

$$R = R_1 \frac{V_1 - V}{V}.$$

The measurement of the potentials V and V_1 must be effected by means of a condenser (page 320, § 346) or an electrometer, it cannot be done by means of a galvanometer connected direct between the points and earth, since the galvanometer will act itself as a leak, and thereby alter the values of the potentials.

A Method of Testing Batteries.

611. In making tests of batteries which are required to give a large current through a low resistance, it is often found impossible to determine the exact amount of current flowing by the direct use of an "ammeter," as the resistance of the latter, although low, may still be sufficiently great to materially reduce the current which the battery is intended to give out. This difficulty may be overcome by the following arrangement, devised by Mr. I. Probert for testing the value of batteries specially designed for working small incandescent lamps.*

The battery, B (Fig. 208), to be tested, is joined up to the lamp, *l* (which has a "voltmeter," V, across its terminals), the

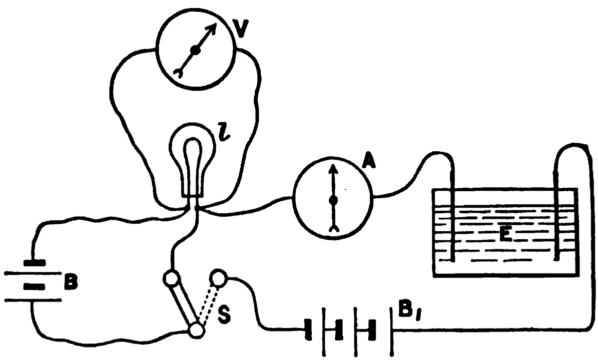


Fig. 208.

switch, S, being turned to the position shown. Under these conditions the battery works direct on to the lamp, which glows at its full brilliancy, and the voltmeter, V, gives the potential or voltage between the lamp terminals. In order to determine the current, the switch, S, is turned to the position shown by the dotted lines; this brings into circuit the auxiliary battery, B_1 (preferably a set of accumulators), the ammeter, A, and the electrolytic cell, E, this cell consisting of 2 copper plates in a sulphate of copper solution. The current from B, although reduced by the resistance of the ammeter, is reinforced by the auxiliary battery, B_1, and by adjusting the distance between the plates in the electrolytic cell the current can be adjusted to the greatest nicety until the deflection on V is the

* 'Electrical Review,' March 6th, 1891.

same as it was previous to the turning of the switch, S, hence the ammeter, A, now shows the current which under the latter conditions was flowing through B. The observation being taken on A the switch, S, is turned back again to the position shown, and the battery, B, continues to work under the practical conditions.

A Method of Measuring Low Resistances.

612. This method is merely an adaptation of the method given on page 420, § 475, and is shown in principle by Fig. 209.

E is a single Daniell cell, R a resistance of 1 ohm, and B C the resistance, x, to be measured. Between B and C a Thomson galvanometer (page 48) in circuit with a resistance is connected.

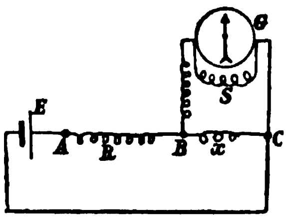


Fig. 209.

Now, taking the resistance of the cell E to be, say, 4 ohms, then if x be $\frac{1}{100}$th of an ohm, the potential between B and C will be approximately $\frac{1}{500}$th of a volt, and the potential between A and B, $\frac{1}{5}$th of a volt, consequently if we can measure these two potentials accurately we can determine the value of a resistance of $\frac{1}{100}$th of an ohm to an equal degree of accuracy. Now a Thomson galvanometer, wound to about 5000 ohms resistance, will give a deflection of 100 divisions with one Daniell cell, there being in circuit a total resistance of 10,000,000 ohms. If there be no resistance in the circuit beyond that of the galvanometer itself (5000 ohms) the deflection would be

$$100 \times \frac{10{,}000{,}000}{5000} = 200{,}000 \text{ divisions.}$$

representing an electromotive force, or potential, of 1 volt approximately; hence 200 divisions would represent a potential of $\frac{1}{1000}$th of a volt. We can easily, therefore, measure a potential of $\frac{1}{500}$th of a volt.

In order to make a measurement we should proceed as follows:—

The battery, resistances, etc., being connected up as shown in Fig. 209, and the shunt being removed from the terminals of the galvanometer, the resistance in circuit with the latter must be varied until a good deflection (about 300 divisions) is obtained. Let d_1 be this deflection, and let G and R_1 be the respective re-

sistances of the galvanometer and the resistance in circuit with the latter; then if v_1 be the difference of potential between B and C, the current c_1 flowing through the galvanometer will be

$$c_1 = \frac{v_1}{R_1 + G}.$$

The galvanometer and the resistance in its circuit are now disconnected from B and C, and are connected to A and B, the $\frac{1}{1000}$th shunt being joined up to the terminals of the instrument. The resistance in its circuit is then varied until a deflection d_2, approximately the same as d_1, is obtained; then if R_2 be this resistance, and if v_2 be the potential between A and B, and further if c_2 be the current producing the deflection d_2, we have

$$c_2 = \frac{v_2}{R_2 + g} \times \frac{1}{1000},$$

where g is the combined resistance of the galvanometer and shunt.

We have therefore

$$\frac{c_1}{c_2} = \frac{(R_2 + g)\,1000}{(R_1 + G)} \times \frac{v_1}{v_2};$$

but

$$v_1 : v_2 :: x : R,$$

or

$$\frac{v_1}{v_2} = \frac{x}{R},$$

and as

$$c_1 : c_2 :: d_1 : d_2,$$

or

$$\frac{c_1}{c_2} = \frac{d_1}{d_2},$$

we get

$$\frac{d_1}{d_2} = \frac{(R_2 + g)\,1000}{(R_1 + G)} \times \frac{x}{R},$$

or

$$x = R\,\frac{(R_1 + G)}{(R_2 + g)\,1000} \cdot \frac{d_1}{d_2}.$$

For example.

The deflection obtained between the points B and C was equal to 320 divisions (d_1), there being a resistance of 8000 ohms (R_1) inserted in the circuit of the galvanometer. When the latter was

connected between A and B, the $\frac{1}{1000}$th shunt was inserted, together with a resistance of 1200 ohms (R_2): the deflection obtained was then equal to 310 divisions (d_2). The resistance of the galvanometer was 5000 ohms (G), and the resistance, R, 1 ohm. What was the value of x?

$$x = 1 \frac{(8000 + 5000)}{(1200 + 5)\,1000} \cdot \frac{320}{310} = \cdot 0111 \text{ ohm.}$$

We are not, of course, necessarily bound to use the $\frac{1}{1000}$th shunt, but in practice it would almost always have to be employed.

613. The degree of accuracy with which the test could be made would depend entirely upon the values of the deflections d_1 and d_2; and as we should endeavour to make them both as high as possible, that is to say, both as nearly equal as possible, the "Percentage of accuracy" would practically be $\frac{\delta\, 200}{d_1}$, where δ is the fraction of a division to which each of the deflections could be read.

Note on the Measurement of Resistance.

614. Dr. W. W. Waghorn has pointed out * that in the case of a measurement made in the general manner indicated in the foregoing test, the deflections obtained on the galvanometer when it is joined first between B and C and then between A and B, will directly indicate the relative values of x and R, *no matter whether the galvanometer has a high or a low resistance*, that is provided the battery E has a negligible resistance. This may be proved as follows:—

Referring to Fig. 209 and assuming that the galvanometer is joined direct between B and C, then the current, C_1, flowing through G will be

$$C_1 = \frac{E}{R + \frac{G\,x}{G + x}} \times \frac{x}{G + x} = \frac{E\,x}{R\,G + R\,x + G\,x}$$

$$= \frac{E\,x}{G\,(R + x) + R\,x}.$$

If now the galvanometer is placed between A and B, i.e. if R and x change places, then the current, C_2, flowing through G will be

$$C_2 = \frac{E\,R}{G\,(x + R) + x\,R};$$

* 'Philosophical Magazine,' April 1889.

but the denominators of the fraction in both cases are the same, hence

$$\frac{C_1}{C_2} = \frac{x}{R},$$

that is, x and R are directly proportional to the current flowing through the galvanometer in the two cases.

Direct Reading Potentiometer.

615. This instrument, manufactured by Messrs. Elliott Bros., and shown in general view by Fig. 210, can be used for measuring either "Current," "Electromotive Force," or "Resistance."

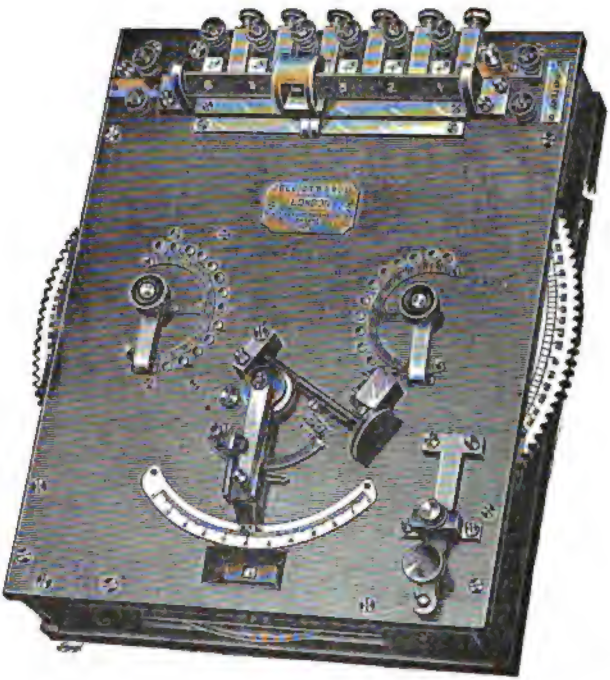


Fig. 210.

The *measurement of current* consists in determining the exact difference of potential in volts between the terminals of a resistance whose value is known in ohms, and through which the current to be measured is passing, the value being given by Ohm's law, $C = \frac{E}{R}$.

The *measurement of electromotive force* consists in its comparison either directly against that of a standard cell, if a value of somewhat the same order is under test—that is to say, not exceeding 1·5 volts (an example given later explains this)—or, in the case of high electromotive forces, in applying the pressure to the two ends of a high resistance divided in some known proportion or ratio, so that the current flowing through the total resistance may cause a fall of potential over a fractional part of it to be comparable with the E.M.F. of a standard cell, and this comparison being determined by the potentiometer, the value of the whole E.M.F. under test may be given in terms of this comparison, multiplied by the ratio into which the resistance is divided.

In practice, a resistance box is so adjusted that electromotive forces of 15, 150, 300, and 600 volts, or any intermediate value, can be measured by choosing the proper terminals on it.

The *measurement of resistance* is effected by placing the resistance to be tested in series with a resistance of known value, and then passing a constant current through the two of them in series, wires being led from their terminals to the potentiometer. The fall of potential across the unknown one can be determined in terms of the fall across the standard; and the current through the two being the same, the resistance values are exactly proportional to these falls of potential.

In the instrument, the measurements and comparisons are effected by opposing the electromotive force, or potential difference, which, as it will be seen above, is the value dealt with in each case, against the potential difference which exists between two points on a suitably arranged and divided wire, by virtue of a constant current passing continuously through the whole length of this wire. A galvanometer placed as an indicator in the connection between the two opposing electromotive forces, serves to show which is the greater, and, when no deflection is obtained, it shows that the opposing forces are equal in value, and therefore balanced. As it is impracticable to use a standard cell to furnish current uniformly through the divided wire, an accumulator cell is employed, suitable resistances, in the form of rheostats, being placed in series with the divided wire, the adjustment of these permits of any fraction of the E.M.F. of the accumulator cell being applied to the divided wire itself.

The divided wire in the instrument is formed of 149 sections, each about 3 inches in length, and they are all adjusted so as to be absolutely equal in resistance one to the other. The slide portion of the wire K M (Fig. 211) is extremely short, as can be

seen in Fig. 210. Inequalities in the drawing of this wire are compensated for by dividing the scale, over which arm L travels, so that the individual divisions of the divided wire R R R are exactly multiplies of the divisions on the scale. Thus for example, the fall of potential over 140·7 sections of the divided wire is exactly five times that over 28 divisions of this wire, and 14 divisions on the scale.

Referring to Fig. 211, A B are two terminals, to which the working battery—i.e. one accumulator cell—should be attached.

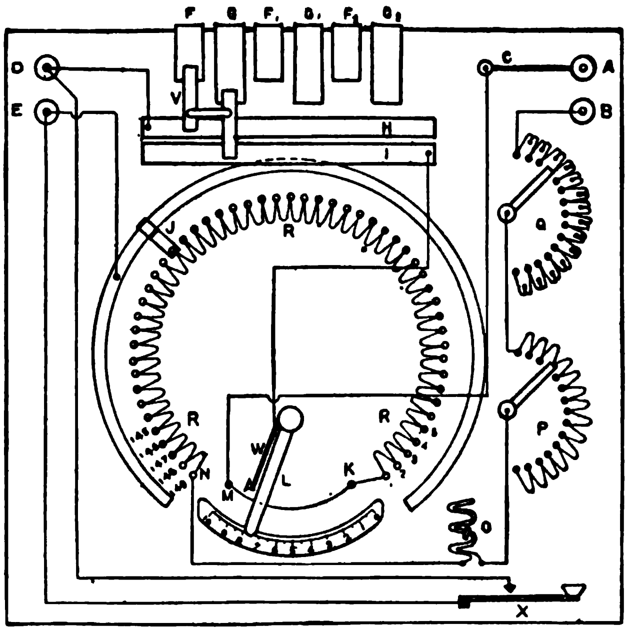


FIG. 211.

At C is a small fuze, which serves to protect the slide wire from injury should too high an E.M.F. be applied to A and B by accident.

D E are the galvanometer terminals, and across these is connected a short-circuit key X, which in its normal, or free, position keeps the galvanometer short-circuited, and thus protected against violent deflections. A d'Arsonval galvanometer (page 68) should always be employed in all these tests.

F, G, F_1, G_1, F_2, G_2, &c., are the terminals to which wires

leading to the various sources of potential differences to be compared should be attached. A multiple double pole switch V permits of the two common bars H and I being connected to any pair of these terminals at will.

The divided wire R R R is laid round in a circular position, and divided in 149 parts of equal resistances, small contacts being placed at each of these 149 points. The whole of this part of the divided wire, which, of course, is the essential part of the whole apparatus, is perfectly protected from mechanical injury by being inside the case.

One extremity N of this wire—at the 149 contact—is connected to O, a small fine adjustment rheostat, and thence to P and Q, to other adjustable rheostats in series, these being so proportioned that the total resistance value of O is rather greater than that of one section of P, and so that the total value of all the sections of P are slightly greater than that of one section of Q. P, Q, and O are not adjusted to any definite values—they serve simply as adjustments. In practice, Q, P, and O has a total resistance of 200 ohms approximately. The divided wire itself, R R R, has a total resistance of about 30 ohms. One end of rheostat Q is joined to terminal B. The other extremity of the wire R R R is taken up through the top of the instrument at K, where it is led round a curved segment, and where the moving contact arm L can travel over it. A scale is fixed to the top of the instrument, and a pointer is attached to L, so that when the moving contact is on the stud K, the pointer attached to the arm L stands at zero on the scale. When the arm L is moved till the pointer stands at the figure 10, then the moving contact has passed over a length of the divided wire exactly equal in resistance to any of the other 149 sections between K and N.

A contact J can travel round the circle of the divided wire and make contact with any of the 149 small contacts fixed to it. This contact J is attached to a large toothed wheel, the edges of which can be seen in Fig. 210 on the right and left of the instrument. This affords a ready means of shifting the position of contact J, and its position with reference to N and K can be seen through a small window in the front of the instrument, through which a number shows corresponding to the number of the contact on which J lies; a device is provided to cause J to make contact definitely on either one or other of any pair of adjacent contact studs on the divided wire. A wire is led from J to one on the galvanometer terminals E.

The travelling contact on arm L is connected through a small

key W to the bar of the multiple switch, the other bar H being connected to the second galvanometer terminal D.

The key is provided with a small clamping device, so that it can be kept down if desired, and the galvanometer deflections manipulated with key X. Care must be taken that no source of E.M.F. is attached by mistake to terminals F, G, &c., as, in the event of key W being clamped down, and switch V being in these terminals, a comparatively powerful current might flow through the galvanometer and the slide wire, probably damaging both. Key W, therefore, should only be clamped down when making a series of tests where there is no chance of a wrong connection having been made outside the instrument.

It will be seen that there exists always between A and B a closed circuit through which the current furnished by the working battery passes—from A through C to M, K, R R R, N, O, P, Q, and so to B. This circuit is of variable resistance, owing to the adjustment of rheostats O, P, and Q; but in all cases the whole of the wire R R R is in circuit.

Resistance Measurement.

616. Having set up a sensitive galvanometer in any convenient position, place the potentiometer on a steady table, and carry wires from the galvanometer terminals to terminals D and E on the instrument, then join two wires coming from a single accumulator cell to A and B, taking care to keep the polarity right, as marked on the instrument.

Suppose it is required to obtain the exact value of a resistance, which is known to be nearly 1 ohm, and that this resistance is capable of carrying 1 ampère of current without heating; then connect this resistance R_x (Fig. 212) in series with a suitable resistance whose value is known (R_s), and across the two in series join a large accumulator cell—if 1 ampère should be too great a current to put through R_x and R_s, then put an adjustable resistance X X X in series with R_x and R_s, and adjust X X X till the current through the circuit has a convenient value.

It should be borne in mind that the greater the fall of potential obtained, the greater the degree of accuracy in making the test, as a larger value is being dealt with, and therefore a small percentage of this becomes a very small value indeed.

Having arranged the resistances, join their terminals to F G as shown.

The resistances of the connecting wires W W W W between the

instrument and R_x and R_s are of no consequence whatever, as the whole method is one of balancing opposing forces, and no currents will flow through these connecting wires W W W W, neither is the resistance of the wires forming part of the circuit joining R_s and R_x together, and to the rheostat X X X and the battery of any consequence, except inasmuch as it varies the total amount of current in the circuit, and hence the *amount* of fall of potential dealt with, but it must affect both R_x and R_s equally.

A B being joined to the working battery, and current flowing through R_x and R_s, all is ready to balance up and make a test.

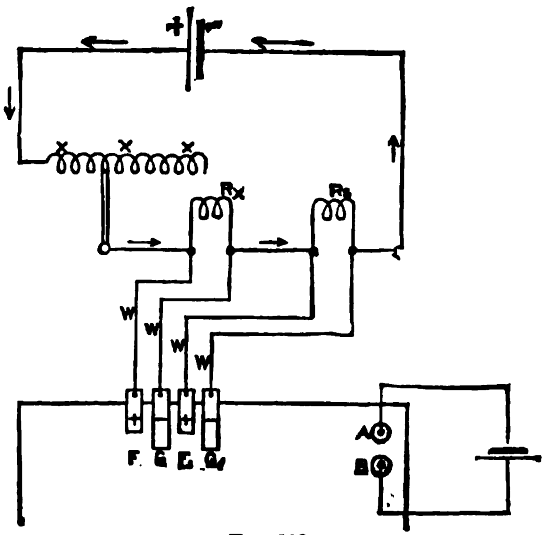


FIG. 212.

For simplicity's sake, assume R_x to be 1 ohm, and that the current be roughly 1 ampère through it, then the fall of potential across it will be about 1 volt; now this is practically equal to one-half of the E.M.F. of the working battery attached to A B, so that for a balance to be obtained, J (Fig. 211) must be in such a position that the fall of E.M.F. on each side of it is about equal.

If R_x (Fig. 212) has a value of 1, then it is convenient to set J (Fig. 211) so that the figure 100 appears at the window, and to put L so that its pointer is at zero; L is, of course, only a fine adjustment, and may be left out of consideration altogether if a rough test only is wanted.

Having set the figure 100 at the window, and the multiple

switch V so that it is in connection with the terminals F G, to which are attached the wires leading to the terminals of R_s, the rheostat Q (Fig. 211) should be adjusted till, on closing key W and pressing down key X, the galvanometer deflection is reduced as small as possible; then rheostat P should be brought into operation to still further effect a balance, and finally a complete balance can be obtained by adjusting the rheostat O. It is obvious, for this adjustment to be of any use, that the currents through the two circuits be constant throughout the tests; it is therefore always advisable to connect on the accumulator cells shortly before beginning to test, so that the current values may have time to settle down.

Having obtained a balance with 100 at the window, and the arm L at zero, release key W and move the multiple switch till it is in contact with F, G, and R_x (Fig. 212); then depress key W. Should the galvanometer move very slightly, then move arm L round by hand, or by means of the fine adjustment screw, until the galvanometer stands at zero once more; suppose a balance is reached with L at 1·3, then the value of $R_x = 1 \cdot 0013$ in terms of R_s. Should moving L to the left, however, increase the galvanometer deflection, then the large toothed wheel must be moved till 99 is at the window, or even further, and the final balance obtained by adjusting the position of L. Suppose the figure 97 shows at the window, and L stands at 8·4, then the value of R_x in terms of $R_s = 0 \cdot 9784$; or, assuming R_s to be 1 ohm, then $R_x = 0 \cdot 9784$ ohm.

It will thus be seen that the readings can be very quickly taken with great accuracy, as in practice a further figure than those given can easily be read.

It is advisable always to move the multiple switch back to F G, and take a reading on R_s with 100 at the window and L at zero, in order to see that no alteration has taken place in the balance at this point owing to variation in either of the currents, due to heating of wires, bad connections, or the like. In practice it is easy to make a large number of tests without any appreciable alteration in the original balance taking place.

Any alteration must, of course, be compensated for by a small movement of rheostat O; or, if necessary, by rheostat P as well.

The foregoing describes how a resistance can be measured when its value is very near to that of the standard with which it is compared; but suppose R_x is only about one-third of an ohm, then, keeping the original conditions of balance, the figure 33 must appear at the window, and the arm L stand about 3 for

R_s to balance—in other words, the instrument becomes direct reading in terms of the standard.

Suppose R_x is about 1·4 ohms, then 140 must appear at the window, and so on.

Suppose R_x is 1·7 ohms, or in fact anything over 1·5, which, of course, is the highest figure obtainable at the window and arm L, then either a 2 ohms standard must replace the 1 ohm standard at R_s, or, if R_s remain 1 ohm, it must be balanced lower down on the potentiometer—say 50—by interposing greater resistances in the rheostats P, Q, and O (Fig. 211); then, assuming a balance on R_s with 50 at the window and arm L at zero, if R_x is 1·710 ohms it will balance with 85 at the window and arm L at 5 on the scale.

From the diagram (Fig. 211) it will be seen that resistance can always be added outside the instrument itself between terminal B and the working battery should rheostats P and Q not afford range enough in some special cases.

Now suppose that in the case of R_s we have a resistance which, instead of being exactly 1 ohm, has a value of 0·9998, and it is desired to get the value of R_x in ohms. All that is necessary is to balance R_s, in the first instance, with 99 at the window and arm L at 9·8; then, evidently, the resistance which would balance with 100 at the window and arm L at zero would be accurately 1 ohm.

The above describes how single ohms can be compared. It is quite obvious that half-a-dozen 1-ohm resistances could be joined in series, a suitable increase in the battery available to pass current through them being made, and wires being joined to the other available terminals F_2, G_2, &c., &c.

A test can be run round the set, and any desired one chosen as a standard, and their values all taken out.

For measurement of lower values—say 0·001 ohm, the method described above is followed out exactly, only naturally when R_s and R_x (Fig. 212) are about 0·001, a much larger current must be employed through them, and special care taken to ensure solid and reliable connections for the main current, and suitably disposed "potential" points for the connection of wires leading to F, G, &c.

Suppose R_s to have a value of 0·001 ohm, and to have 500 ampères through it, then the fall of potential across it will be ½ volt.

Balance with rheostats P, Q, and O as before, with 100 at the window and arm L at zero; then when R_x is connected on, if balance

is obtained with 99 at the window and arm L at 8·1, the value of R_x is equal to 0·0009981 ohm.

This measurement is made with accuracy quite equal to that of the tests of single ohms, as it is the falls of potential which are compared, and not the resistances themselves; but the resistances are proportional to these falls, inasmuch as the resistances being joined in series, the same current passes through both.

Current Measurement.

617. In Fig. 213 the working battery is joined to A and B, as in the previous test, a standard cell is joined up to F G, through a high resistance to prevent any appreciable current being taken off

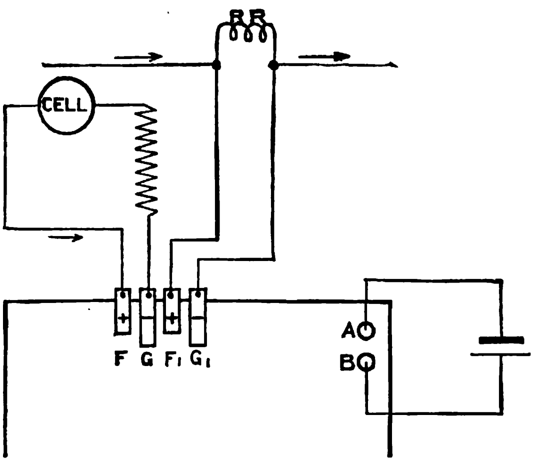


FIG. 213.

the cell. The current to be measured passes through the resistance standard R R.

First, if the electromotive force of the standard cell be assumed, at the temperature at the time of test, at 1·434 volts, then set figures 143 at the window, and arm L at 4 on the scale. Rheostats P, Q, and O must be manipulated till a balance is obtained on depressing key W, when the multiple switch V is on the terminals F G, then each movement of the toothed wheel means 0·01 volt, and the movement of arm L from 1 to 2 or 2 to 3 on the scale means 0·001 volt.

The fall of potential across resistance R R can be rapidly determined. Suppose the current is about 500 ampères, and we

know the value of R R to be 0·001 ohm exactly, then if a balance is obtained with the multiple switch V on F G, with 50 at the window and arm L at 1·65 on the scale, the current through R R is equal to 501·65 ampères.

Suppose, again, that the current is about 1 ampère, and that we know the value of R R to be 1 ohm exactly, then, the potentiometer being balanced as before, we may find, when the multiple switch V is put at F G, that a balance is obtained with 101 at the window and the arm L at 3·65 on the scale—the current through R R is then equal to 1·01365.

Electromotive Force Measurement.

618. Fig. 214 shows the connections.

The working battery is connected to A B as usual, the standard cell and resistance to F G.

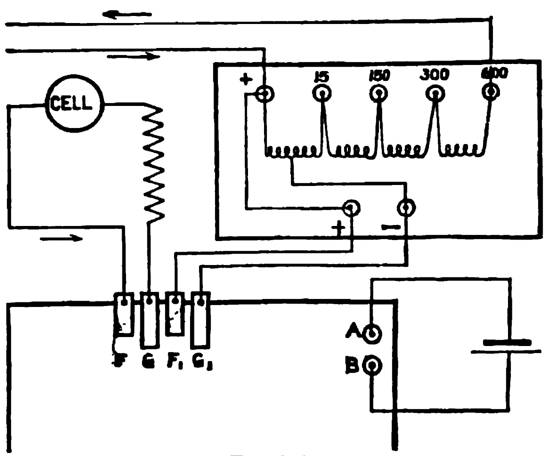


FIG. 214.

The small separate resistance box is employed, the two terminals marked "Potentiometer" being connected to F G, wires attached to the source of electromotive force to be measured being connected to terminals on this box, according to the various ranges used. The terminals marked "Potentiometer" have between them a small fractional part of the whole resistance in the box.

Setting the figure 143 at the window, and arm L at 4 on the scale as before, when a balance is obtained with the multiple switch on the standard cell at F G, then each movement of the toothed

wheel means 0·01 volt, and 0·0001 corresponds to the small whole divisions on the scale.

Suppose an electromotive force of about 100 volts is to be measured, the positive wire should be joined to the terminal marked + on the separate resistance box, and the other to the terminal marked 150. Then, having balanced as above, the instrument is direct reading in volts, that is to say, if a balance is obtained with 101 at the window and arm L at 5·3 on the scale, then the electromotive force under test equals 101·53 volts. For readings using the + to 15 terminals on the separate resistance box, the readings must be divided by 10; if using terminals + to 300, readings must be multiplied by 2; if using + to 600, multiply by 4.

Supposing it is desired to determine exactly an electromotive force of about 2·2 volts. Obviously, if the standard cell is balanced with 143 at the window and 4 on the scale, the range of the potentiometer is insufficient to compare this directly. If the terminals + to 15 on the separate resistance box are used, then the electromotive force of 2·2 volts would balance with figure showing at the window. For great accuracy, the following method may be adopted:—

Connect two accumulator cells in series to A B; then with switch V (Fig. 211) on F G, corresponding to the terminals of the standard cell, adjust P, Q, and O so that a balance is obtained with 71 at the window and arm L at 7 on the scale $\left(\frac{1 \cdot 434}{2} = 0 \cdot 717\right)$. Assuming that the electromotive force of the standard is taken at 1·434 volts at the working temperature, then let wires be brought from the source of electromotive force to be tested to terminals F G; then, if a balance is obtained on moving the multiple switch to F G, with 111 at the window and arm L at 4 on the scale, the value of the electromotive force under test $= 1 \cdot 114 \times 2 = 2 \cdot 228$ volts; that is to say, with the standard balanced at the position corresponding to half its value, all readings at the window and scale must be double to obtain correct values in volts.

This process can be carried further, but not more than 6 volts should ever be applied to A B.

The Silvertown Compound Key for Cable Testing.

619. This key, designed by Mr. J. Rymer Jones, and which is in general use in the testing rooms of the India Rubber, Gutta Percha and Telegraph Works Company, Silvertown, is an excellent arrangement, and greatly facilitates the execution of the "Inductive capacity" and "Insulation" tests of insulated wires or of cables; it is particularly useful when a large number of wires have to be tested. The apparatus (Fig. 215) consists of two keys,

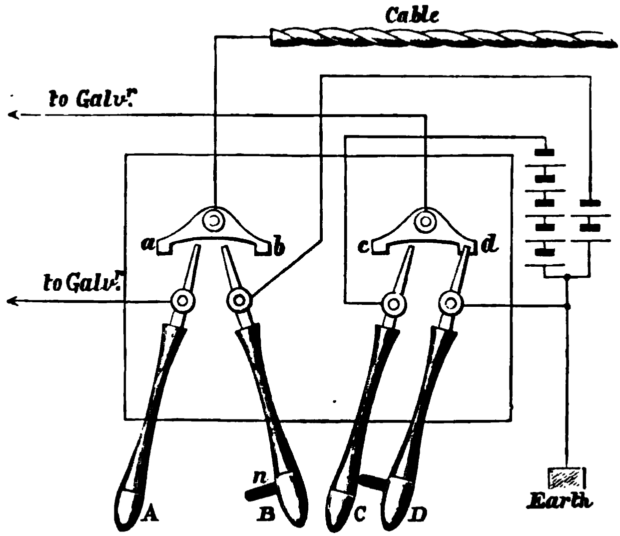


Fig. 215.

of the form shown by Figs. 148 and 149, pages 317 and 318, mounted on one base.

Supposing the connections to be made as shown by the figure, then in order to measure the "discharge" from the cable, levers C and D are set in the positions shown. Lever B is now pressed to the left so that its projecting piece *n* comes in contact with lever A; the brass tongue of lever B is then in contact with *b*, so that the small battery (about 10 Daniell cells), whose zinc pole is joined to lever B, is connected to the cable. If now lever A is pressed over to the right, then lever B is also moved and the tongue of the latter consequently leaves *b* whilst the tongue of A

comes in contact with *a*, and thus puts the cable in connection with the galvanometer. As the second terminal of the galvanometer is connected to the piece *c d*, the circuit is completed to earth through *d* and the tongue of lever D.

To measure the discharge from a condenser, one terminal of the former would be connected to the piece *a b* and the other terminal to earth; the manipulation of the levers would of course be the same as in the case of the cable.

To take the "Insulation" test (page 408) of the cable, levers A and B would be set over to the right so that the tongue of lever A is in contact with *a* whilst the tongue of B is disconnected from *b*. The short-circuit key of the galvanometer being closed, lever C is now pressed over to the right, so that the tongue of lever C comes in contact with *c*, whilst the tongue of lever D becomes disconnected from *d*; the zinc pole of the large battery thus becomes connected through *c* with one terminal of the galvanometer, and as the other terminal is connected (through lever A and *a*) with the cable, the circuit is complete. The short-circuit key of the galvanometer is now depressed, and the deflections noted in the usual manner (page 410). As soon as the observations are completed the short-circuit key of the galvanometer is raised, and lever D being pressed over to the left the battery becomes disconnected from the galvanometer terminal, and the latter is connected to earth, so that the cable discharges itself.

Particular care must be taken that the short-circuit key of the galvanometer is raised before lever D is pressed over to the left, otherwise the whole discharge from the cable will pass through the galvanometer coils, and the needles may either be demagnetised or at least the "constant" of the instrument be altered.

Method of Testing Batteries in the Postal Telegraph Department.

Direct Reading Battery Testing Instruments.

620. One form of apparatus (devised by Mr. A. Eden) employed in the Postal Telegraph Department for battery testing, is shown by Figs. 216 and 217. It consists of two adjustable sets of resistance coils R_1 and R_2 (R and B, Fig. 216), R_1 being in the direct circuit of one coil of a tangent galvanometer * G, and R_2 being a shunt between the terminals of the battery E when the key *k* is de-

* This galvanometer is the same as that employed for making the daily morning tests (Figs. 16 and 202, pages 24 and 553).

pressed. There is also a shunt S of fixed resistance connected through the second coil of the galvanometer and to the key *k*, as shown. The values of the resistance coils forming R_1 are 429, 858, 1716, 3432, 6864, and 13,728 ohms respectively; that is, they

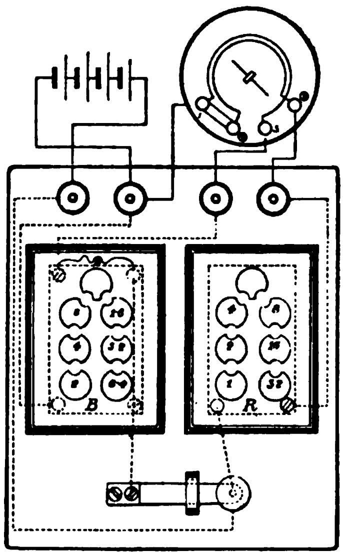


Fig. 216.

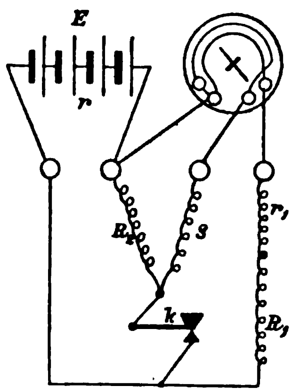


Fig. 217.

are in the proportion of 1 : 2 : 4 : 8 : 16 : 32. In circuit with R_1 is a fixed resistance r_1 of 269 ohms which, together with the resistance of the half coil of the galvanometer, namely 160 ohms, makes up $269 + 160 = 429$ ohms; this resistance r_1 is external to R_1, so that when all the plugs are inserted in the latter, r_1 still remains in circuit.

Electromotive Force Test.

621. The principle of the method of testing for electromotive force is as follows:—

The adjustment of the galvanometer is made such, that $2\frac{1}{2}$ milliampères of current passing through the half coil will give a deflection of 100 divisions. The electromotive force of a Daniell cell is 1·08 volts; if, therefore, such a cell were in circuit with a total resistance of 432 ohms, the resulting current will be

$$\frac{1 \cdot 08 \times 1000}{432} = 2 \cdot 5 \text{ milliampères.}$$

A Daniell cell, therefore, assuming its resistance to be 3 ohms approximately, would if joined up as shown by Figs. 216 and 217, that is with a resistance in circuit of

$$160 + 269 + 3 = 432 \text{ ohms},$$

give on the galvanometer a deflection of 100 divisions, that is provided the electromotive force of the cell were fully up to 1·08 volts; any less deflection than 100 divisions will indicate directly the percentage value of the actual electromotive force. If, say, 5 Daniell cells were in circuit, and also a total resistance of 5×432 ohms, then the deflection obtained should still be 100 divisions, provided each of the five cells had an electromotive force of 1·08 volts; and it is evident that if with a still larger number of cells, there were placed in circuit a total resistance as many times greater than 432 ohms as there are cells to be tested, then if the average electromotive force per cell of the battery were 1·08 volts, the deflection obtained would still be 100 divisions. If the deflections were less than 100, it would show that the average electromotive force per cell of the battery must be proportionately less than 1·08 volts.

A Bichromate battery would, when tested in the same manner, give a deflection of 200 divisions, if in good condition, since the electromotive force of a Bichromate cell is exactly double that of a Daniell, i.e. the force is 2·16 volts, whilst its resistance is approximately the same.

Similarly a Leclanché battery, which has an electromotive force per cell of 1·62 volts approximately, would if in good condition give a deflection of 150 divisions, since

$$1 \cdot 08 : 100 :: 1 \cdot 62 : 150.$$

In order to enable the percentage of fall in force, to which a deflection less than 150 divisions corresponds, to be at once determined, a table is provided.

Resistance Test.

622. Referring to Fig. 217, when the key k is raised, the whole of the current from the battery will flow through the galvanometer. If C be this current, then

$$C = \frac{E}{r + R'},$$

R' being equal to $R_1 + r_1 + \frac{g}{2}$, $\frac{g}{2}$ being the resistance of the half coil of the galvanometer.

When key k is depressed, then the total current, C', passing through the two coils of the galvanometer will be

$$C' = \frac{E}{r + \frac{R_2 A}{R_2 + A}} \times \frac{R_2}{R_2 + A} = \frac{E R_2}{r(R_2 + A) + R_2 A},$$

where

$$A = \frac{R' S'}{R' + S'},$$

S' being equal to $S + \frac{g}{2}$, so that

$$C' = \frac{E R_2}{r\left(R_2 + \frac{R' S'}{R' + S'}\right) + R_2 \frac{R' S'}{R' + S'}}$$

$$= \frac{E R_2 (R' + S')}{r[R_2 (R' + S') + R' S'] + R_2 R' S'}.$$

If R_2 be adjusted so that

$$C = C',$$

then we get

$$\frac{E}{r + R'} = \frac{E R_2 (R' + S')}{r[R_2 (R' + S') + R' S'] + R_2 R' S'},$$

or

$$(r + R') R_2 (R' + S') = r[R_2 (R' + S') + R' S'] + R_2 R' S',$$

therefore

$$r R_2 (R' + S') + R_2 R'^2 + R_2 R' S' = r R_2 (R' + S') + r R' S' + R_2 R' S',$$

therefore

$$R_2 R'^2 = r R' S',$$

therefore

$$r = R' \frac{R_2}{S'}.$$

Since

$$R' = R_1 + r_1 + \frac{g}{2}, \quad \text{and,} \quad S' = S + \frac{g}{2},$$

we have

$$r = \left(R_1 + r_1 + \frac{g}{2}\right) \frac{R_2}{S' + \frac{g}{2}}.$$

Now r (the *total* resistance of the battery) is equal to the resistance per cell, r', multiplied by the number, n, of cells of which the battery is composed; if, therefore, we make $R_1 + r_1 + \frac{g}{2}$ directly proportional to the number of cells, as we do in the case of the electromotive force test, that is, if we make

$$R_1 + r_1 + \frac{g}{2} = \kappa\, n,$$

where κ is a constant; then since

$$r = n\, r'$$

we get

$$n\, r' = \kappa\, n \frac{R_2}{S' + \frac{g}{2}},$$

or

$$r' = \frac{\kappa\, R_2}{S' + \frac{g}{2}}.$$

Now for every cell being tested the resistance included in circuit in making the electromotive force test is 432 ohms, that is $\kappa = 432$, so that

$$r' = R_2 \frac{432}{S + \frac{g}{2}}.$$

If now a fixed value be given to $S + \frac{g}{2}$, then r' will be in direct proportion to R_2. The fixed value is preferably the highest value which $R_1 + r_1 + \frac{g}{2}$ will have, which will be 60×432, or 25,920 ohms, so that

$$r' = R_2 \frac{432}{60 \times 432} = \frac{R_2}{60};$$

that is to say, the *resistance per cell* of the battery being tested is $\frac{1}{60}$th the resistance of the shunt R_2. If, therefore, the resistances of which R_2 is composed are marked with values which are $\frac{1}{60}$th of their actual values, then those marked values will give at once the resistance per cell of the battery under test.

623. An improved form of apparatus based on the foregoing, has been devised by Mr. A. Eden, and is shown by Figs. 218 and 219.

In Fig. 218, R are the resistances which are inserted in circuit in proportion to the number of cells to be tested. B are the resistances for shunting the battery. *b* is a switch which can be

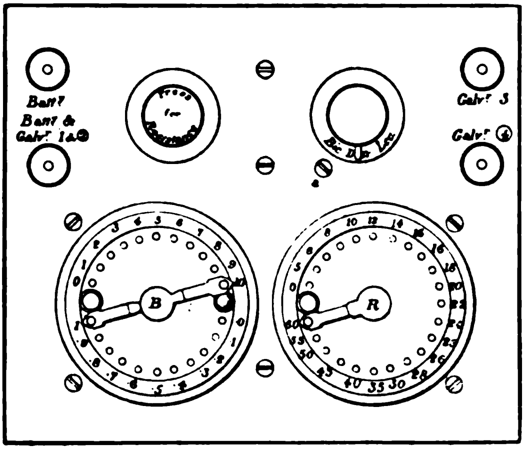


Fig. 218.

turned to three different positions according as Bichromate, Daniell, or Leclanché batteries have to be tested. This switch shunts the galvanometer, so that the normal deflection for each of the three

Fig. 219.

descriptions of batteries shall be 100, instead of 200, 100, and 150, as is the case with the form of apparatus shown by Fig. 216. *a* is a plunger key corresponding to key *k* in the apparatus last referred to.

Combined Resistances.

624. **Problem.**—Required the joint resistance of the resistances a, b, x, d, and g, between the points A and B (Fig. 220).

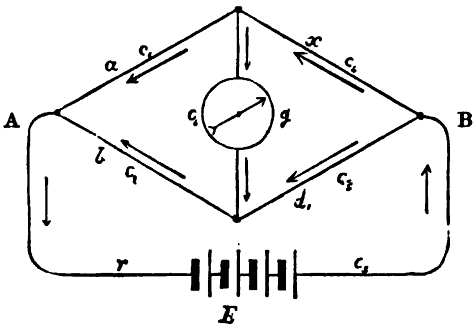


Fig. 220.

If we call R the resistance of the combined resistances between the points A and B, then what we have to do is to obtain an equation of the form

$$c_5 = \frac{E}{r + R}.$$

Now it is obvious that the value of R can be in no way dependent upon the value of r, hence in order to simplify the problem we may assume r to be equal to 0.

By Kirchoff's laws (page 178) we have the following six equations, showing the connection between the resistances a, b, c, d, and g, the current strengths, c_1, c_2, c_3, c_4, c_5, and c_6, and the electromotive force E :—

$$c_5 - c_1 - c_2 = 0 \qquad [1]$$
$$c_4 - c_6 - c_1 = 0 \qquad [2]$$
$$c_3 + c_6 - c_2 = 0 \qquad [3]$$
$$c_3\,d + c_2\,b - E = 0 \qquad [4]$$
$$c_1\,a - c_2\,b - c_6\,g = 0 \qquad [5]$$
$$c_3\,d - c_4\,x - c_6\,g = 0 \qquad [6]$$

In order to determine the value of c_5 from these six equations, we must first find the value of c_1 from, say, equation [1], and substitute this value in the other equations, thereby getting rid of c_1; again in like manner, if we find the value of c_2 from, say, equation [3], and substitute throughout, we get rid of c_2, and so on. As it

will be unnecessary to show all these substitutions, we shall confine ourselves to one or two only; thus from equation [1] we have

$$c_5 - c_1 - c_2 = 0, \quad \text{or,} \quad c_1 = c_5 - c_2;$$

therefore we get

$$c_4 - c_6 - c_5 + c_2 = 0 \qquad [2]$$
$$c_3 + c_6 - c_2 = 0 \qquad [3]$$
$$c_3\,d + c_2\,b - \mathrm{E} = 0 \qquad [4]$$
$$c_5\,a - c_2\,a - c_2\,b - c_6\,g = 0 \qquad [5]$$
$$c_3\,d - c_4\,x - c_6\,g = 0. \qquad [6]$$

By continuing this process, we at length get

$$c_5\,a - c_6\,a - c_6\,b - c_6\,g - (a+b)\frac{\mathrm{E} - c_6\,b}{b+d} = 0$$

and

$$c_5\,x + c_6\,g - (d+x)\frac{\mathrm{E} - c_6\,b}{b+d} = 0\,;$$

therefore

$$c_6\,(a\,d + b\,d + b\,g + d\,g) = c_5\,(a\,b + a\,d) - \mathrm{E}\,(a+b)$$

and

$$c_6\,(b\,g + d\,g + b\,x + b\,d) = -\,c_5\,(b\,x + d\,x) + \mathrm{E}\,(d+x).$$

By dividing one equation by the other, c_6 is eliminated, that is, we get

$$\frac{a\,d + b\,d + b\,g + d\,g}{b\,g + d\,g + b\,x + b\,d} = \frac{c_5\,(a\,b + a\,d) - \mathrm{E}\,(a+b)}{-\,c_5\,(b\,x + d\,x) + \mathrm{E}\,(d+x)},$$

or

$$c_5 = \frac{\mathrm{E}}{\dfrac{(a\,b+a\,d)(b\,g+d\,g+b\,x+b\,d) + (b\,x+d\,x)(a\,d+b\,d+b\,g+d\,g)}{(d+x)(a\,d+b\,d+b\,g+d\,g) + (a+b)(b\,g+d\,g+b\,x+b\,d)}}.$$

By dividing the numerator and denominator of the fraction below the thick line by $a+x$, we finally get

$$c_5 = \frac{\mathrm{E}}{\dfrac{g.[(a+x)(b+d)] + a\,b\,(d+x) + d\,x\,(a+b)}{g\,[(a+x)+(b+d)] + (a+b)(d+x)}};$$

that is to say,

The combined resistance of the resistances, a, b, c, d, x, and g, between A and B $\Big\} =$

$$\frac{g\,[(a+x)(b+d)] + a\,b\,(d+x) + d\,x\,(a+b)}{g\,[(a+x)+(b+d)] + (a+b)(d+x)}.$$

It will be observed that if $g = \infty$, that is to say, if we remove g, then we get

$$\left.\begin{array}{l}\text{Combined}\\ \text{resistance}\end{array}\right\} = \frac{g\,[(a+x)(b+d)]}{g\,[(a+x)+(b+d)]} = \frac{(a+x)(b+d)}{(a+x)+(b+d)},$$

which is the joint resistance of $(a + x)$ and $(b + d)$.

If we have $g = 0$, that is to say, if we join together the two points connected by g, then we get

$$\left.\begin{array}{l}\text{Combined}\\ \text{resistance}\end{array}\right\} = \frac{a\,b\,(d+x) + d\,x\,(a+b)}{(a+b)(d+x)} = \frac{a\,b}{a+b} + \frac{d\,x}{d+x},$$

which is the joint resistance of a and b, added to the joint resistance of d and x.

The truth of these simplifications is obvious.

Combined Condensers.

625. **Problem.**—Required the joint electrostatic capacity of two or more condensers joined up in "cascade."

Let a, b, and c, f, Fig. 221, be the plates of the two condensers, then if we suppose these plates to be of equal size, and d_1 and d_2

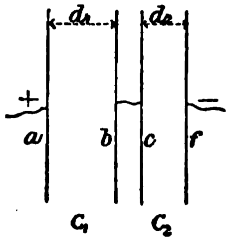


Fig. 221.

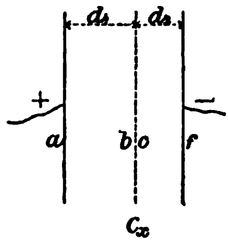


Fig. 222.

to be the distances separating them, the respective capacities C_1 and C_2 will be in the proportion

$$C_1 : C_2 :: d_2 : d_1,$$

or

$$\frac{d_2}{C_1\,d_1} = \frac{1}{C_2}.$$

Now the plates b and c, being joined together, may be considered to be one plate, as shown by the dotted line $b\,c$, Fig. 222; moreover as the latter plate is in no way connected with either of

the charging wires + and −, it practically does not affect the joint capacity of the arrangement; hence we can represent this joint capacity as being due to a condenser formed of the plates a and f, separated by a distance $d_1 + d_2$. The capacity C_x of the combination must therefore be given by the proportion

$$C_x : C_1 :: d_1 : d_1 + d_2,$$

or

$$C_x = \frac{C_1\,d_1}{d_1 + d_2} = \frac{1}{\dfrac{1}{C_1} + \dfrac{d_2}{C_1\,d_1}} = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2}}.$$

If we had a third condenser of a capacity C_3, in the circuit of C_1 and C_2, then the joint capacity C'_x, of this condenser in combination with C_x, must be

$$C'_x = \frac{1}{\dfrac{1}{C_x} + \dfrac{1}{C_3}} = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}},$$

and so on with any number of condensers. Hence we have the law:—

The joint electrostatic capacity of any number of condensers joined together in "cascade" is equal to the reciprocal of the sum of the reciprocals of their respective capacities.

Electrostatic Capacity of Aerial Lines.

626. If we have two wires having diameters d_1 and d_2 respectively, suspended in space, the distance between their axes being $2\,h$; then the electrostatic capacity between the two wires will be

$$\frac{\kappa\, l}{\log \dfrac{16\, h^2}{d_1\, d_2}},\text{*}$$

where l is the length of the wires, and κ a constant, whose numerical value is dependent upon the units in which l, h, d_1, and d_2 are expressed.

If the wires are of equal diameter, that is, if both wires are, say, of a diameter d, then the above equation becomes

$$\frac{\kappa\, l}{\log \dfrac{16\, h^2}{d^2}} = \frac{\kappa\, l}{\log \left(\dfrac{4\, h}{d}\right)^2} = \frac{\kappa\, l}{2 \log \dfrac{4\, h}{d}}.$$

* The proof of this equation is complex, and is beyond the scope of this book.

Since at every point in a plane equidistant from the two wires the potential must be zero, and the electrostatic capacity between this plane and either of the wires (since the distance is one-half that between the wires) must be twice that between the wires, it follows that the capacity of a wire of diameter d suspended *alone* at height h above the ground (this representing the plane referred to, and h being *half* the distance between the wires), must be

$$\frac{\kappa l}{2 \log \frac{4h}{d}} \times 2 = \frac{\kappa l}{\log \frac{4h}{d}}.$$

The mean value of κ obtained from actual experiments on lines in this country is ·0616; l being expressed in miles, and h and d in mils, the resulting answer being in microfarads.

For example.

What is the electrostatic capacity per mile of an aerial wire 112 mils in diameter and 30 feet above the ground?

$d = 112$ mils. $h = 30 \times 12 \times 1000 = 360{,}000$ mils. $l = 1$.

$$\left.\begin{array}{c}\text{Electrostatic}\\ \text{capacity per}\\ \text{mile}\end{array}\right\} = \frac{\cdot 0616 \times 1}{\log \frac{4 \times 360{,}000}{112}} = \frac{\cdot 0616}{4 \cdot 109} = \cdot 0150 \text{ microfarads.*}$$

Practical Measurement by Kempe's Method.

627. The practical measurement of the electrostatic capacity of an aerial line by an ordinary discharge method is often a matter of considerable difficulty, owing to the loss of charge which takes place during the time the lever of the discharge key is moving from the battery to the galvanometer contact, the insulation resistance of an aerial wire, except in very dry weather, being comparatively low. It is best, therefore, to make the test by the method shown by Fig. 223. The line whose electrostatic capacity is required, is joined up to the point c of a Wheatstone bridge, earth being connected to F, as shown. K is a double key which, in its normal position, closes the battery circuit, *and also short-circuits the galvanometer*; the contacts of this key are so set that on depressing the latter the galvanometer short circuit is opened a moment before

* This value, it may be stated, is that of an aerial copper wire weighing 200 lbs. per mile.

the battery circuit. *k* is a supplementary key which enables the short circuit to be opened without opening the battery circuit.

The following, then, is the method of making the measurement:—

Key *k* being depressed, the resistances are adjusted until balance is obtained as nearly as possible on the galvanometer; *k* is then raised, and the galvanometer being thus short-circuited the needle comes exactly to zero. On depressing key K the short circuit is opened and immediately afterwards the battery circuit is broken, the result being that although exact resistance balance may not have been obtained as regards the four arms of the bridge, yet the interval between the opening of the galvanometer short circuit and the battery circuit is so extremely brief, that the needle has no impulse given to it from this want of

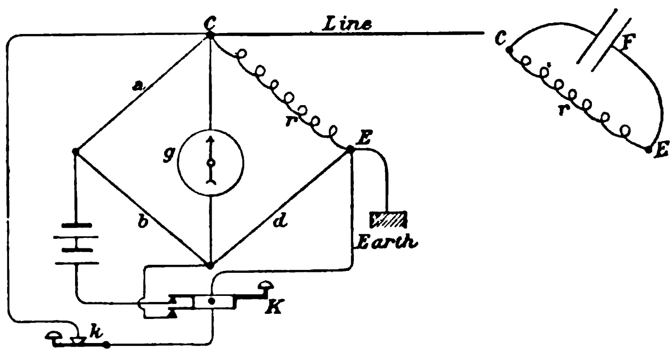


Fig. 223.

balance, but the throw which it receives, and which is due to the discharge from the line, is practically entirely due to this discharge. Moreover, as the latter takes place at the instant that the battery circuit is broken, the loss due to the low insulation of the line is reduced to a minimum; and moreover this loss cannot be material unless the insulation resistance is extremely low, as the whole charge would practically discharge through the combined resistances formed by the bridge, since these resistances would be small compared with the insulation resistance.

After the discharge from the line has been noted, the latter is disconnected from *c*, and a condenser is joined across *c* and E, as shown by the side figure; a discharge is then taken from this condenser, which discharge, compared with that obtained from the line, gives by direct proportion the capacity of the line in terms of the capacity of the condenser, as will be well understood.

It may be added that in practice this method of measurement is found to give very satisfactory and reliable results.

The values which should be given to the arms a, b, d, and r of the bridge will depend upon the length of the line being tested, and also upon the sensitiveness of the galvanometer; the values are best found by trial.

Measurement of the Coefficent of Self-Induction. Kempe's Method.*

628. If r (Fig. 224) be an electromagnet of resistance r in circuit with an external resistance R, then if we excite this magnet by a current of strength C, the discharge which will take place from the magnet if it be allowed to discharge itself the moment the exciting current is taken off, will be directly proportional to C, inversely proportional to the total resistance $R + r$, through which the discharge takes place, and directly proportional to a coefficient which is dependent upon the make of the magnet, i.e. its shape, number of convolutions of its coils, &c.; this constant which is called L, is the coefficient of self-induction of the magnet.

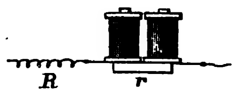


Fig. 224.

If q be the quantity in coulombs (page 77) discharged from the magnet, then

$$q = \frac{C}{R + r} L. \qquad [A]$$

Supposing now we have a condenser (Fig. 225) of F farads capacity shunted by a shunt of S ohms resistance, and let there be a current of C ampères, flowing through R_1 and S, then if e be the potential difference, in volts, between the terminals of the condenser, the charge in the condenser will be

F e coulombs.

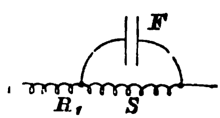


Fig. 225.

If the charging current be now cut off and the condenser be allowed to discharge itself through R_1 and S, the quantity q_1 discharging through R_1 will be

$$q_1 = F e \frac{S}{R_1 + S};$$

* 'Electrical Review,' April 12th, 1899.

but since the current flowing through R_1 and S is equal to C, we have

$$C = \frac{e}{S}, \quad \text{or,} \quad e = C\,S;$$

therefore

$$q_1 = F\frac{C\,S^2}{R_1 + S}. \qquad [B]$$

By combining [A] and [B] we get

$$\frac{q}{q_1} = \frac{L}{F\,S^2} \cdot \frac{R_1 + S}{R + r};$$

therefore

$$L = F\,S^2 \frac{q}{q_1} \cdot \frac{R + r}{R_1 + S}.$$

If $R + r$ is made equal to $R_1 + S$, then

$$L = F\,S^2 \frac{q}{q_1}.$$

629. In order to measure q and q_1 the arrangement shown by Fig. 226 was devised and is found to answer its purpose very

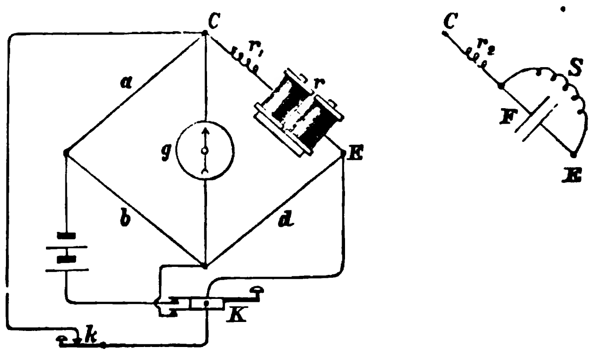


Fig. 226.

satisfactorily. It is an arrangement similar to that devised for measuring electrostatic capacities, page 592.

The magnet r (whose coefficient of self-induction is required) and a resistance r_1 are joined up in one arm of a Wheatstone Bridge, as shown.

The key k being depressed, the resistance d is adjusted until balance is obtained as nearly as possible on the galvanometer;

key k is then raised, and when the galvanometer needle has come exactly to zero, key K is depressed and the throw, d_1, due to the discharge from the magnet r is noted. A shunted condenser (see right-hand figure) in circuit with a resistance r_2, is then substituted in the place of the electromagnet and resistance r_1, and the throw, d_2, with the same is obtained in a similar manner to that in the case of the electromagnet. Then

$$L = F\,S^2\,\frac{d_1}{d_2},$$

since d_1 and d_2 are proportional to q and q_1.

The resistances r_1 and r_2 (which must have no self-induction) are used in order to enable the throws in the two cases to be made approximately the same if the condenser F is not an ádjustable one; but these resistances must be such that

$$r_1 + r = r_2 + S.$$

It is evident that by increasing r_2 and diminishing S, the portion of the discharge from the condenser which passes through the galvanometer can be reduced to any required extent. If it should happen that the discharge from the condenser when r_2 is equal to 0, is smaller than the discharge from the electromagnet, then we can diminish the latter discharge by increasing r_1, S being of course increased so as to make $S = r_1 + r$.

Although, strictly speaking, the equation

$$L = F\,S^2\,\frac{d_1}{d_2}$$

is only correct when

$$a\,d = b\,(r_1 + r),$$

yet the error due to d being slightly out is extremely small, and practically does not affect the correctness of the final result.

As in practice F is expressed in microfarads and not farads, the exact value of L is given by the expression

$$L = F\,S^2\,\frac{d_1}{d_2} \div 1{,}000{,}000.$$

The name given to the unit of self-induction by Professor Ayrton is the "secohm," but the name "henry" is now assigned to this unit.

As the submultiple, l, of the henry, the "millihenry" ($\frac{1}{1000}$th henry) is found more convenient for practical use, we have

$$l = F\,S^2\,\frac{d_1}{d_2} \div 1000 \text{ millihenries.}$$

For example.

In an experiment made with an electromagnet of 400 ohms resistance (r), the values of F, d_1, d_2, r_1, r_2, and S were 1 microfarad, 45 divisions, 275 divisions, 600 ohms, 200 ohms, and 800 ohms, respectively; what was the value of l?

$$l = 1 \times 800 \times 800 \times \frac{45}{275} \div 1000 = 104{\cdot}7 \text{ millihenries.}$$

If the self-induction is such that the discharge current lasts for an appreciable time, it may be necessary to employ a ballistic galvanometer (page 74) in making the measurements.

Measurement of the Self-Induction of a Telegraph-Wire Loop.

630. The measurement of the self-induction of a telegraph-wire loop presents the difficulty of there being the factor of inductive capacity, as well as of self-induction, to be dealt with, and the problem is to separate the one from the other. This may be done in the following manner:—

First join up the two wires (which will afterwards form the loop) as shown by Fig. 227; R being a resistance equal to that

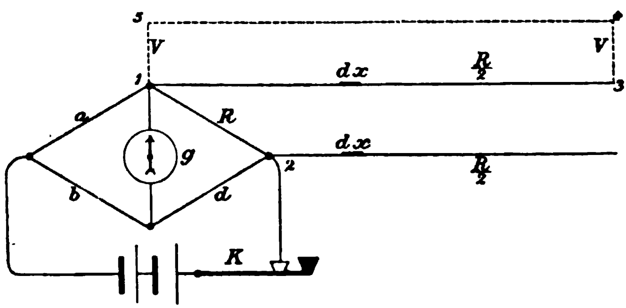


Fig. 227.

of the wire loop. In this case, regarding the two wires as the two plates of a condenser, then the potential difference between any one point on one wire and the adjacent point on the other wire will be the same all along the length of the wires. If then we call V this potential difference, the charge held by the wires will be represented by the area 1, 3, 4, 5, that is by

$$V \frac{R}{2}.$$

Now on depressing key, K, the whole of this charge will discharge itself at the ends 1, 2, of the wires; the portion, Q, of this flowing through the combined resistance of *a*, *b*, *g*, and *d* (which combined resistance we will call R_1), being

$$Q = V \frac{R}{2} \cdot \frac{R}{R_1 + R}. \qquad [A]$$

Next loop the wires as shown by Fig. 228, and remove the resistance R between 1 and 2. Under these conditions, since the resistance between the points 1 and 2 remains unaltered (for the resistance R has been replaced by the resistance of the looped

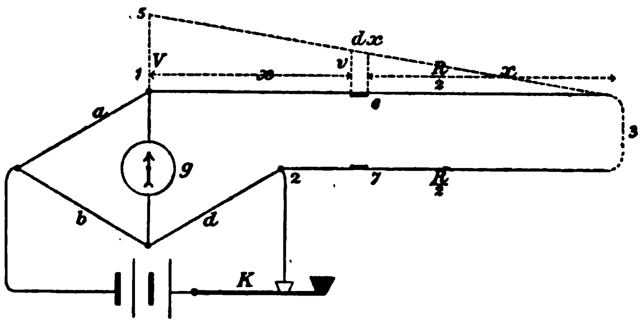


FIG. 228.

wires, to which R is equal), the potential difference between points 1 and 2 will remain unaltered; but this potential difference will decrease along the loop up to 3, at which point (since two adjacent points on the two wires are at this place joined together by the looping of the wires) the potential difference will be zero. The charge held by the two wires will, in fact, in this case, be represented by the triangle 1, 3, 5.

Now if $v\,dx$ be a differential part of the charge, then when the key, K, is depressed, since $2\left(\frac{R}{2} - x\right)$ is the resistance of the loop 6, 3, 7, and $R_1 + R$ is the resistance of the whole looped circuit (R_1 being the combined resistance of *a*, *b*, *g*, and *d*), a portion of this part, namely

$$dQ = v \frac{2\left(\frac{R}{2} - x\right)}{R_1 + R}\, dx,$$

will flow through the combination of resistances R_1. And since

$$V : v :: \frac{R}{2} : \frac{R}{2} - x,$$

or,

$$v = V \frac{\frac{R}{2} - x}{\frac{R}{2}},$$

therefore

$$d\,Q = V \frac{2\left(\frac{R}{2} - x\right)^2}{(R_1 + R)\frac{R}{2}}\,d\,x$$

and the integral of this between the limits $x = R$ and $x = 0$ will give the quantity Q flowing out at 1. As the foregoing equation is of the same form as equation [1], page 498, we can see at once (from [2], page 498) that integration will give the value of Q as

$$Q = \frac{V}{3} \cdot \frac{2\left(\frac{R}{2}\right)^3}{(R_1 + R)\frac{R}{2}} = \frac{V}{3} \cdot \frac{R}{2} \cdot \frac{2\frac{R}{2}}{R_1 + R} = \frac{V}{3} \cdot \frac{R}{2} \cdot \frac{R}{R_1 + R}, \quad [B]$$

which is one-third of the quantity [A] discharged in the case of Fig. 227.

Hence we see that if the discharges in cases Fig. 227 and Fig. 228 are due to electrostatic capacity only, then the discharge in case Fig. 227 will be exactly three times as great as in the case of Fig. 228.

Now actually in case Fig. 228 we have also a discharge due to self-induction, this discharge being in the *reverse* direction to the static discharge; hence if D be the observed discharge obtained in case Fig. 227, and d that in case Fig. 228, then if d_1 be the amount of the discharge due to self-induction, we must have

$$d_1 = \frac{D}{3} - d = \frac{D - 3\,d}{3}.$$

In order to reduce the result corresponding to d_1, to millihenries, we must obtain a discharge deflection d_2 with a condenser of F

microfarads capacity, this condenser being connected in the place of the two wires $\frac{R}{2}$, $\frac{R}{2}$, Fig. 227. We then get

$$l = F\,R^2\,\frac{d_1}{d_2} \div 1000 = F\,R^2\,\frac{D - 3\,d}{3000\,d_2} \text{ millihenries.}$$

For example.

On a looped underground telegraph line, 23 miles long, the following values of F, R, D, d, and d_2 were noted.

$F = \frac{1}{3}$rd mf. R = 200 ohms. D = 96. $d = -54$.* $d_2 = 30$.

What was the self-induction per mile of the loop?

$$l = \tfrac{1}{3} \times 200 \times 200 \times \frac{96 - (3 \times -54)}{3000 \times 30} = \frac{200 \times 200 \times 258}{3 \times 3000 \times 30}$$

$$= 38 \cdot 22 = \frac{38 \cdot 22}{23} = 1 \cdot 662 \text{ millihenries.}$$

631. Tests for self-induction of long lengths of line cannot be made satisfactorily by this method, as on long lines the static discharge so greatly preponderates that it swamps, as it were, the self-induction discharge, and the effect of the latter on the readings becomes barely noticeable. On the other hand, if the line is extremely short, and the self-induction preponderates, yet its value is so small that it gives but a very small discharge effect, and moreover, this small effect is barely noticeable on the galvanometer in consequence of the fact that the conductor resistance of the line being low (through the line being short) it acts as a shunt and allows only a small portion of this small discharge to pass through the galvanometer; recourse must be had in such a case to the "Secohmmeter" (page 601).

Measurement of Mutual Induction.

632. This is a comparatively easy measurement to make.

Let a and b be two wire loops, through one of which, a, a current is sent by means of a battery E, there being a resistance R in the circuit.

If C = current flowing in the loop, μ the coefficient of mutual induction between the two loops, then if q be the quantity discharged through b on raising or depressing key K, we have

$$q = \frac{C\,\mu}{g + r}.$$

* This deflection must be taken as *minus*, as it was in the opposite direction to the other deflections.

Now if V be the potential difference between the ends of R, then

$$C = \frac{V}{R},$$

or,

$$q = \frac{V\mu}{R(g+r)}.$$

Having noted, then, the deflection due to q, take the galvanometer off and connect it between the ends of R, there being in circuit with the galvanometer a condenser of F microfarads capacity (see

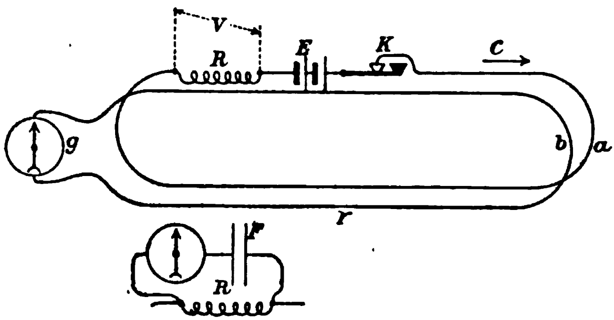


FIG. 229.

small figure). In this case on raising or depressing key K, a discharge deflection will pass through the galvanometer due to the condenser F being charged to a potential V, with a quantity q_1, so that

$$q_1 = V F, \quad \text{or,} \quad V = \frac{q_1}{F}.$$

Hence

$$q = \frac{q_1 \mu}{F R (g+r)},$$

or

$$\mu = F R (g+r) \frac{q}{q_1}.$$

For q and q_1 may, of course, be substituted the discharge deflections d and d_1 due to those quantities, and if F is in microfarads, then

$$\mu = \frac{F R (g+r)}{1000} \cdot \frac{d}{d_1} \text{ millihenries.}$$

For example.

If in a measurement, $F = \frac{1}{3}$ microfarad; $R = 100$ ohms; $g + r = 160$ ohms; $d = 70$ divisions; $d = 120$ divisions; then

$$\mu = \frac{\frac{1}{3} \times 100 \times 160 \times 70}{1000 \times 120} = 3 \cdot 11 \text{ millihenries.}$$

The Secohmmeter.

633. This instrument (devised by Professors Ayrton and Perry) is a most valuable apparatus for measuring self-induction, especially when the latter is of small dimension.

The apparatus consists, in the first place, of two rotary commutators, each with four stationary brushes. The commutators are on the same spindle, one at the front and the other at the back of the apparatus (one seen in Fig. 230), but for convenience they

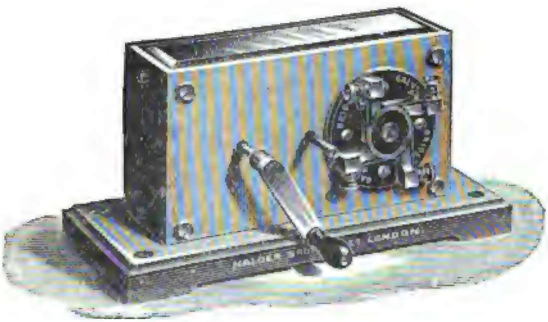


FIG. 230.

are shown in Figs. 231 and 234 as if they were in the same horizontal plane. One commutator, G C, is for periodically reversing the galvanometer connections, and the other, B C, for reversing the battery connections. An adjustment is provided for enabling the relative positions of the two sets of brushes to be varied, so that both sets of reversals can be made to occur simultaneously, or one a little before or after the other, or one reversal midway between two successive reversals of the other.

The commutators can be driven at one or other of two speeds relatively to that of the driving handle. With one arrangement there are four reversals of both the galvanometer and of the battery for one revolution of the handle, and with the other twenty-four reversals of each for one revolution of the handle. The apparatus

can be driven by hand, or more conveniently by a small electromotor, so as to obtain a constant speed of reversal varying from 200 to 6000 reversals per minute of both the galvanometer and the battery. There are two shafts, on either side of which the handle fits, for high or low speed ratios.

The principle of the instrument is as follows:—If a coil having self-induction be joined up in the usual way to a Wheatstone Bridge, and balance be obtained when the battery key is first depressed and then the galvanometer key, it will be found that on reversing the operations, i.e. first depressing the galvanometer key and then the battery key, there will be a sudden impulse on the galvanometer in one direction on completing the battery circuit,

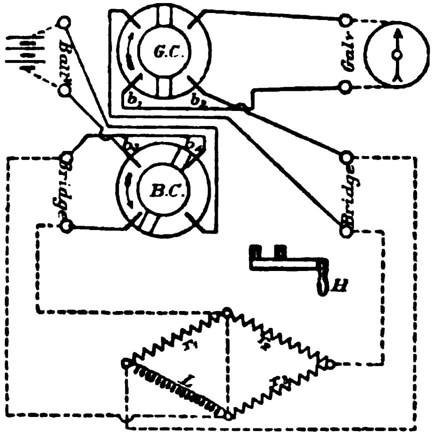


FIG. 231.

and an equal impulse in the opposite direction on breaking that circuit, which impulses are a measure of the self-induction of the coil. If instead of making and breaking, the battery is reversed, the same effect is produced, but the impulses are double the former ones.

As at first explained, by means of one rotating commutator the battery circuit is reversed periodically, so that a succession of impulses would be given to the galvanometer. As these impulses, however, are alternately in opposite directions, no permanent deflection would be obtained on the galvanometer unless its terminals were periodically reversed between successive reversals of the battery. The second commutator is provided for this purpose, and being on the same shaft always reverses the galvanometer

between the reversals of the battery, so that a succession of impulses in the same direction are given; this would result in a steady deflection being obtained, whose magnitude would depend on the self-induction of the coil, and on the number of impulses per second. If, however, there be placed in the adjacent arm of the Bridge a second coil exactly similar, as regards self-induction and resistance, to the first coil, then balance, that is to say, no deflection on the galvanometer, will be produced. If, therefore, this adjacent arm is provided with self-inductive resistances whose values both as regards resistance and also self-induction can be

FIG. 232.

varied, the arrangement forms a zero method of measurement, as in an ordinary Wheatstone Bridge test.

To Compare Two Coefficients of Self-Induction.

We thus have the following method :—

Join up the apparatus as shown in Fig. 231, and then if the resistances r_1 and r_2 of the non-inductive arms of the bridge be adjusted to give balance with a steady current, balance will also be obtained on rotating the commutators, provided

$$\frac{L_1}{L_2} = \frac{r_1}{r_2},$$

L_1 and L_2 being the coefficients of self-induction of the inductive branches.

One of these branches is an adjustable standard of self-induction shown by Fig. 232. The arrangement consists of two coils connected in series. The smaller coil revolves inside the other so that it may be inclined at any angle with it, and by this means the self-induction may be varied continuously between a minimum and a maximum value. The apparatus is, in fact, a modification of that used by Professor Hughes (and suggested as a variable standard of self-induction by Lord Rayleigh). The coils, however, are much wider axially and shallower radially, and in shape are parts of concentric spheres, so that the inner coil may be very close to the outer and yet be free to revolve inside it. By this means a large range of values has been secured. Two scales are provided, one reading directly in quadrants or secohms, and the other being a proportional scale which is merely used by the manufacturer in

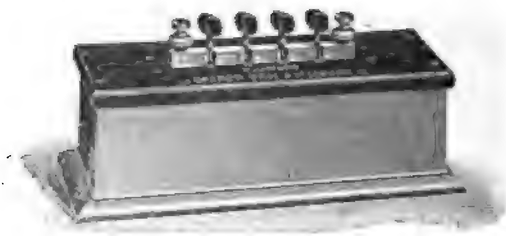


FIG. 233.

the calibration of the instrument. The wire is of copper, so that the resistance may be as low as possible for a given value of the self-induction.

The range of the instrument as made is usually from about ·004 to ·040 secohms or henries.

In order to increase the range of the variable standard it is convenient to use in series with the standard, coils of known self-induction. For this purpose single coils can be used which are made to any value from 10 to 1000 millihenries. To still further increase the facility of working, a box of four coils has been designed, as shown in Fig. 233, in which any one, or more, of the coils can be used, by unplugging as in an ordinary resistance box. The coils being carefully arranged so as not to interfere with one another, the self-induction is simply the sum of that of the coils unplugged. The usual form has coils of 10,

20, 30 and 40 millihenries, so that the range is from 10 to 100 millihenries.

The speed at which the secohmmeter is driven need not be known, but the greater the speed the more sensitive the test; the rate cf reversal must not, however, be so great as to prevent the currents from reaching their steady values between the consecutive reversals of the battery, an excessive rate which can very seldom be attained in simple measurements.

Having selected two resistances in the ratio arms, balance is first obtained for steady currents by turning the secohmmeter handle slightly until contact is made by both commutators, and then altering the resistance in the bridge in the usual manner.

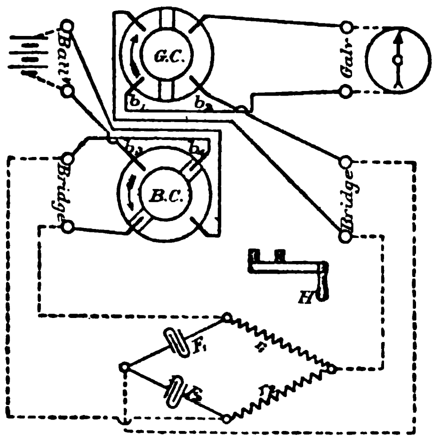


FIG. 234.

The final adjustment is obtained by moving the sliding contact, by which perfect balance can be secured.

When the resistance balance has been obtained, the commutators are rotated slowly, and balance is again restored by turning the handle of the variable standard, Fig. 232. It will, however, probably be found that at the first attempt, balance cannot be obtained by the variable standard, as the self-induction of the coil tested may be outside the limits of the standard. This can be determined by turning the variable standard to its extreme positions in both directions while the commutators are rotated. If these two positions give deflections in opposite directions, balance can be secured at some point of the range; but if both deflections are in the same direction, the self-induction of the coil is either

below the lowest of the standard or, more probably, greater than its highest value, and balance must be got either by altering the ratio coils of the bridge and repeating the above operations, or by the employment of auxiliary fixed standards of self-induction to increase the range of the variable standard.

When taking the finalr eading, it is advisable to run the commutators at a fairly high speed to gain sensitiveness, and it is also desirable when the reading has been obtained, to disturb balance slightly, so as to be certain that contact is maintained. In all such tests it is necessary that the connections should be as short and straight as possible, and that the coil tested should be kept as far away as convenient from the standard. If an auxiliary self-induction box is used in series with the standard it should also be kept at a distance.

To Compare Two Capacities.

Join up the apparatus as in Fig. 234, then balance will be obtained on rotating the commutators, when

$$\frac{F_1}{F_2} = \frac{r_2}{r_1},$$

F_1 and F_2 being the capacities of the condensers, and r_1 and r_2 the resistances of the non-inductive branches of the bridge. As before, increasing the speed at which the commutators are driven merely increases the sensibility of the test without affecting the ratio connecting the capacities with the resistances.

In similar ways two coefficients of mutual induction may be compared with one another, or a coefficient of mutual induction with a coefficient of self-induction, or either of these coefficients with the capacity of a condenser shunted by a non-inductive resistance. The coefficient of self-induction of a coil may also be found absolutely by knowing the speed at which the secohmmeter is run.

To Measure the Resistance of a Polarisable Electrolyte.

Measure the resistance in the ordinary way with a bridge, but connect the galvanometer and battery to the bridge through the commutators, instead of direct; and using the higher speed ratio for the gearing, turn the handle at the highest convenient speed while taking a reading.

It is important that the commutators be set so as to make and break as nearly as possible simultaneously.

634. Mr. J. E. Taylor (of the Engineer-in-chief's Office, General Post Office), who has had large experience of the secohmmeter, makes the following observations with reference to the use of the instrument:—

The ordinary form of Wheatstone Bridge must not be used, as its coils are not neutral in respect to capacity or self-induction. A very convenient form of auxiliary apparatus, arranged as a Wheatstone Bridge with high resistance coils and a slide wire, is one that consists of two similar 3-dial sets, with unit, ten, and hundred ohm coils, and of ratio arms, galvanometer and battery keys. There is also a separate box of high-resistance coils of 1000, 2000, 3000, 4000, and 10,000 ohms, specially wound in the manner stated below, and a slide wire also in a separate box. This box of high-resistance coils and the slide wire can then be used in conjunction with either of the two sets of dials.

It is very necessary to check the resistance coils, especially the higher ones, for capacity effects between ingoing and outcoming wires or between layer and layer. Thus, it will invariably be found that high-resistance bobbins, say 1000 ohms and upwards, which are wound non-inductively in the ordinary way, will have so much capacity between the ingoing and outcoming wires as to seriously affect any measurements, if used indiscriminately with this apparatus. For this reason these coils should either be distributed on a large number of small bobbins, or should be singly wound with the direction of winding reversed at the commencement of each layer; this keeps the outgoing and incoming wires, which have a comparatively high difference of potentials between them, well apart. To check these capacity effects, a series of observations should be taken with all self-induction coils excluded from the bridge, but with the secohmmeter joined up as for a self-induction measurement. It is then best to proceed as follows.

Use a testing battery of five or six volts and first test the ratio arms set at say 1 : 10. Balance these arms in the two sets of dials in the ordinary way, using only low resistance coils in each. Then rotate the commutators rapidly and note whether the galvanometer remains undeflected. If the latter is the case the coils used are correct. Next make tests in the same way with all the other ratios, viz. 1 : 100, 10 : 1 and 100 : 1. After this has been done, test in a similar manner the separate coils in the high resistance box joining it up in series with one of the sets of dials for the purpose. Any want of inductive neutrality will always be negligible in coils of 100 ohms or less.

Some importance must be attached to the correct setting of the galvanometer and battery commutators on the secohmmeter itself, as otherwise small changes of resistance in the bridge, or want of exact resistance balance, may produce serious error in the measurement of self-induction. To effect this correct setting, all self-induction should be excluded from the bridge, and after adjusting the arms with, say, 100 ohms in each, and using equal ratios, the balance should be upset by adding, say, one ohm in one of the arms, and then the commutators should be rotated as before. If the commutators are not accurately set the galvanometer will be deflected, owing to the duration of the reversal contacts with the battery being unequal. By turning one or other of the commutators, little by little, the correct adjustment, which gives no deflection, can be found. With such adjustment a small error in resistance balance is of no moment in the self-induction measurement, as the current strength in the arms containing the unknown and the standard of self-induction is but very slightly affected by a small want of balance, and the discharge due to self-induction is, in coils not having iron cores, strictly proportional to the current strength.

It should be remarked that when using a reflecting galvanometer of high sensibility, a small deflection will be obtained without depressing the galvanometer key, if the commutators be rotated rapidly. This deflection produces little or no error in measurements, as when the galvanometer key is depressed it is rendered inoperative, or nearly so. The deflection is due to two causes: first and chiefly, to the electrification and diselectrification (as the battery reversals take place) of the insulating surfaces of the bridge, and of the commutators, where terminals or contact pieces connected to the galvanometer are fixed; secondly, to the electrostatic capacity of the leads from the galvanometer commutator to the bridge. These effects can be minimised by designing the apparatus so that any parts connected directly to the galvanometer reverser, and the parts on the reverser itself, have as little surface contact with the ebonite, and as much air-insulation, as possible.

It is advisable when obtaining new apparatus, or when any doubt as to the accuracy of the standards (due to warping or otherwise) exists, to check the same in the following way:—

Take a standard condenser shunted by a resistance given by the formula

$$R = \sqrt{\frac{L}{K \times 10^3}},$$

where

R = resistance required, in ohms,
L = self-induction under test in millihenries,
K = capacity of standard condenser in microfarads.

Join the shunted condenser in series with the self-induction standard to be checked and place the two in one arm of the bridge. Balance for resistance, and rotate the commutator. If the self-induction standard is accurate, no deflection will be produced. Prove the sensitiveness of the test by varying the shunting resistance by a small amount, and without upsetting the resistance balance of the bridge. Care must be taken to use a standard condenser of very high insulation resistance, and the shunting resistance coils must be devoid of capacity effects. If this is not the case the test is of no value.

The shunted condenser can also be used, either separately or in conjunction with the self-induction standards, for extreme measurements of very high or very low self-induction; or it can take the place of the standards entirely, at the sacrifice, however, of direct reading and quick balancing.

It is necessary when using high resistance shunts with the condenser to ascertain that the commutator is not being rotated at too high a speed. This is best done by lowering the speed and noting if the result is the same as before.

For the measurements of self-induction coefficients of coils without iron cores and devoid of reactance due to mutual induction, the resistance of the coils not being very excessive, the instrument is dead-accurate. With iron-cored coils the measurements may often be affected somewhat by eddy-current effects in the cores, if the latter are solid.

Under any circumstances the current flowing through the coils under test should be recorded, but especially when iron core coils are being measured, as in such cases the self-induction coefficient varies with the current strength.

Strictly speaking the apparatus measures the total "magnetic flux per unit current strength" in the coil under test, rather than the "coefficient of self-induction." Consequently where mutual inductance reactions come into play, the measurement is no longer that of the coefficient of self-induction. This renders measurements of coils with metallic bobbins or checks misleading, unless due allowance be made.

A Method of Determining the Relative Positions, and the Top and Bottom Ends, of several Cables in a Tank.

635. In the event of the ends of several sections of cable, of similar type and in the same tank, being either marked wrongly, or their labels being defaced or detached, it is important to have some certain method of ascertaining the locality of the top and bottom end of each section, so that no mistake can be made when splicing one section to another prior to paying out. Also, in the event of no records being available of the order in which several sections were coiled down into a tank, some reliable means of ascertaining their relative positions in the tank is desirable. These difficulties are met by Mr. H. W. Sullivan in the following manner.

To Determine the Relative Positions of Several Lengths.

1. "Trace all ends in the tank by means of a detector and battery, and mark them in pairs 1–1, 2–2," &c. This would be best done by a conductor resistance test, so as to obtain at the same time the approximate length of each piece, for reasons mentioned further on.

2. Connect up the top section (the ends of which are always traceable by sending a man down into the tank) in circuit with a small battery (say three cells) and a "make and break" key, first looping together in pairs the ends of the other cables. At the instant of making battery contact so as to complete the circuit of the top section, which may be called the *primary* coil, a more or less momentary current, in the opposite direction, will be induced in the other cables—each acting as a *secondary* coil; but distance being a factor in induction, this induced current will be strongest in the cable immediately underneath the top piece, and weakest in that farthest away from it.

In the case of three cables, A, B, and C, the relative lengths of the pieces may possibly be such that the inductive effect in C might be greater than in B, i.e. C might be of such length as compared with B as to more than make up for the closer proximity of B to A, thus erroneously indicating A and C to be adjacent lengths. To avoid possible error of this kind, the induced current is measured under two conditions; firstly with the ends of all the other cables *looped*, and secondly with all ends *free*. Should the *supposed* second section from the top of the tank be really the one immediately below the top section, the current induced in it will

be the same in each case; but should a length intervene, the effect of looping the intervening length will be to diminish the induced discharge in what was *supposed* to be the second section, the interposed looped cable appearing to act as a partial screen to the inductive energy given out by the top section.

The second cable in the tank having been thus located, it is then in its turn treated as the *primary*, and the third cable is similarly ascertained, and so on. In this way, be the relative lengths what they may, the positions of the several cables are determinable with certainty.

To Identify the Top and Bottom Ends.

First Method.—Join up cable A (as the primary) in circuit with a key and battery and a shunted galvanometer, and note the *direction* of the deflection (D), and also observe to which terminal of the galvanometer the *top* end of the cable is connected. Next remove the galvanometer, leaving only the battery and key in the primary circuit, and join the two ends of the secondary cable B, to the same galvanometer, and at the moment of making battery contact again through A, note on the galvanometer the direction of the current (*d*) induced in B.

If D and *d* are in *opposite* directions, the ends of cables A and B have been correspondingly joined up to the galvanometer terminals; that is, the top end of each will have been connected to the same terminal; but if D and *d* are in the *same* direction, the ends of B will have been reversed to those of A. B is then made the primary, and the ends of C are similarly identified.

Second Method.—The above method of identifying ends depends on the assumption that both the top section and other lengths of cable to be compared with it *are coiled in the same direction*, which is generally the case. For certain special reasons, however, this may not always be so, and therefore, to meet such altered conditions, Mr. Sullivan has devised a second test. This is based upon the fact that upon charging a cable, the distant end does not at once assume the full potential, and an induced current must necessarily follow the same law. The primary cable is charged as before, and the current induced thereby in the secondary is measured first at one end and then at the other, the distant end being free in each instance, and the galvanometer being connected between the end of the cable and the tank. The end of the cable from which the *greatest* deflection is obtained is the *top* end.

Proof of the Principle of the Wheatstone Bridge.

636. Let a, b, x, d be the four arms of the bridge; g the galvanometer, and E the battery; and let V be the potential at the point A. This potential V will fall regularly along the resistances forming the arms of the bridge down to B, at which

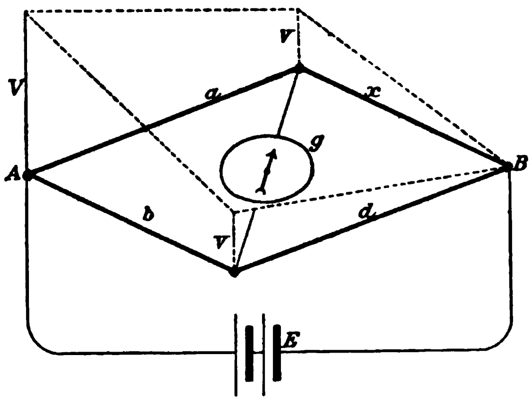


Fig. 235.

point it may be taken to be zero. When balance is effected, then the potentials at the two terminals of the galvanometer will be the same, let each of these potentials be v, we then have the following proportions :—

$$V : v :: a + x : x,$$

and

$$V : v :: b + d : d,$$

therefore

$$\frac{a + x}{x} = \frac{b + d}{d},$$

or

$$\frac{a}{x} + 1 = \frac{b}{d} + 1,$$

that is

$$\frac{a}{x} = \frac{b}{d},$$

or

$$x = \frac{a\,d}{b}.$$

Sullivan's Universal Shunt.

637. For use with his well known galvanometer (page 70), Mr. Sullivan has designed a special universal shunt of the Ayrton-Mather form (page 96), in which he applies the principle of the Thomson-Varley slide resistances (page 231), with the result that a very fine degree of subdivision is obtained.

The ordinary universal shunt gives only a limited number of ratios, while each of the 101 coils of 1000$^\omega$ each in the Thomson-Varley slide is only subdivisible into 100 equal parts.

In the Sullivan shunt, only 11 (instead of 101) coils of 1000$^\omega$ each are employed and the subdivision is carried down to one ohm, a very wide range of ratios or multiplying powers being consequently attained.

Fig. 236 shows the theoretical connections of this shunt box. The actual connections are made by means of sliding contacts over

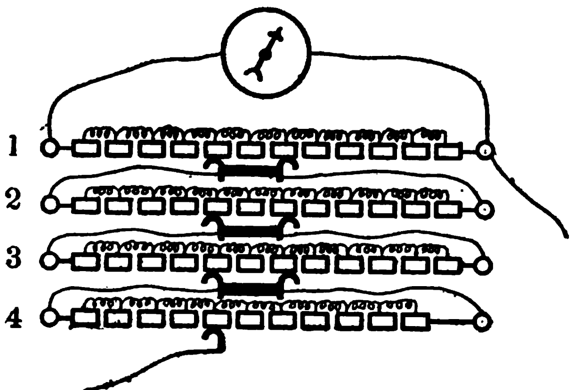


Fig. 236.

the studs of the four sets of coils, No. 1 set of which contains 11 coils of 1000$^\omega$ each, No. 2, 11 coils of 200$^\omega$, No. 3, 11 coils of 40$^\omega$, and No. 4, 10 coils of 8$^\omega$ each.

Two sliding contact springs, insulated from each other, ride over and embrace two coils of Nos. 1, 2 and 3 sets, and each pair of coils so embraced is permanently in parallel with all the coils in the next lower set. No. 4 sliding contact is a single one. Thus we have derived circuit upon derived circuit, the function and the action being similar to that of a vernier on a scale, only that the

subdivision is carried further, as each 1000^{ω} coil can be subdivided by means of the second set of coils into 10 equal parts, by means of the third set of coils into 100 equal parts, and finally by means of the fourth set of coils into 1000 equal parts.

It will thus be seen that this shunt reads differences of one ohm, and allows of the flow of current through the galvanometer being regulated with the greatest degree of nicety. To ascertain the ratio or multiplying power of any shunt reading, it is simply necessary to take its reciprocal number, the shunt readings being

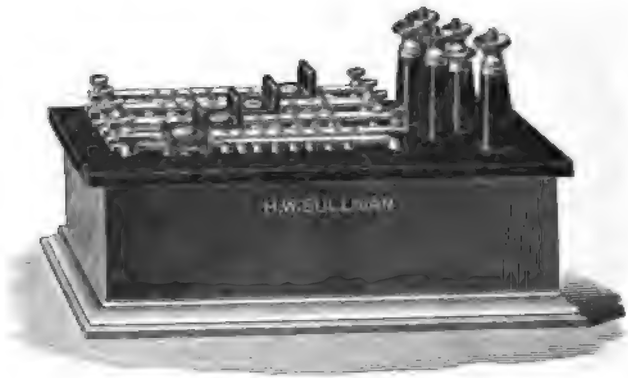


Fig. 237.

of course treated as decimals. Thus, for example, a shunt reading of 4554 would be ·4554, the reciprocal number of which is 2·19587; or the ratio of the reading to 10,000 is,

$$\frac{10{,}000}{4554} = 2{\cdot}19587.$$

This shunt box can also be very conveniently used as a proportional bridge in resistance measurements, or to serve as the resistances in Thomson's electrostatic capacity test (page 369).

It should be mentioned that the box contains, in addition to the other resistances, an independent coil of $100{,}000^{\omega}$ which forms a very convenient standard for the comparison of high resistances by the direct deflection method (page 404), or for localising faults in cables by the fall of potential test (page 431).

Fig. 237 shows the actual form in which the shunt box is made.

TESTING DRY-CORE CABLES.

638. The testing of dry-core cables—i.e. cables with the conductors insulated with paper, and enclosed in lead piping—involves somewhat elaborate arrangements in order to ensure the testing being satisfactorily and quickly carried out. As some dry-core cables have as many as 600 wires, the insulation, &c., of each of which has to be measured, the necessity of a systematic arrangement by which the tests can be rapidly made is obvious.

639. In the case of short lengths of less than, say 440 yards, a readable deflection can rarely be obtained on any single wire of the cable, even with the most delicate form of mirror galvanometer and the highest battery power which it would be safe to use. It has, therefore, become the practice to join the wires in certain groups when testing at the cable works, these groups being so arranged as to provide the longest possible length of conductor in the short length of cable, with due regard to the provision of a completely satisfactory test.

It being necessary to ascertain the insulation of each wire from every other wire of the cable with which it is physically possible for it to be in contact, no wire may be tested in the same group as:—

(*a*) Its companion wire in the same pair.

(*b*) Either wire of the pair on either side of it in the same layer.

(*c*) Any wire of the layer above or beneath that of which it forms a part.

Taking the case of the 74-wire cable, Fig. 238, for example, the following wires can be tested together in the outside layer:—Nos. 39, 43, 37, 51, 55, 59, 63, 67 and 71, provided the wires which lie between them in the same layer are to earth whilst the test is being taken. With these can be tested in the same group, the wires numbered 3, 7, and 11, provided all the remaining wires in the same layer, and all the wires in the second layer, are to earth whilst the test is being made.

Similarly, the following wires in the second layer can be tested together, viz.:—Nos. 15, 19, 23, 27, 31, and 35, and with them can be associated No. 1 from the centre pair, provided, as before, that all the other wires in the cable are to earth whilst the test is being made, and the same remark applies to the next row of wires, thus forming two groups of seven wires each. The remaining two groups must consist of six wires only, as no wires are left to test with which to associate them.

It will be seen then that a complete test can be made of a 74-wire cable by arranging the conductors as follows :—

4 groups of 12 wires each;
2 „ 7 „
2 „ 6 „

FIG. 238.

In the same way a 54-wire cable can be tested in—

4 groups of 8 wires each;
4 „ 4 „
2 „ 2 „
2 single wires.

With a 30-wire cable the necessary tests are actually more numerous, as the eight wires forming the four central pairs must all be tested separately, it being possible for any one of them to be in contact with any of the seven remaining wires; also two of the wires in the outer layer of the cable must also be tested separately, but the remaining wires of the outer layer can be tested in four groups of five wires each. Fourteen tests are, in fact, actually necessary to prove the insulation of the 30 wires.

On the other hand, the larger cables do not involve a corre-

sponding increase in the number of tests necessary. Thus, the 122, 182, 254, 338, 434, and 542 wire cables can each be fully tested by means of eight tests, in the same way as in the case of the 74-wire cable; the additional layers would in this case be each formed into four groups of wires, each group being tested with a group in another layer not immediately contiguous to it: in the same way, in fact, as the wires in the third layer were added to the wires in the first layer in the example before given. For instance, in a 542-wire cable the wires in the first, third, fifth, seventh, ninth, and eleventh layers could be tested together, forming four groups consisting of 75 wires each; the wires in the second, fourth, and sixth layers would be formed into four groups of 60 wires each, and to each of two of these groups would be added a single wire from the central pair, the entire cable thus being tested in:— 4 groups of 75 wires each, 2 groups of 62 wires each, and 2 groups of 60 wires each.

The same principle applies to the other standard types of larger size, viz. 96, 150, 216, 294, 384, 486, and 600 wire cables; but in consequence of each of the layers having an odd number of pairs the arrangement will have to be somewhat different.

The general method of testing will be made clear by describing in detail the most convenient arrangement of the cable for the purpose of making the test.

Preparation of Cable for Testing.

640. The ends having been sawn off, the sheathing is carefully cut by means of pipe-cutters, about 15 inches from the sawn end, and then pulled of, leaving the stranded conductors bare.

Cord is now tied around the outside layers close to the end of the sheathing, and the alternate pairs of the layer are bent up at right angles, the two bottom pairs forming a diameter across the end of the cable, and with the remaining pairs forming a fan-shaped structure.

The remainder of the cable is now tied with string, about one and a half inch nearer the end, and the remaining pairs of the outer layers are also bent out at right angles to their normal direction into a similar fan-like formation.

A third tie with string is now made around the next layer of the cable, one and a half inch from the former tie, and the alternate pairs of this layer are bent out as before. After a fourth tie has been made, this process is repeated with the remaining pairs of that layer.

The remaining layer of the cable is twice tied with strings,

and its six pairs bent out alternately as before, the central pair being left unbent, and occupying its normal position.

Each pair is now divided about two inches from the end, and, the individual conductors being separated by about one inch, the paper insulation is removed for about half an inch, and the conductors themselves are cleaned. The separation is made in the direction of the original line of the conductors, that is, in line with the cable itself, and the end so prepared is then ready for wiring in parallel lines to form the eight groups of conductors referred to above, forming semicircles of diminishing radii from the outer layer towards the centre. At the end of each row of conductors a tailpiece of binding wire is left for connecting the groups together, so as to secure either the tested group, or the remaining groups bunched for connection, to the sheathing as earth.

The free end of the cable, as tested at the factory, is simply stripped for a few inches, and the pairs are carefully separated out so as to ensure that no small pieces of thread or string shall bridge the conductors.

Testing.

641. In order to test the insulation resistance of dry-core cables a high battery power and a delicate galvanometer are requisite, even when the conductors are arranged in groups, as the insulation resistance, after 1 minute electrification, should never be less than 10,000 megohms per mile at a temperature of not less than 50° F., and it frequently reaches ten times that figure when the cables leave the factory. The rate of electrification is less than that of gutta-percha.

The grouped ends must be well covered with hot paraffin wax.

Electrostatic Capacity.

642. It may be added that the electrostatic capacity of each wire in a dry-core cable, measured against all the remaining wires of the cable, and with the lead sheathing as earth, does not exceed, in the case of 20-lb. and 40-lb. conductors, ·08 microfarad per mile, and in the case of 150-lb. conductors, ·1 microfarad per mile; the temperature of the cable during the test being not less than 50° Fahr.

The mean electrostatic capacity from *wire to wire* of each pair in any length of cable (all the wires in the cable, and the testing battery and apparatus being insulated) should not exceed 70 per cent. of the mean wire-to-earth capacity, this latter capacity being measured as just described.

TABLES.

TABLE I.—NATURAL TANGENTS.

Degrees.	Tangents.	Degrees.	Tangents.	Degrees.	Tangents.	Degrees.	Tangents.	Degrees.	Tangents.	Degrees.	Tangents.
1·00	·0175	16·00	·2867	31·00	·6009	46·00	1·0355	61·00	1·8040	76·00	4·0108
1·25	.0218	16·25	·2915	31·25	·6068	46·25	1·0446	61·25	1·8228	76·25	4·0867
1·50	·0262	16·50	·2962	31·50	·6128	46·50	1·0538	61·50	1·8418	76·50	4·1653
1·75	·0306	16·75	·3010	31·75	·6188	46·75	1·0630	61·75	1·8611	76·75	4·2468
2·00	·0349	17·00	·3057	32·00	·6249	47·00	1·0724	62·00	1·8807	77·00	4·3315
2·25	·0393	17·25	·3105	32·25	·6310	47·25	1·0818	62·25	1·9007	77·25	4·4194
2·50	·0437	17·50	·3153	32·50	·6371	47·50	1·0913	62·50	1·9210	77·50	4·5107
2·75	·0480	17·75	·3201	32·75	·6432	47·75	1·1009	62·75	1·9416	77·75	4·6057
3·00	·0524	18·00	·3249	33·00	·6494	48·00	1·1106	63·00	1·9626	78·00	4·7046
3·25	·0568	18·25	·3298	33·25	·6556	48·25	1·1204	63·25	1·9840	78·25	4·8077
3·50	·0612	18·50	·3346	33·50	·6619	48·50	1·1303	63·50	2·0057	78·50	4·9152
3·75	·0655	18·75	·3395	33·75	·6682	48·75	1·1403	63·75	2·0278	78·75	5·0273
4·00	·0699	19·00	·3443	34·00	·6745	49·00	1·1504	64·00	2·0503	79·00	5·1446
4·25	·0743	19·25	·3492	34·25	·6809	49·25	1·1606	64·25	2·0732	79·25	5·2672
4·50	·0787	19·50	·3541	34·50	·6873	49·50	1·1708	64·50	2·0965	79·50	5·3955
4·75	·0831	19·75	·3590	34·75	·6937	49·75	1·1812	64·75	2·1203	79·75	5·5301
5·00	·0875	20·00	·3640	35·00	·7002	50·00	1·1918	65·00	2·1445	80·00	5·6713
5·25	·0919	20·25	·3689	35·25	·7067	50·25	1·2024	65·25	2·1692	80·25	5·8197
5·50	·0963	20·50	·3789	35·50	·7133	50·50	1·2131	65·58	2·1943	80·50	5·9758
5·75	·1007	20·75	·3789	35·75	·7199	50·75	1·2239	65·75	2·2199	80·75	6·1402
6·00	·1051	21·00	·3839	36·00	·7265	51·00	1·2349	66·00	2·2460	81·00	6·3138
6·25	·1095	21·25	·3889	36·25	·7332	51·25	1·2460	66·25	2·2727	81·25	6·4971
6·50	·1139	21·50	·3939	36·50	·7400	51·50	1·2571	66·50	2·2998	81·50	6·6912
6·75	·1184	21·75	·3990	36·75	·7467	51·75	1·2685	66·75	2·3276	81·75	6·8969
7·00	·1228	22·00	·4040	37·00	·7536	52·00	1·2799	67·00	2·3559	82·00	7·1154
7·25	·1272	22·25	·4091	37·25	·7604	52·25	1·2915	67·25	2·3850	82·25	7·3479
7·50	·1317	22·50	·4142	37·50	·7673	52·50	1·3032	67·50	2·4142	82·50	7·5958
7·75	·1361	22·75	·4193	37·75	·7743	52·75	1·3151	67·75	2·4443	82·75	7·8606

8·00	·1405	23·00	·4245	38·00	·7813	53·00	1·3270	68·00	2·4751	83·00	8·1443
8·25	·1450	23·25	·4296	38·25	·7883	53·25	1·3392	68·25	2·5065	83·25	8·4490
8·50	·1495	23·50	·4348	38·50	·7954	53·50	1·3514	68·50	2·5386	83·50	8·7769
8·75	·1539	23·75	·4400	38·75	·8026	53·75	1·3638	68·75	2·5715	83·75	9·1309
9·00	·1584	24·00	·4452	39·00	·8098	54·00	1·3764	69·00	2·6051	84·00	9·5144
9·25	·1629	24·25	·4505	39·25	·8170	54·25	1·3891	69·25	2·6395	84·25	9·9310
9·50	·1673	24·50	·4557	39·50	·8243	54·50	1·4019	69·50	2·6746	84·50	10·3854
9·75	·1718	24·75	·4610	39·75	·8317	54·75	1·4150	69·75	2·7100	84·75	10·8829
10·00	·1763	25·00	·4663	40·00	·8391	55·00	1·4281	70·00	2·7475	85·00	11·4301
10·25	·1808	25·25	·4716	40·25	·8466	55·25	1·4415	70·25	2·7852	85·25	12·0346
10·50	·1853	25·50	·4770	40·50	·8541	55·50	1·4551	70·50	2·8239	85·50	12·7062
10·75	·1899	25·75	·4823	40·75	·8617	55·75	1·4687	70·75	2·8636	85·75	13·4566
11·00	·1944	26·00	·4877	41·00	·8693	56·00	1·4826	71·00	2·9042	86·00	14·3007
11·25	·1989	26·25	·4931	41·25	·8770	56·25	1·4966	71·25	2·9460	86·25	15·2571
11·50	·2035	26·50	·4986	41·50	·8847	56·50	1·5108	71·50	2·9887	86·50	16·3499
11·75	·2080	26·75	·5040	41·75	·8925	56·75	1·5253	71·75	3·0326	86·75	17·6106
12·00	·2126	27·00	·5095	42·00	·9004	57·00	1·5399	72·00	3·0777	87·00	19·0811
12·25	·2171	27·25	·5150	42·25	·9083	57·25	1·5547	72·25	3·1240	87·25	20·8188
12·50	·2217	27·50	·5206	42·50	·9163	57·50	1·5697	72·50	3·1716	87·50	22·9038
12·75	·2263	27·75	·5261	42·75	·9248	57·75	1·5849	72·75	3·2205	87·75	25·4517
13·00	·2309	28·00	·5317	43·00	·9325	58·00	1·6003	73·00	3·2709	88·00	28·6363
13·25	·2355	28·25	·5373	43·25	·9407	58·25	1·6160	73·25	3·3226	88·25	32·7303
13·50	·2401	28·50	·5430	43·50	·9490	58·50	1·6319	73·50	3·3759	88·50	38·1885
13·75	·2447	28·75	·5486	43·75	·9573	58·75	1·6479	73·75	3·4308	88·75	45·8294
14·00	·2493	29·00	·5543	44·00	·9657	59·00	1·6643	74·00	3·4874	89·00	57·2900
14·25	·2540	29·25	·5600	44·25	·9742	59·25	1·6808	74·25	3·5457	89·25	76·3900
14·50	·2586	29·50	·5658	44·50	·9827	59·50	1·6977	74·50	3·6059	89·50	114·5887
14·75	·2633	29·75	·5715	44·75	·9913	59·75	1·7147	74·75	3·6680	89·75	229·1817
15·00	·2679	30·00	·5774	45·00	1·0000	60·00	1·7321	75·00	3·7321	90·00	∞
15·25	·2726	30·25	·5832	45·25	1·0176	60·25	1·7450	75·25	3·7983		
15·50	·2778	30·50	·5890	45·50	1·0088	60·50	1·7675	75·50	3·8667		
15·75	·2820	30·75	·5949	45·75	1·0265	60·75	1·7856	75·75	3·9375		

TABLE II.*—RESISTANCE OF A KNOT-POUND OF COPPER WIRE OF VARIOUS CONDUCTIVITIES, AT 75° FAHR., IN STANDARD OHMS.

Percentage of Conductivity.	Resistance. S. Ohms.	Percentage of Conductivity.	Resistance. S. Ohms.	Percentage of Conductivity.	Resistance. S. Ohms.	Percentage of Conductivity.	Resistance. S. Ohms.
105·0	1126·7	102·0	1159·9	99·0	1195·0	96·0	1232·4
104·9	1127·8	101·9	1161·0	98·9	1196·2	95·9	1233·7
104·8	1128·9	101·8	1162·1	98·8	1197·4	95·8	1234·9
104·7	1129·9	101·7	1163·3	98·7	1198·7	95·7	1236·2
104·6	1131·0	101·6	1164·4	98·6	1199·9	95·6	1237·5
104·5	1132·1	101·5	1165·6	98·5	1201·0	95·5	1238·8
104·4	1133·2	101·4	1166·7	98·4	1202·3	95·4	1240·1
104·3	1134·3	101·3	1167·9	98·3	1203·5	95·3	1241·4
104·2	1135·4	101·2	1169·0	98·2	1204·7	95·2	1242·7
104·1	1136·4	101·1	1170·2	98·1	1206·0	95·1	1244·0
104·0	1137·5	101·0	1171·3	98·0	1207·2	95·0	1245·3
103·9	1138·7	100·9	1172·5	97·9	1208·4	94·9	1246·6
103·8	1139·8	100·8	1173·7	97·8	1209·7	94·8	1248·0
103·7	1140·8	100·7	1174·8	97·7	1210·9	94·7	1248·1
103·6	1142·0	100·6	1176·0	97·6	1212·2	94·6	1250·6
103·5	1143·1	100·5	1177·1	97·5	1213·3	94·5	1251·9
103·4	1144·1	100·4	1178·3	97·4	1214·6	94·4	1253·2
103·3	1145·2	100·3	1179·5	97·3	1215·8	94·3	1254·5
103·2	1146·4	100·2	1180·7	97·2	1217·1	94·2	1255·9
103·1	1147·4	100·1	1181·9	97·1	1218·4	94·1	1257·2
103·0	1148·6	100·0	1183·1	97·0	1219·6	94·0	1258·5
102·9	1149·7	99·9	1184·2	96·9	1220·9	93·9	1259·9
102·8	1150·9	99·8	1185·4	96·8	1222·2	93·8	1261·3
102·7	1151·9	99·7	1186·6	96·7	1223·4	93·7	1262·6
102·6	1153·1	99·6	1187·8	96·6	1224·7	93·6	1264·0
102·5	1154·2	99·5	1189·0	96·5	1226·0	93·5	1265·3
102·4	1155·3	99·4	1190·2	96·4	1227·2	93·4	1266·7
102·3	1156·4	99·3	1191·3	96·3	1228·5	93·3	1268·0
102·2	1157·6	99·2	1192·5	96·2	1229·8	93·2	1269·4
102·1	1158·7	99·1	1193·8	96·1	1231·1	93·1	1270·7

Resistance of "statute-mile-pound" equals resistance of "knot-pound" multiplied by ·752422. log. ·752422 = $\bar{1}$·8764614.

* See page 459.

Table IV.*—Clark, Forde and Taylor's Coefficients for Correcting the Observed Resistance of High Conductivity Copper Wire, at any Temperature, to 75° Fahr., or at 75° to any Temperature.

Temp. Fahr.	Co-efficient.	Logarithm.	Temp. Fahr.	Co-efficient.	Logarithm.	Temp. Fahr.	Co-efficient.	Logarithm.
85	·9787	$\bar{1}$·99063	67	1·0177	·00762	49	1·0598	·02521
84·5	·9797	·99109	66·5	1·0188	·00810	48·5	1·0610	·02571
84	·9808	·99156	66	1·0200	·00858	48	1·0622	·02621
83·5	·9818	·99202	65·5	1·0211	·00906	47·5	1·0634	·02670
83	·9829	·99249	65	1·0222	·00955	47	1·0646	·02720
82·5	·9839	·99295	64·5	1·0233	·01003	46·5	1·0659	·02770
82	·9850	·99342	64	1·0245	·01051	46	1·0671	·02820
81·5	·9861	·99389	63·5	1·0257	·01099	45·5	1·0683	·02870
81	·9871	·99436	63	1·0268	·01148	45	1·0695	·02920
80·5	·9882	·99482	62·5	1·0279	·01196	44·5	1·0708	·02970
80	·9892	·99529	62	1·0291	·01245	44	1·0720	·03021
79·5	·9903	·99576	61·5	1·0302	·01293	43·5	1·0733	·03071
79	·9914	·99623	61	1·0310	·01324	43	1·0745	·03121
78·5	·9924	·99670	60·5	1·0325	·01390	42·5	1·0757	·03171
78	·9935	·99717	60	1·0337	·01439	42	1·0770	·03222
77·5	·9946	·99764	59·5	1·0349	·01487	41·5	1·0783	·03272
77	·9950	·99811	59	1·0360	·01536	41	1·0795	·03323
76·5	·9967	·99858	58·5	1·0372	·01585	40·5	1·0808	·03373
76	·9978	·99905	58	1·0384	·01634	40	1·0821	·03424
75·5	·9989	·99952	57·5	1·0395	·01683	39·5	1·0833	·03475
75	1·0000	·00000	57	1·0407	·01732	39	1·0846	·03526
74·5	1·0011	·00047	56·5	1·0419	·01781	38·5	1·0858	·03576
74	1·0022	·00095	56	1·0430	·01830	38	1·0871	·03627
73·5	1·0033	·00142	55·5	1·0442	·01879	37·5	1·0884	·03678
73	1·0044	·00189	55	1·0454	·01928	37	1·0897	·03729
72·5	1·0055	·00236	54·5	1·0466	·01977	36·5	1·0910	·03780
72	1·0066	·00284	54	1·0478	·02026	36	1·0922	·03831
71·5	1·0077	·00332	53·5	1·0490	·02075	35·5	1·0935	·03882
71	1·0088	·00380	53	1·0501	·02125	35	1·0948	·03933
70·5	1·0099	·00427	52·5	1·0513	·02174	34·5	1·0961	·03984
70	1·0110	·00475	52	1·0525	·02223	34	1·0974	·04036
69·5	1·0121	·00523	51·5	1·0537	·02272	33·5	1·0987	·04087
69	1·0132	·00571	51	1·0549	·02322	33	1·1000	·04138
68·5	1·0144	·00618	50·5	1·0561	·02372	32·5	1·1013	·04189
68	1·0155	·00666	50	1·0573	·02422	32	1·1026	·04241
67·5	1·0166	·00714	49·5	1·0585	·02471	31·5	1·1039	·04292

* See page 467.

TABLE V.*—COEFFICIENTS FOR CORRECTING THE OBSERVED RESISTANCE OF "SILVERTOWN" GUTTA-PERCHA, AT ANY TEMPERATURE, TO 75° FAHR.

Temp. Fahr.	Co-efficient.	Logarithm.	Temp Fahr.	Co-efficient.	Logarithm.	Temp. Fahr.	Co-efficient.	Logarithm.
100	·1494	$\bar{1}$·1744650	77	·8589	$\bar{1}$·9339572	54	4·937	0·6934494
99·5	·1552	·1909757	76·5	·8922	·9504679	53·5	5·128	·7099601
99	·1612	·2074864	76	·9267	·9669786	53	5·327	·7264708
98·5	·1675	·2239971	75·5	·9627	·9834893	52·5	5·533	·7429815
98	·1740	·2405078	75	1·000	0·0000000	52	5·748	·7594922
97·5	·1807	·2570185	74·5	1·039	·0165107	51·5	5·970	·7760029
97	·1877	·2735292	74	1·079	·0330214	51	6·202	·7925136
96·5	·1950	·2900399	73·5	1·121	·0495321	50·5	6·442	·8090243
96	·2026	·3065506	73	1·164	·0660428	50	6·692	·8255350
95·5	·2104	·3230613	72·5	1·209	·0825535	49·5	6·951	·8420457
95	·2186	·3395720	72	1·256	·0990642	49	7·220	·8585564
94·5	·2270	·3560827	71·5	1·305	·1155749	48·5	7·500	·8750671
94	·2358	·3725934	71	1·355	·1320856	48	7·791	·8915778
93·5	·2450	·3891041	70·5	1·408	·1485963	47·5	0·093	·9080885
93	·2545	·4056148	70	1·463	·1651070	47	8·406	·9245992
92·5	·2643	·4221255	69·5	1·519	·1816177	46·5	8·732	·9411099
92	·2746	·4386362	69	1·578	·1981284	46	9·070	·9576206
91·5	·2852	·4551469	68·5	1·639	·2146391	45·5	9·422	·9741313
91	·2962	·4716576	68	1·703	·2311498	45	9·787	·9906420
90·5	·3077	·4881683	67·5	1·769	·2476605	44·5	10·17	1·0071527
90	·3197	·5046790	67	1·837	·2641712	44	10·56	·0236634
89·5	·3320	·5211897	66·5	1·908	·2806819	43·5	10·97	·0401741
89	·3449	·5277004	66	1·982	·2971926	43	11·39	·0566848
88·5	·3583	·5542111	65·5	2·059	·3137033	42·5	11·84	·0731955
88	·3722	·5707218	65	2·139	·3302140	42	12·29	·0897062
87·5	·3866	·5872325	64·5	2·222	·3467247	41·5	12·77	·1062169
87	·4016	·6037432	64	2·308	·3632354	41	13·27	·1227276
86·5	·4171	·6202539	63·5	2·397	·3797461	40·5	13·78	·1392383
86	·4343	·6367646	63	2·490	·3962568	40	14·31	·1557490
85·5	·4501	·6532753	62·5	2·587	·4126675	39·5	14·87	·1722597
85	·4675	·6697860	62	2·687	·4292782	39	15·44	·1887704
84·5	·4856	·6862967	61·5	2·792	·4457889	38·5	16·04	·2052811
84	·5044	·7028074	61	2·889	·4622996	38	16·66	·2217918
83·5	·5240	·7193181	60·5	3·012	·4788103	37·5	17·31	·2383025
83	·5443	·7358288	60	3·128	·4953210	37	17·98	·2548132
82·5	·5654	·7523395	59·5	3·250	·5118317	36·5	18·68	·2713239
82	·5873	·7688502	59	3·376	·5283424	36	19·40	·2878346
81·5	·6100	·7853609	58·5	3·506	·5448531	35·5	20·15	·3043453
81	·6337	·8018716	58	3·642	·5613638	35	20·93	·3208560
80·5	·6582	·8183823	57·5	3·783	·5778745	34·5	21·74	·3373667
80	·6837	·8348930	57	3·930	·5943852	34	22·59	·3538774
79·5	·7102	·8514037	56·5	4·082	·6108959	33·5	23·46	·3703881
79	·7378	·8679144	56	4·240	·6274066	33	24·37	·3868938
78·5	·7663	·8844251	55·5	4·405	·6439173	32·5	25·32	·4034095
78	·7960	·9009358	55	4·575	·6604280	32	26·30	·4199202
77·5	·8296	·9174465	54·5	4·753	·6769387	31·5	27·32	·4364309

* See page 471.

TABLE VI.* — COEFFICIENTS FOR CORRECTING THE OBSERVED RESISTANCE OF "WILLOUGHBY SMITH'S" GUTTA-PERCHA, AT ANY TEMPERATURE, TO 75° FAHR.

Temp.	Co-efficient.	Logarithm.	Temp.	Co-efficient.	Logarithm.	Temp.	Co-efficient.	Logarithm.
100	·1992	$\bar{1}$·2992893	77	·8789	$\bar{1}$·9439395	54	5·083	0·7061201
99·5	·2057	·3132343	76·5	·9077	·9579423	53·5	5·284	·7229628
99	·2125	·3273589	76	·9375	·9719713	53	5·492	·7397305
98·5	·2194	·3412366	75·5	·9682	·9859651	52·5	5·709	·7565600
98	·2266	·3552599	75	1·000	0·0000000	52	5·934	·7733475
97·5	·2340	·3692159	74·5	1·039	·0166155	51·5	6·168	·7901444
97	·2417	·3832767	74	1·080	·0334238	51	6·412	·8069935
96·5	·2497	·3974185	73·5	1·123	·0503798	50·5	6·665	·8238002
96	·2579	·4114513	73	1·167	·0670709	50	6·928	·8406079
95·5	·2667	·4260230	72·5	1·213	·0838608	49·5	7·201	·8573928
95	·2751	·4394906	72	1·261	·1007151	49	7·485	·8741918
94·5	·2841	·4534712	71·5	1·296	·1126050	48·5	7·781	·8910354
94	·2934	·4674601	71	1·363	·1344959	48	8·088	·9078411
93·5	·3030	·4814426	70·5	1·417	·1513699	47·5	8·407	·9246410
93	·3130	·4955443	70	1·473	·1682027	47	8·739	·9414617
92·5	·3232	·5094714	69·5	1·531	·1849752	46·5	9·084	·9582771
92	·3338	·5234863	69	1·591	·2016702	46	9·442	·9750640
91·5	·3448	·5375673	68·5	1·654	·2185355	45·5	9·815	·9918903
91	·3561	·5515720	68	1·719	·2352759	45	10·203	1·0087279
90·5	·3678	·5656117	67·5	1·787	·2521246	44·5	10·606	·0255516
90	·3798	·5795550	67	1·858	·2690457	44	11·024	·0423392
89·5	·3923	·5936183	66·5	1·931	·2857823	43·5	11·460	·0591846
89	·4051	·6075622	66	2·007	·3025474	43	11·911	·0759842
88·5	·4184	·6215917	65·5	2·086	·3193143	42·5	12·382	·0927908
88	·4321	·6355843	65	2·169	·3362596	42	12·870	·1095785
87·5	·4463	·6496269	64·5	2·254	·3529539	41·5	13·378	·1263912
87	·4609	·6636067	64	2·343	·3697723	41	13·906	·1432022
86·5	·4761	·6776982	63·5	2·436	·3866773	40·5	14·455	·1600181
86	·4917	·6917002	63	2·532	·4034637	40	15·025	·1768145
85·5	·5078	·7056927	62·5	2·632	·4202859	39·5	15·618	·1936254
85	·5245	·7197455	62	2·736	·4371161	39	16·235	·2104523
84·5	·5417	·7337588	61·5	2·844	·4539296	38·5	16·876	·2272695
84	·5594	·7477225	61	2·956	·4707044	38	17·542	·2440791
83·5	·5778	·7617775	60·5	3·073	·4875626	37·5	18·235	·2609058
83	·5967	·7757560	60	3·194	·5043349	37	18·954	·2777009
82·5	·6163	·7897922	59·5	3·320	·5211381	36·5	19·702	·2945103
82	·6365	·8037984	59	3·451	·5379450	36	20·480	·3113300
81·5	·6574	·8178297	58·5	3·587	·5547314	35·5	21·288	·3271349
81	·6789	·8318058	58	3·729	·5715924	35	22·128	·3449422
80·5	·7012	·8458419	57·5	3·876	·5883838	34·5	23·002	·3617656
80	·7227	·8589585	57	4·029	·6051973	34	23·910	·3785796
79·5	·7480	·8739016	56·5	4·188	·6220067	33·5	24·853	·3953788
79	·7725	·8878985	56	4·354	·6388884	33	25·834	·4121917
78·5	·7978	·9018940	55·5	4·526	·6557145	32·5	26·854	·4290090
78	·8240	·9159272	55	4·704	·6724673	32	27·913	·4458065
77·5	·8510	·9299296	54·5	4·890	·6893089	31·5	29·014	·4626076

* See page 471.

Table VII*.—Coefficients for Correcting the Observed Resistance of "Hooper's" India-rubber, at any Temperature, to 75° Fahr.

Temp.	Co-efficient.	Logarithm.	Temp.	Co-efficient.	Logarithm.	Temp.	Co-efficient.	Logarithm.
100	·526	$\bar{1}$·7212464	77	·950	$\bar{1}$·9776997	54	1·715	0·2341530
99·5	·533	·7268215	76·5	·963	·9832748	53·5	1·737	·2397281
99	·540	·7323965	76	·975	·9888499	53	1·759	·2453032
98·5	·547	·7379716	75·5	·987	·9944249	52·5	1·782	·2508782
98	·554	·7435467	75	1·000	0·0000000	52	1·805	·2564533
97·5	·561	·7491218	74·5	1·013	·0055751	51·5	1·828	·2620284
97	·568	·7546968	74	1·026	·0111501	51	1·852	·2676035
96·5	·576	·7602719	73·5	1·039	·0167252	50·5	1·876	·2731785
96	·583	·7658470	73	1·053	·0223003	50	1·900	·2787536
95·5	·591	·7714220	72·5	1·066	·0278754	49·5	1·925	·2843287
95	·598	·7769971	72	1·080	·0334504	49	1·949	·2899037
94·5	·606	·7825722	71·5	1·094	·0390255	48·5	1·975	·2954788
94	·614	·7881473	71	1·108	·0446006	48	2·000	·3010539
93·5	·622	·7937223	70·5	1·122	·0501756	47·5	2·026	·3066290
93	·630	·7992974	70	1·137	·0557507	47	2·052	·3122040
92·5	·638	·8048725	69·5	1·152	·0613258	46·5	2·079	·3177791
92	·646	·8104476	69	1·167	·0669009	46	2·106	·3233542
91·5	·655	·8160226	68·5	1·182	·0724759	45·5	2·133	·3289292
91	·663	·8215977	68	1·197	·0780510	45	2·160	·3345043
90·5	·672	·8271728	67·5	1·212	·0836261	44·5	2·188	·3400794
90	·680	·8327478	67	1·228	·0892012	44	2·216	·3456545
89·5	·691	·8383229	66·5	1·244	·0947762	43·5	2·245	·3512295
89	·698	·8438980	66	1·263	·1003513	43	2·274	·3568046
88·5	·708	·8494731	65·5	1·276	·1059264	42·5	2·303	·3623797
88	·716	·8550481	65	1·293	·1115014	42	2·333	·3679548
87·5	·726	·8606232	64·5	1·309	·1170765	41·5	2·363	·3735298
87	·735	·8661983	64	1·326	·1226516	41	2·394	·3791049
86·5	·745	·8717733	63·5	1·343	·1282267	40·5	2·425	·3846800
86	·754	·8773484	63	1·361	·1338017	40	2·456	·3902550
85·5	·764	·8829235	62·5	1·378	·1393768	39·5	2·488	·3958301
85	·774	·8884986	62	1·396	·1449519	39	2·520	·4014052
84·5	·784	·8940736	61·5	1·414	·1505269	38·5	2·553	·4069803
84	·794	·8996487	61	1·433	·1561020	38	3·586	·4125553
83·5	·804	·9052238	60·5	1·451	·1616771	37·5	2·619	·4181304
83	·814	·9107988	60	1·470	·1672522	37	2·653	·4237055
82·5	·825	·9163739	59·5	1·489	·1728272	36·5	2·687	·4292806
82	·836	·9219490	59	1·508	·1784023	36	2·722	·4348556
81·5	·846	·9275241	58·5	1·527	·1839774	35·5	2·757	·4404307
81	·857	·9330991	58	1·547	·1895524	35	2·793	·4460058
80·5	·868	·9386742	57·5	1·567	·1951275	34·5	2·829	·4515808
80	·880	·9442492	57	1·587	·2007026	34	2·865	·4571559
79·5	·891	·9498244	56·5	1·608	·2062777	33·5	2·902	·4627310
79	·902	·9553994	56	1·629	·2118527	33	2·940	·4683060
78·5	·914	·9609745	55·5	1·650	·2174278	32·5	2·978	·4738811
78	·926	·9665496	55	1·671	·2230029	32	3·013	·4794562
77·5	·938	·9721246	54·5	1·693	·2285780	31·5	3·055	·4850312

* See page 471.

TABLE VIII.*—OF THE MULTIPLYING POWER OF SHUNTS EMPLOYED WITH A GALVANOMETER OF 6000 OHMS RESISTANCE.

Resistance of Shunt	Multiplying Power.	Logarithm of Multiplying Power.	Combined Resistance of Galvanometer and Shunt.	Resistance of Shunt.	Multiplying Power.	Logarithm of Multiplying Power.	Combined Resistance of Galvanometer and Shunt.	Resistance of Shunt.	Multiplying Power.	Logarithm of Multiplying Power.	Combined Resistance of Galvanometer and Shunt.
ohms.			ohms.	ohms.			ohms.	ohms.			ohms.
1	6001	3·7782236	1·0	75	81·00	1·9084850	74·1	950	7·316	·8642618	818·3
2	3001	3·4772660	2·0	80	76·00	1·8808136	79·0	1000	7·000	·8450980	857·2
3	2001	3·3012471	3·0	85	71·59	1·8548402	83·8	1100	6·455	·8098626	229·6
4	1501	3·1763807	4·0	90	67·67	1·8303769	88·7	1200	6·000	·7781513	1000·0
5	1201	3·0795430	5·0	95	64·16	1·8072508	93·5	1300	5·615	·7493807	1068·5
6	1001	3·0004341	6·0	100	61·00	1·7853298	98·4	1400	5·286	·7231107	1135·7
7	858·2	3·9335581	7·0	110	55·55	1·7446450	108·0	1500	5·000	·6989700	1200·0
8	751·0	2·8756399	8·0	120	51·00	1·7075702	117·7	1600	4·750	·6766936	1263·2
9	667·7	2·8245619	9·0	130	47·15	1·6735185	127·2	1700	4·529	·6560407	1324·7
10	601·0	2·7788745	10·0	140	43·86	1·6420488	136·8	1800	4·333	·6368188	1384·6
11	546·5	2·7375504	11·0	150	41·00	1·6127839	146·3	1900	4·158	·6188636	1443·1
12	501·0	2·6998377	12·0	160	38·50	1·5854607	155·9	2000	4·000	·6020600	1500·0
13	462·6	2·6651493	13·0	170	36·29	1·5598348	165·3	2200	3·727	·5713943	1609·7
14	429·6	2·6320441	14·0	180	34·33	1·5357118	174·8	2400	3·500	·5440680	1714·3
15	401·0	2·6031444	15·0	190	32·58	1·5129244	184·2	2600	3·308	·5195261	1814·0
16	376·0	2·5751878	16·0	200	31·00	1·4913617	193·6	2800	3·143	·4973306	1909·1
17	354·0	2·5489296	17·0	220	28·27	1·4513719	212·2	3000	3·000	·4771213	2000·0
18	334·4	2·5241753	17·9	240	26·00	1·4149733	230·8	3300	2·818	·4499718	2129·0
19	316·8	2·5007578	18·9	260	24·08	1·3816024	249·2	3600	2·667	·4259742	2250·0
20	301·0	2·4785665	19·9	280	22·43	1·3508099	267·5	4000	2·500	·3979400	2400·0
22	273·7	2·4373224	21·9	300	21·00	1·3222193	285·7	4300	2·395	·3793780	2504·8
24	251·0	2·3996737	23·9	330	19·18	1·2828939	312·8	4600	2·304	·3625579	2603·7
26	231·8	2·3650572	25·9	360	17·67	1·2471628	340·4	5000	2·400	·3424227	2727·3
28	215·3	2·3330239	27·9	400	16·00	1·2041200	375·0	5500	2·081	·3182929	2883·1
30	201·0	2·3031961	29·9	430	14·95	1·1747574	401·2	6000	2·000	·3010300	3000·0
33	182·8	2·2620237	32·8	460	14·00	1·1461280	428·6	6500	1·923	·2840019	3120·0
36	167·7	2·2244554	35·8	500	13·00	1·1139434	461·5	7000	1·857	·2688353	3230·8
40	151·0	2·1789769	39·7	550	11·81	1·0722867	508·0	7500	1·800	·2552725	3333·3
43	140·5	2·1477999	42·7	600	11·00	1·0413927	545·5	8000	1·750	·2430380	3428·6
46	131·4	2·1187276	45·6	650	10·31	1·0131744	582·1	8500	1·706	·2319536	3517·2
50	121·0	2·0827854	49·6	700	9·574	·9809755	617·6	9000	1·667	·2218574	3600·0
55	110·1	2·0378646	54·5	750	9·000	·9542425	666·7	9500	1·632	·2126137	3677·4
60	101·0	2·0043214	59·5	800	8·500	·9294189	705·9	10000	1·600	·2041200	3750·0
65	91·30	1·9699189	64·3	850	8·059	·9062704	744·5	10500	1·571	·1962946	3818·2
70	86·70	1·9380892	69·2	900	7·667	·8846085	782·6	11000	1·545	·1890562	3882·3

* See page 418.

TABLE IX.*—OF THE MULTIPLYING POWER OF SHUNTS EMPLOYED WITH A GALVANOMETER OF 10,000 OHMS RESISTANCE.

Resistance of Shunt.	Multiplying Power.	Logarithm of Multiplying Power.	Combined Resistance of Galvanometer and Shunt.	Resistance of Shunt.	Multiplying Power.	Logarithm of Multiplying Power.	Combined Resistance of Galvanometer and Shunt.	Resistance of Shunt.	Multiplying Power.	Logarithm of Multiplying Power.	Combined Resistance of Galvanometer and Shunt.
ohms.			ohms.	ohms.			ohms.	ohms.			ohms.
1	10001	3·0000434	1·0	75	134·3	2·1281838	74·4	950	11·53	1·0616905	863·6
2	5001	3·6990569	2·0	80	126·0	2·1003705	79·4	1000	11·00	1·0413927	909·0
3	3333	3·5230090	3·0	85	118·6	2·0742570	84·3	1100	10·09	1·0039303	982·1
4	2501	3·3981137	4·0	90	112·1	2·0496487	89·2	1200	9·333	·9700368	1071·5
5	2001	3·3012471	5·0	95	106·3	2·0263827	94·1	1300	8·692	·9391350	1140·4
6	1668	3·2221092	6·0	100	101·0	2·0043214	99·0	1400	8·143	·9107769	1228·1
7	1430	3·1552059	7·0	110	91·91	1·9633585	108·8	1500	7·667	·8846065	1304·4
8	1251	3·0972573	8·0	120	84·33	1·9259993	118·6	1600	7·250	·8603380	1379·3
9	1112	3·0461482	9·0	130	77·92	1·8916660	128·3	1700	6·882	·8377370	1453·0
10	1001	3·0004341	10·0	140	72·43	1·8599100	138·1	1800	6·556	·8166095	1525·4
11	910·1	2·9590848	11·0	150	67·67	1·8303747	147·8	1900	6·263	·7967934	1596·8
12	834·3	2·9213396	12·0	160	63·50	1·8027737	157·5	2000	6·000	·7781512	1666·7
13	770·2	2·8866208	13·0	170	59·82	1·7768721	167·2	2200	5·545	·7439371	1803·3
14	715·3	2·8544796	14·0	180	56·56	1·7524753	176·8	2400	5·167	·7132105	1935·5
15	667·7	2·8245597	15·0	190	53·63	1·7294206	186·5	2600	4·846	·6853972	2063·5
16	626·0	2·7965743	16·0	200	51·00	1·7075702	196·1	2800	4·571	·6600520	2187·5
17	589·2	2·7702888	17·0	220	46·45	1·6670282	215·3	3000	4·333	·6368221	2307·7
18	556·6	2·7455085	18·0	240	42·67	1·6300888	234·4	3300	4·030	·6053377	2481·2
19	527·3	2·7220708	19·0	260	39·46	1·5961741	253·4	3600	3·778	·5772364	2647·1
20	501·0	2·6998377	20·0	280	36·71	1·5648351	272·4	4000	3·500	·5440680	2857·1
22	455·6	2·6585317	22·0	300	34·33	1·5357159	291·3	4300	3·326	·5218675	3007·0
24	417·7	2·6208299	23·9	330	31·30	1·4955864	319·5	4600	3·174	·5015951	3150·7
26	385·6	2·5861544	25·9	360	28·78	1·4590573	347·5	5000	3·000	·4771213	3333·3
28	358·1	2·5540563	27·9	400	26·00	1·4149733	384·6	5500	2·818	·4499690	3548·4
30	334·3	2·5241796	29·9	430	24·27	1·3848158	412·2	6000	2·667	·4259687	3750·0
33	304·0	2·4829169	32·9	460	22·74	1·3567739	439·8	6500	2·538	·4045705	3939·4
36	278·8	2·4452582	35·9	500	21·00	1·3222193	476·2	7000	2·429	·3853509	4117·6
40	251·0	2·3996737	39·8	550	19·18	1·2828898	521·3	7500	2·333	·3679767	4285·7
43	233·6	2·3683950	42·8	600	17·67	1·2471546	556·0	8000	2·250	·3521825	4444·4
46	218·4	2·3392354	45·8	650	16·38	1·2144362	610·3	8500	2·177	·3377528	4594·6
50	201·0	2·3031961	49·8	700	15·29	1·1842858	654·2	9000	2·111	·3245111	4736·8
55	182·8	2·2620194	54·7	750	14·33	1·1563472	697·7	9500	2·053	·3123110	4871·8
60	167·7	2·2244467	59·6	800	13·50	1·1303338	740·7	10000	2·000	·3010300	5000·0
65	154·8	2·1899004	64·6	850	12·77	1·1060108	784·1	10500	1·952	·2905646	5122·0
70	143·9	2·1579315	69·5	900	12·11	1·0831840	825·7	11000	1·909	·2808266	5238·1

* See page 418.

TABLE X*.—FOR THE MANCE, KENNELLY, AND SCHAEFER, FAULT TESTS.

n	$\frac{1}{n-1}$	$\frac{1}{\sqrt{n}-1}$	$\frac{1}{\sqrt[1.3]{n}-1}$	n	$\frac{1}{n-1}$	$\frac{1}{\sqrt{n}-1}$	$\frac{1}{\sqrt[1.3]{n}-1}$	n	$\frac{1}{n-1}$	$\frac{1}{\sqrt{n}-1}$	$\frac{1}{\sqrt[1.3]{n}-1}$	n	$\frac{1}{n-1}$	$\frac{1}{\sqrt{n}-1}$	$\frac{1}{\sqrt[1.3]{n}-1}$	n	$\frac{1}{n-1}$	$\frac{1}{\sqrt{n}-1}$	$\frac{1}{\sqrt[1.3]{n}-1}$
2·00	1·000	2·414	1·420	2·20	·8333	2·069	1·119	2·40	·7143	1·821	1·041	2·60	·6250	1·633	·9212	2·80	·5556	1·485	·8279
2·01	·9901	2·394	1·407	2·21	·8264	2·055	1·190	2·41	·7092	1·810	1·034	2·61	·6211	1·624	·9161	2·81	·5525	1·479	·8238
2·02	·9804	2·374	1·394	2·22	·8197	2·041	1·181	2·42	·7042	1·797	1·027	2·62	·6173	1·617	·9109	2·82	·5495	1·472	·8197
2·03	·9709	2·354	1·381	2·23	·8130	2·207	1·172	2·43	·6993	1·789	1·021	2·63	·6135	1·608	·9059	2·83	·5464	1·466	·8157
2·04	·9615	2·335	1·369	2·24	·8065	2·013	1·163	2·44	·6944	1·779	1·014	2·64	·6098	1·600	·9008	2·84	·5435	1·459	·8117
2·05	·9524	2·316	1·357	2 25	·8000	2·000	1·155	2·45	·6897	1·769	1·008	2·65	·6061	1·502	·8959	2·85	·5405	1·453	·8078
2·06	·9434	2·297	1·346	2·26	·7937	1·987	1·146	2·46	·6849	1·759	1·002	2·66	·6024	1·585	·8909	2·86	·5376	1·447	·8038
2·07	·9346	2·279	1·335	2·27	·7874	1·974	1·138	2·47	·6802	1·749	·9952	2·67	·5988	1·577	·8861	2·87	·5348	1·441	·7999
2·08	·9259	2·261	1·322	2·28	·7813	1·961	1·130	2·48	·6757	1·740	·9891	2·68	·5952	1·570	·8813	2·88	·5319	1·435	·7960
2·09	·9174	2·244	1·310	2·29	·7752	1·948	1·122	2·49	·6711	1·730	·9830	2·69	·5917	1·562	·8766	2·89	·5291	1·429	·7923
2·10	·9091	2·226	1·299	2·30	·7692	1·936	1·114	2·50	·6667	1·721	·9770	2·70	·5882	1·555	·8719	2·90	·5263	1·422	·7885
2·11	·9009	2·210	1·288	2·31	·7634	1·923	1·106	2·51	·6623	1·712	·9712	2·71	·5848	1·548	·8673	2·91	·5236	1·416	·7848
2·12	·8929	2·193	1·278	2·32	·7576	1·912	1·098	2·52	·6579	1·702	·9653	2·72	·5814	1·540	·8627	2·92	·5208	1·411	·7811
2·13	·8850	2·176	1·267	2·33	·7519	1·900	1·091	2·53	·6536	1·693	·9596	2·73	·5780	1·533	·8582	2·93	·5181	1·405	·7775
2·14	·8772	2·160	1·257	2·34	·7463	1·888	1·083	2·54	·6494	1·684	·9539	2·74	·5747	1·526	·8537	2·94	·5155	1·399	·7739
2·15	·8696	2·145	1·247	2·35	·7407	1·876	1·076	2·55	·6452	1·675	·9483	2·75	·5714	1·519	·8493	2·95	·5128	1·394	·7703
2·16	·8621	2·129	1·237	2·36	·7353	1·865	1·069	2·56	·6410	1·667	·9428	2·76	·5682	1·512	·8449	2·96	·5102	1·388	·7667
2·17	·8547	2·114	1·227	2·37	·7299	1·854	1·061	2·57	·6369	1·658	·9373	2·77	·5650	1·505	·8406	2·97	·5076	1·382	·7632
2·18	·8471	2·099	1·218	2·38	·7246	1·843	1·054	2·58	·6329	1·649	·9319	2·78	·5618	1·498	·8363	2·98	·5051	1·377	·7568
2·19	·8403	2·084	1·208	2·39	·7104	1·832	1·047	2·59	·6289	1·641	·9265	2·79	·5587	1·492	·8321	2·99	·5025	1·372	·7563

* See pages 266, 280, and 282.

TABLE XI.—STANDARD WIRE GAUGE.†

No.	Diameters.			No.	Diameters.		
	Mils.*	Differences.	Millimetres.		Mils.*	Differences.	Millimetres.
0,000,000	500		12·70	23	24	4	·610
000,000	464	36	11·78	24	22	2	·559
00,000	432	32	10·97	25	20	2	·508
0,000	400	32	10·16	26	18	2	·457
000	372	28	9·45	27	16·4	1·6	·417
00	348	24	8·84	28	14·8	1·6	·376
0	324	24	8·23	29	13·6	1·2	·345
1	300	24	7·62	30	12·4	1·2	·315
2	276	24	7·01	31	11·6	·8	·295
3	252	24	6·40	32	10·8	·8	·274
4	232	20	5·89	33	10·0	·8	·254
5	212	20	5·38	34	9·2	·8	·234
6	192	20	4·88	35	8·4	·8	·213
7	176	16	4·47	36	7·6	·8	·193
8	160	16	4·06	37	6·8	·8	·173
9	144	16	3·66	38	6·0	·8	·152
10	128	16	3·25	39	5·2	·8	·132
11	116	12	2·95	40	4·8	·4	·122
12	104	12	2·64	41	4·4	·4	·112
13	92	12	2·34	42	4·0	·4	·102
14	80	12	2·03	43	3·6	·4	·0914
15	72	8	1·83	44	3·2	·4	·0813
16	64	8	1·63	45	2·8	·4	·0711
17	56	8	1·42	46	2·4	·4	·0610
18	48	8	1·22	47	2·0	·4	·0508
19	40	8	1·016	48	1·6	·4	·0406
20	36	4	·914	49	1·2	·4	·0305
21	32	4	·813	50	1·0	·2	·0254
22	28	4	·711				

* Mil = $\frac{1}{1000}$th of an inch.
† This gauge is the only legal standard wire gauge for the United Kingdom.

TABLE XII.—ELECTRICAL AND MECHANICAL DATA OF RECENT SUBMARINE CABLES.

Cable.	Date laid.	Length.	Res. per N.M. of Conductor at 75° F.	Specific Conductivity of Conductor.	Res. per N.M. of Dielectric at 75° F.	Inductive Capacity per N.M. at Nom. Temp.	Weights per N.M. (dry) in air. Copper.	Gutta-percha.	Jute.	Iron.	Hemp.	Outer covering.	Total (including Brass Tape).	Type.
	A.D.	N.M.	ohms.	per cent.	megs.	mfds.	lbs.	lbs.	tons.	tons.	tons.	tons.	tons.	
Aden—Bombay, No. 3.. ..	April, 1891	1850	8·004	99·2	734	·392	400	300	1·2	14·2	..	..	15·8	Shore end, No. 1.
									·6	10·2	..	1·7	12·9	" " 2.
									·2	5·7	..	·9	7·1	Heavy { inter-
									·2	3·5	..	·5	4·5	Light } mediate
									·1	1·1	..	·4	1·9	Main, No. 1.
									·1	·9	..	·4	1·7	" " 2.
Rio Janeiro—Bahia	August, 1891	760	9·339	98·2	633	·364	130	130	·5	17·4	..	1·8	19·7	Shore end, No. 1.
									·5	10·9	..	1·1	13·0	" " 3.
									·1	2·4	..	·3	3·0	Main, No. 1.
									·1	·9	..	·3	1·4	" " 2.
Bahia—Pernambuco	August, 1891	428·12	9·33	98·3	672	·363			Weights as above.					
Zanzibar—Seychelles	Nov. 1893 ..	1120	9·133	100·4	740	·352	130	130	·8	12·3	..	1·2	14·5	Shore end.
									·3	5·7	..	·9	7·1	Heavy { inter-
									·1	2·9	..	·4	3·6	Light } mediate
									·1	·7	..	·4	1·3	Main.
Seychelles—Mauritius	Nov. 1893 ..	1065	9·106	100·7	776	·353			Weights as above.					
Singapore—Labuan	April, 1894	733·12	9·181	99·9	709	·350	130	130	·6	10·0	..	1·1	11·9	Shore end, No. 1.
									·3	5·6	..	·8	6·8	" " 2.
									·1	2·4	..	·4	3·1	Intermediate.
									·1	·7	..	·4	1·3	Main cable.
Labuan—Hong Kong	May, 1894 ..	1238·39	9·206	99·7	562	·345			Weights as above.					
Anglo-American—Atlantic	July, 1894 ..	1847·54	1·834	100·0	636	·420	650	400			..		14·8	Shore end, No. 1.
											..		7·1	" " 2.
											..		4·5	Intermediate.
											..		1·8	Main.
Bermudas—Turks Islands ..	Jan. 1898 ..	824·67	9·179	99·9	913	·319	130	130	·9	12·2	..	1·4	14·7	Shore end, No. 1.
									·3	5·6	..	·8	6·8	" " 2.
									·1	3·0	..	·6	3·9	Intermediate.
									·1	1·0	..	·3	1·5	Main.
Turks Islands—Jamaica ..	Jan. 1898 ..	452·88	9·184	99·9	919	·316			Weights as above.					

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